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Generalized rigid-body dynamics system simulation and the application to the behavior of suspended agricultural tractors

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GENERALIZED RIGID-BODY DYNAMICS SYSTEM SIMULATION AND THE APPLICATION TO THE BEHAVIOR OF SUSPENDED AGRICULTURAL TRACTORS

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Generalized rigid-body dynamics system simulation

and the application to

the behavior of suspended agricultural tractors

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GENERAL INTRODUCTION

The trend in the design of agricultural equipment toward increased functional efficiency, reliability, productivity, and operating speed was firmly established in the past two decades. The productivity of the equipment however was limited by the operator's ability to tolerate the dynamic conditions imposed upon him by the machine. It has become essential that the manufacturer place more emphasis on the operator's health, safety and environment during the development of the equipment. Progress has been made in recent years in protecting the operator during an overturn of a vehicle and in reducing the noise exposure levels in the operator's cab. Vehicle ride comfort has been studied as a means for increasing the vehicle's operating speed over rough terrains and for reducing operator health hazards.

Two decades ago, farming operations were based on speeds in the range 3-6 kilometers per hour (km/h) while current farming operations are based on speeds in the range 8-11 km/h (Buckingham, 1981a,b). In the next decade, Simpson (1979) has projected that farming operations could possibly be based on speeds in the range 13-16 km/h. This trend toward increased operating speeds has led to the development of "high speed farming systems". The advantages of increased speeds which were listed by Wilkins and Coleman (1971) and Reece (1971) are: (1) lower capital investment, (2) simplified transport, (3) more compact machinery units, and (4) simplified machinery servicing.

Medical research has shown that driving agricultural tractors contributes to high incidence of spinal and stomach disorders of operators. An important contributory factor is the excessive levels of low frequency vibrations of the tractor conveyed by the seat to the operator. Stayner et al. (1975) and Graef (1979) reported vibration levels received by tractor operators for different farming operations. These studies showed that the vibration exposure levels of the operator were increased with higher tractor operating speeds when driving over rough terrains.

During the past two decades, research quantified human tolerance to whole-body vibration. Numerous guidelines and comfort criteria were proposed. The vibration exposure guidelines were derived exclusively by subjective response to simulated vibration in the laboratory. This process involved exposing a groups of human subjects to vibration levels at different frequencies, objectively measuring the vibration, and drawing comparisons between the vibration measurements and the subjective response to bodily discomfort.

The International Standards Organization (ISO) drafted Standard 2631-"Guide for the Evaluation of Human Exposure to Whole-Body Vibration" (Matthews, 1973, and Stikeleather, 1976). This guideline recommends frequency exposure boundaries for reduced comfort, fatigue-decreased proficiency, and exposure limits from human health and safety aspects. The guideline standardizes the analyzing and reporting of the exposure data. The guideline reflects the increased sensitivity of the human body to vertical vibration in the range 4-8 Hertz (Hz), and lateral and longitudinal vibrations below 2 Hz.

The Society of Automotive Engineers (SAE) drafted Recommended

Practice J1013-"Measurement of Whole-Body Vibration of the Seated Operator of Agricultural Equipment". The recommendation encouraged reporting of the data in accordance with the ISO 2631 standard which specifies the instrumentation, the analytical procedure for weighting the vibration level at different frequencies, and the description of the site and operating conditions (Stikeleather, 1976).

Stikeleather (1976) discussed two additional guidelines which were concerned with whole-body vibration. The first draft standard ISO 5007-"Agricultural Wheeled Tractors, Operator Seat Measurement of Transmitted Vibration and Seat Dimensions" established a means for evaluating the performance of tractor seats including transmitted vibration, operator weight range, and seat dimensions and lateral stability. The second draft ISO 5008-"Agricultural Tractors and Field Machinery Measurement of Whole-Body Vibration of the Operator" established a procedure for measuring whole-body vibration which was essentially equivalent to SAE J1013.

With current tractor operating speeds in the range 8-11 km/h, the ISO 2631 and SAE J1013 guidelines can be met by the manufacturer with low natural frequency seat suspension systems. A good seat suspension can attenuate 50 percent of the vertical terrain-induced vehicle vibration to the operator (Matthews, 1980). A seat suspension is effective because it allows relative motion between the operator and the controls.

But as the tractor operating speeds are increased to the range 13-16 km/h, the manufacturer will have difficulty meeting the guidelines with a seat suspension system. This difficulty will arise because the seat suspension system must allow more relative motion between the operator and the controls. This excessive relative motion will transmit vibration to the operator through his arms and legs, and will impair his ability to control the tractor.

Therefore, alternative types of suspension systems need to be considered for attenuating the terrain-induced vibrations. Two viable alternatives to the seat suspension are the cab suspension, and the chassis or whole vehicle suspension. Both suspension types offer advantages and disadvantages as a vibration isolation system, and each type has its corresponding performance characteristics.

For this study a chassis suspension system was considered for improving the tractor ride comfort and attenuating the terrain-induced vibration. Analytical system modelling techniques were used to predict and gain insights into the dynamic behavior of the suspended tractor (Mitschke, 1962; Sisson and Wiley, 1975; Majcher et al., 1976). Also, the effect in the suspension design parameter changes will be evaluated more easily for determining the relative performance characteristics of the suspension and for making the suspension design more nearly optimal.

OUTLINE OF DISSERTATION FORMAT AND OBJECTIVES

This dissertation consists of three sections. Each section was written as a separate paper. The candidate (Paul W. Claar II) conducted the research, and authored the papers under the supervision of his major professor (Dr. Wesley F. Buchele) and the editing assistance of Drs. Stephen J. Marley and Pradip N. Sheth. Section I (Off-Road Vehicle Ride: Review of Concepts and Design Evaluation with Computer Simulation) was formally presented as SAE paper number 801023 in September at the 1980 International Off-Highway Meeting of the Society of Automotive Engineers in Milwaukee, Wisconsin. Section II (A Matrix-Computer Procedure for Vibration Analysis of Articulated Mechanical Systems) will be presented at the 1984 Design Engineering Division Meeting of the American Society of Mechanical Engineers. Section III (Agricultural Tractor Chassis Suspension System for Improved Ride Comfort) was formally presented as refereed paper SAE paper number 801020 in September at the 1980 Off-Highway Meeting of the Society of Automotive Engineers in Milwaukee, Wisconsin; as ASAE paper number 80-1565 in December at the 1980 Winter Meeting of the American Society of Agricultural Engineers in Chicago, Illinois; and as a paper for the Seventh International Conference (1981) of the International Society for Terrain-Vehicle Systems in Calgary, Alberta, Canada.

Section I reports on the development of suspension systems for improving the operator ride comfort on American and European off-road vehicles. Analytical techniques are described for predicting the dynamic response of the off-road vehicle and the human ride response criteria. The approach includes the selection of a terrain input for exciting the vehicle. An example of an agricultural tractor and plow system is presented for illustrating the analytical simulation techniques.

Section II reports on the development of analytical techniques for computing the transfer function - frequency domain response and the time domain response of constrained mechanical systems. Analytical methods are developed for formulating the linearized dynamic matrices for the vibrational behavior of a mechanical system, and for determining the force and base motion transfer functions in the frequency domain. These analytical techniques have been incorporated into the Integrated Mechanisms Program (IMP) which simulates the dynamic behavior of generalized rigid body, multiloop, multidegree of freedom, planar and spatial generalized mechanisms with associated constraint paths and loops. With the numerical results from IMP, the frequency response and the mode shape(s) of the system were computed. With the Fast Fourier Transform (FFT), the arbitrary force and/or base motion histories were combined with the transfer functions to simulate the frequency and time domain responses of the system. These linearized dynamic frequency and time domain techniques are applicable to the vibrational analysis of articulated machinery and vehicle suspension systems. Specific applications to planar linkages and an agricultural wheel tractor are illustrated.

Section III reports on an exploratory concept for a chassis suspension system for improving the operator ride comfort of an agricultural wheel tractor. The criteria and the concepts that have been incorporated into the design of a hybrid leading and trailing arm suspension system are discussed. The chassis suspension system and its parameters are evaluated by simulating nine different tractor models, nine different tractor-plow models, and eighteen different tractor-farm trailer models which have been derived from the various combinations of the suspension configurations and operator cab locations. The generalized mechanical system simulation program IMP is utilized for predicting the dynamic linear transfer function behavior of each model. With frequency domain analysis techniques and the FFT algorithm, the dynamic vehicle response to the ISO 5007 smooth track excitation is computed. Finally, ISO 2631 and SAE J1013 guidelines are used to compute comparative ride numbers, and it is shown that, in theory, a suspended chassis would provide improved ride comfort. Additional design factors, which must be considered prior to hardware development, are then summarized for further consideration.

The study addressed five major objectives:

1. Investigate the dynamic response of an agricultural wheel tractor with a chassis suspension system, with or without an attached implement.
2. Formulate analytical models of the tractor configurations with three types of chassis suspension systems and three operator cab positions, with or without an attached implement.
3. Establish performance trends and design parameters for each

of the three suspension systems for isolating the tractor chassis and the operator from the terrain-induced vibration.

4. Use a general purpose mechanical systems simulation program to predict the dynamic response of the tractor and tractor-implement systems.
5. Formulate a frequency domain modeling technique and incorporate it into the computer simulation program.

SECTION 1: OFF-ROAD VEHICLE RIDE: REVIEW OF THE CONCEPTS AND DESIGN EVALUATION WITH COMPUTER SIMULATION

INTRODUCTION

The off-road vehicle - e.g. agricultural wheel tractor or rubber-tired earthmoving scraper, is designed for providing a predetermined drawbar pull and tractive effort. With the increased speed, size, and power output of these vehicles, the operator has been subjected to increasing levels of shock and vibration from the terrain and the attached implements. To reduce operator fatigue, ergonomic considerations have become an important design criteria for vehicle development. While these improvements - e.g. operator platform and controls, and seat suspensions, help in operator convenience and comfort, further development is needed to extend the productivity of the man-machine combination. The primary challenge in this development is to achieve the conflicting design goals of reducing transmitted vibration to the operator while keeping the relative motion between the operator and the control devices to an acceptable minimum. To design an off-road vehicle along basic automotive design theory becomes difficult because the vehicle must provide the base support for the implement in order to regulate the implement depth and level during its operation and to control the implement position during the transport mode.

The first objective of this section is to discuss the design parameters and research activities for improving the dynamic behavior and ride comfort for increased vehicle productivity by American and European off-road equipment manufacturers, and university and government research institutes during the last 25 years. The second objective is to review available dynamic modeling and computer simulation techniques for evaluating the effect of design changes on off-road vehicle ride characteristics, in particular for an agricultural wheel tractor.

DESIGN CONSIDERATIONS FOR OFF-ROAD VEHICLE RIDE COMFORT

To increase the travel speed and to reduce the vibration of an off-road vehicle, the dynamic behavior of the vehicle must be modified for improving ride comfort. One of the limiting factors to increased operating speeds has been the operator's tolerance to a harsh ride. The vehicle ride quality depends upon the terrain surface roughness, the tractor-implement interactions, the travel speed, and the vehicle geometry and suspension characteristics. The most objectionable levels of acceleration, as experienced by the operator, are in the frequency range 2-10 Hz and are caused by the terrain and the implement loads (Mitschke, 1972; and Soehne, 1965). The vibration frequency should be in the range 1-2 Hz for minimizing resonance and discomfort to the human body. The following design factors, as outlined by Radforth (1978), must be considered for good vehicle ride characteristics:

Cab and seat location

The operator cab and seat should be configured at a point on the vehicle where the vertical, longitudinal, and lateral vibrations are minimized. The vehicle configuration should take into account the dynamic interaction between the tractor and the implement.

Large tires

The largest diameter tires should be used for the best ride comfort and obstacle negotiation because they do not follow the rough terrain exactly but filter the terrain profile. The larger wheel has more time to raise the center of the axle by an amount equal to the bump's height over which it is traveling, which, in turn, lowers the upward acceleration force on the chassis.

Arm suspension

A sprung rigid axle leading or trailing arm suspension should be used for reducing the unsprung weight and suspension power consumption and for transmitting the motion from the wheels to the chassis through the suspension elements. The wheel can swing upward and rearward as it engages the bump, thereby allowing more time for it to travel to its new elevation and reducing the upward wheel velocity. This motion decreases the force transmitted to the chassis which reduces the chassis acceleration rate.

Since the rough terrain acts as a form of motion resistance to the vehicle, power is consumed in the suspension system and in the small deformations of the terrain surface. This type of motion resistance is dependent upon the unsprung mass, the ability of the suspension system to absorb the vertical wheel travel while minimizing the force transmitted to the chassis, and the vehicle travel speed.

Springs and dampers

The springs should have low stiffness rates to allow for a high spring preload to support the vehicle weight at the midposition of wheel travel when the vehicle is in the static equilibrium position. With the springs extending and compressing, the motion of the wheels can be controlled so that the spring forces will not be transmitted to the chassis while traveling over the rough terrain. As a starting point, the spring rate should be approximately 1/8 times the tire stiffness rate (Cole, 1972; Mola, 1974).

Preloaded adjustable and nonlinear springs should be used for operating the vehicle with and without implement loads over rough terrains to maintain good ride comfort.....

Shock absorbers should be used to viscously damp the low frequency vehicle oscillations. As a starting point, the damping to critical damping ratio should be approximately 1/4 (Cole, 1972; Mola, 1974).

Wheel travel

The most critical criterion for the suspension system is to allow sufficient vertical wheel motion for negotiating the terrain roughness. The vertical wheel motion should be two to three times the static deflection of the tire for the initial suspension design. When a wheel contacts a bump and the suspension does not allow sufficient vertical motion, the wheel will continue its upward motion, taking the chassis with it at the same high velocity, causing large accelerations on the operator.

Unsprung mass

The "unsprung mass" segment of the vehicle should be minimized while the sprung to unsprung mass ratio should be maximized for achieving a smooth ride and higher travel speeds. When a vehicle passes over a bump, the unsprung mass is rapidly accelerated and must be decelerated by the suspension system and the inertia of the chassis. Minimization of the unsprung mass segment of an off-road vehicle is a very difficult design problem.

Low pressure tires with soft spring rates act as springs between the masses of the wheels and axles and the ground for reducing the dynamic forces on the axles. This is an effective means for isolating the vehicle chassis from the rough terrain surface and achieving a high sprung to unsprung mass ratio. A main suspension is necessary for damping the large amplitude, low frequency oscillations that arise with the low pressure tires and large chassis inertia.

REVIEW OF MEANS TO IMPROVE RIDE COMFORT

To improve the ride comfort and increase the travel speed and productivity of the vehicle, several alternative suspension concepts have been implemented and evaluated for improving ride comfort: (1) seat suspension; (2) vehicles with operator cab suspension systems; (3) vehicles with front axle suspension systems; (4) vehicles with front and rear axle suspension systems.

Seat suspension

Tractor seat suspensions have been used to help the operator counteract the low frequency vibrations caused by the vehicle traveling over an undulating terrain. During the last 25 years, a variety of commercial and experimental seat suspensions have been developed for improved ride characteristics. Haack (1952) determined the natural frequencies of 14 commercial seat suspensions. Dupuis (1960) extended this work to include the effects of varying operator weights. From these studies, they concluded that the most important seat suspension parameters were: (1) the natural frequency - less than 1.5 Hz; (2) the velocity damping - 0.5 times the critical damping; (3) vertical seat movement - total of 150 millimeters (mm); (4) the linear mode of vertical movement; (5) the inclusion of horizontal resilience; and (6) sufficient seat room and position adjustment (vertical and longitudinal) for the different ranges of operator weights. Matthews (1964a) has compiled an extensive summary of the early tractor research that has been done in the United States and Germany.

Stikeleather and Suggs (1970) developed an active seat suspension for isolating the operator from low frequency bounce vibrations. The design objective for this system was to maintain an instantaneous dynamic equilibrium for the operator. Basically, the system comprised a vibration-sensing transducer, an electronic circuit for modifying and amplifying the transducer output, an electrohydraulic servovalve, a hydraulic position actuator, a hydraulic power source, and a feedback transducer. The system was constructed to evaluate its performance in the laboratory. The investigation indicated that the suspension isolated the operator from the input frequencies in the range 1.5-8 Hz and provided approximately 65 to 75 percent acceleration attenuation. The researchers have suggested that this active suspension system should have use for isolating the operator's platform.

Young and Suggs (1973) further developed the active seat suspension for isolating the low frequency roll and pitch vibrations. The suspension incorporated a rotary actuator for measuring the rotational displacements. Laboratory studies indicated that the suspension provided approximately 70 to 80 percent vibration displacement isolation in the range 1.0-2.0 Hz.

Zack (1971) described the development of the semi-active seat suspension for isolating the operator. The suspension used a knee-hinge linkage for allowing the operator to move a total of 175 mm in the vertical direction. The suspension was capable of maintaining a constant natural frequency of 0.9 Hz with varying operator weights and providing adjustable damping for improved ride comfort.

Graef (1974, 1979) has investigated tractor seat suspensions as a means for improving ride comfort. Initially, a set of mathematical models was developed for simulating various combinations of passive elements - masses and linear springs and dampers with kinematic couplings. The optimum suspension configuration from the study served as the basis for constructing a laboratory model of the seat suspension. In the laboratory, four suspension configurations were investigated: (1) seat with constant spring and damping rates, (2 and 3) seats with variable spring and constant damping rates, and (4) seat with variable spring and damping rates. The investigation has shown the last configuration provided the best vibration isolation for varying operator weights.

The Institut National de Recherche et de Secuite (INRS) (1974) has developed the OP-7000 oil-pneumatic semi-active seat suspension. The suspension has improved the ride comfort in the frequency range 2.5-3.5 Hz. Thompson (1977) has described the development of a semi-active seat suspension for the John Deere agricultural tractors.

<u>Vehicles with cab suspensions</u>

The National Institute of Agricultural Engineering (NIAE) has developed a suspended operator's cab for an agricultural tractor. Initially, computer simulation techniques were used for formulating a model of the linear suspension and studying its dynamic behavior. From this study, an experimental suspension was constructed and was mounted on a tractor. The suspension was designed for providing isolation in the bounce, pitch, longitudinal, roll and lateral modes. The suspension was evaluated in the laboratory and the field, and on the test track. Tractor operators have been satisfied with the suspension, but they preferred that isolation from the roll and lateral modes not be provided (Hilton and Moran, 1976).

In the Netherlands, a passive cab suspension system comprised of springs and dampers has been developed for a tractor ('t Hart, 1977). The suspension provided vibration isolation in five of the six degrees of freedom - bounce, pitch, longitudinal, lateral, and roll. The tractor with the suspended cab was operated under actual field conditions for evaluating the ride comfort. The tractor operators indicated an improvement in ride comfort thereby allowing increased operating speeds and productivity. The operators have reported adverse effects from the large relative motion between the cab and the chassis. The research has indicated that the soft bounce, and roll and lateral modes were most responsible for the improved ride comfort.

Graef (1976) has investigated a cab suspension for improving ride comfort. Computer simulation techniques were used for optimizing the spring and damping rates for the cab suspension. Based on the computer simulation study, a suspension was constructed for providing vibration isolation in the coupled bounce and pitch modes. The laboratory tests showed that the cab suspension was effective in reducing the vibration levels on the operator.

Suggs and Huang (1969) investigated the feasibility of a cab suspension for isolating the operator and his control devices from the terrain-induced vibrations. A 3/8 scale model was constructed for evaluating and optimizing the suspension. The suspension consisted of two multiple leaf laminated torsion-bar springs which were mounted in needle bearings and enclosed in tubes. The spring assemblies were mounted on the rear side of the cab, rather than underneath, for providing the cab support and saving needed deflection space. A pneumatic shock absorber damped the cab resonance. The suspension parameters were: (1) 2.25 Hz natural frequency and (2) 0.3 damping to critical damping ratio. The cab suspension provided significant vibration isolation at the higher frequencies.

Roley (1975) developed a set of passive, semi-active, and active cab suspension models for comparing their relative performance for typical tractor cab vibration inputs. A four degree-of-freedom (DOF) - bounce, pitch, longitudinal, and lateral - cab model with interchangeable passive, semi-active, and active elements was developed. The models were excited by real time vibration accelerations that were measured on the ISO 5007 smooth track. The active suspension system with a 0.25 Hz natural frequency provided attentuation of over 90 percent for all vibration modes. The semi-active suspension provided 25 percent improvement when compared to the passive suspension with the same natural frequencies.

Owen et al. (1973a,b) developed a passive suspension system for damping the shock and vibration which is inherently transmitted by the chassis to the cab on a conventional draft-type agricultural vehicle. In this system, the operator's platform was suspended on a set of hydraulic shock absorbers and torsion bar springs for isolating the vibration transmitted to the platform. The suspension prevented tilting and provided stability for the platform. This suspension allowed the platform and the operator to move in unison which allowed the operator to have a sense of security and complete control while operating the tractor with varying implement loads over undulating terrain.

Vehicles with front axle suspension systems

During the 1950s and early 1960s, agricultural tractors with a flexible front axle suspension unit for improving ride comfort were quite common on the European continent. One example of this tractor configuration was the IBH Hanomag tractor with a leaf spring suspension on the front axle.

Haack (1952) developed a mathematical model of a tractor with a sprung front axle for analyzing the bounce and pitch modes of vibration. From this theoretical study, some important ride comfort criteria have been established. First, the natural frequencies of the coupled system (bounce and pitch modes) should be far enough apart to insure that the front axle will not produce additional pitching as a result of resonance. Second, the front axle suspension should not give rise to "negative spring coupling"; that is, the moment of the front axle springing is smaller than that of the rear axle, but rather to "positive spring coupling"; that is, the moment of the front axle springing is greater than that of the rear axle. Third, the total spring deflection must be 2.25 times the static deflection of the front tire. Otherwise, the terrain-induced vibration and the dynamic work loads will cause displacement amplification of the sprung mass. Finally, if the tractor experiences pitching from "negative spring coupling", it can be reduced by increasing the air pressure in the front tires or decreasing the air pressure in the rear tires. If the suspension is tuned for a tractor-implement system, the suspension will give rise to "negative spring coupling" for the tractor alone.

Matthews (1967) has investigated the front axle suspension as a means for improving ride comfort. A mathematical model of a tractor with a front axle suspension system was developed for analyzing the bounce and pitch modes of vibration and was excited by an idealized undulating surface. The following conclusions were drawn: First, the bounce mode of vibration at the seat point was not significantly affected by the front axle suspension. This conclusion is likely to apply for all conventional tractor designs, however a small effect may be noticed where the seat is located more than 300 mm ahead of the rear axle centerline. Second, the pitch vibration, and hence fore-to-aft motion, was reduced with the front axle suspension. The suspension must permit 75-100 mm static spring deflection for this vibration reduction to be attained.

Patil et al. (1977) determined the optimum suspension parameters and the effectiveness of front axle and seat suspension systems together and a seat suspension system alone for minimizing the bounce and pitch modes of vibration in the frequency range 0.5-11 Hz. This study used computer simulation techniques to model the tractor, the front axle suspension, and the operator as a lumped mass system interconnected by linear springs and dampers. The models were subjected to a sinusoidal vibration at the tire contact points. The study concluded that the front axle and seat suspensions together were only marginally better than the seat suspension alone. From an economic standpoint, it was recommended that the tractor should only have a seat suspension.

Daimler-Benz AG (1978a) currently markets the MB-TRAC 1300 agricultural tractor which has a front axle suspension system. The suspension is comprised of mechanical coil springs and hydraulic shock absorbers. The suspension should provide improved ride comfort for the following reasons. First, the operator cab is located at the midpont of the tractor wheelbase. Second, the static deflection of the springs is approximately 150-200 mm.

Schulze (1961) investigated a rubber spring front axle suspension system. The suspension provided the following advantages: (1) good behavior of the characteristic spring curve, (2) good damping efficiency, and (3) rapid elimination of hard shocks. Its disadvantages were: (1) small spring displacement, (2) steep spring characteristic curve, (3) limited adjustment to varying loads with spring coupling, (4) reduction of spring displacement by static spring compression under varying loads, (5) varying natural frequency, (6) internal heating due to continuous loading, (7) greater spring displacement requiring large dimensions, (8) hardening of the spring deflection curve under dynamic loading, (9) material resistant to aging and creep. The research did conclude that the front axle suspension did improve the tractor ride comfort.

Winters (1950) has been granted a patent for a front spring suspension system for an agricultural tractor. The suspension system consisted of a supporting member attached to the steering post and a carrier member attached to the chassis. A coil spring and hydraulic shock absorber were attached between the wheels and the carrier member for attenuating the terrain-induced vibration.

The CATERPILLAR "Cushion Hitch" has been developed for reducing the bounce and pitch vibration levels on the operator of a tractor-scraper unit (Carter et al., 1968). The suspension used a parallelogram linkage to connect the tractor to the scraper and to allow vertical movement between the units. The movement is controlled and damped by a hydraulic cylinder which is attached between the bottom tractor pivot and the top scraper pivot. When a bump is encountered by the wheels, the cylinder cushions the impact by pushing oil out of the cylinder which, in turn, pushes a piston upward in an oil over nitrogen accumulator. The accumulator compresses the nitrogen gas until enough force is built up to push the piston downward against the oil to reverse the action. The suspension used this process of oil flow and nitrogen gas compression for dissipating the rapid succession of impact loads. The suspension parameters were optimized by the use of analytical computer simulation techniques. From laboratory simulation tests, the suspension was successful in improving the ride comfort.

The TEREX TS24-B tractor-scraper unit used a leading arm front axle suspension system for improving ride comfort and attenuating the terrain-induced vibration (Mason, 1976; IBH-TEREX, 1977; Cadou and Bower, 1978). The suspension is comprised of two leading arms rigidly attached to the front differential and axle assembly and pin-connected to the rear of the tractor frame. Two single-acting ride hydraulic cylinders are connected to the front axle and the tractor frame for cushioning the movement of the front axle assembly as it oscillates about a transverse horizontal axis. When the wheels encounter a bump, the ride cylinders are forced upward which, in turn, forces oil out of the cylinders. The oil flows to a nitrogen over oil accumulator which, in turn, compresses the nitrogen gas which acts as the "cushion" or spring in the suspension system. The compression of the gas continues until sufficient pressure is developed to push the oil back out into the ride cylinders. This process of oil flow and gas compression absorbed the terrain shocks and improved the ride comfort. Laboratory and field tests have shown that the suspension is effective for improving the operator ride comfort.

Hawk (1979) has been granted a patent for a similar type of leading arm front axle suspension system on a tractor-scraper unit for improving operator ride comfort.

The Clark MICHIGAN 110-15 tractor-scraper unit used the Hydra-Ride suspension system for isolating the drive axle from the tractor chassis and improving the operator ride comfort, the productivity and reliabity of the unit, and the structural integrity (Clark Equipment, 1977; Swanson, 1967). The suspension is comprised of the front drive assembly being attached to a twin-beam structure which, in turn, is attached by two pins to the rear of the tractor frame. A stabilizer bar is connected between the tractor chassis and the axle for eliminating any lateral movement, and rubber bounce stops limit the vertical movement between the axle and the frame. Two hydraulic ride cylinders and a nitrogen over oil accumulator are used for attenuating the terrain-induced vibrations.

The DYNAFLOAT elastomeric suspension with geometric spring rates has become an alternative to the nitrogen over oil accumulator and the passive mechanical spring and hydraulic shock absorber suspensions (McClelland, 1970). The suspension is comprised of a series of rubber bonded to metal pads that are stacked inside a columnar suspension tube. The geometric increase in stiffness of the suspension allows the ride comfort to not become excessively soft at maximum vehicle loads nor excessively harsh at minimum vehicle loads. The rubber pads dissipate approximately 50 percent of the impact energy for damping the vehicle oscillations in four to eight cycles without requiring additional shock absorbers, thus increasing the vehicle's stability and control.

<u>Vehicles with front and rear axle suspension systems</u>

The English firm, Taylor Engineering Ltd., has developed the TRANTOR vehicle for agricultural field and transport work (Lucas, 1978). This vehicle has a front and rear axle suspension system which is comprised of independently suspended front wheels and leaf springs and telescopic hydraulic shock absorbers for the rear axle.

The Daimler-Benz UNIMOG vehicle has been developed for agricultural field and transport work (Daimler-Benz AG, 1978b). The passive front and rear axle suspension system is comprised of coil springs and hydraulic shock absorbers for attenuating the terrain-induced vibrations. The vehicle was not developed for high-speed field work.

Zezula (1965) has described the development of the Zetor tractor with a passive chassis suspension system for improving the operator ride comfort. The unsprung segment of the tractor was comprised of the front and rear axles and wheels, the 3-point implement hitch, and the tubular frame connecting the front and rear axles. The sprung segment of the tractor was comprised of the engine, transmission, and the operator platform and controls. The suspension comprised four coil springs and four hydraulic shock absorbers. The most important design feature was that the suspension did not change the position of the implement in relation to the ground. In this manner, the suspension could be tuned for maximum ride comfort because the implement forces could be transmitted to the unsprung segment of the tractor.

Laboratory and field measurements indicated that the tractor with the experimental chassis suspension provided better ride comfort and showed greater stability on slopes and when turning than a standard tractor. The measurements indicated that the suspension reduced the acceleration levels by 2.5 times which had 2.1 times less duration, and the vibration frequencies were reduced by 50 percent.

The German manufacturing firm, Kloekner-Humboldt-Deutz AG, has developed an agricultural tractor for field and transport work. The tractor was equipped with a front and rear axle suspension system which was comprised of solid rubber, hyperbolic cross-section springs and hydraulic shock absorbers for attenuating the terrain-induced vehicle vibration (Gego and May, 1974 and 1975; Breuer, 1975; Kloekner-Humboldt-

Deutz AG, 1974).

Several statements should be made about a chassis suspension system. First, the front axle suspension systems on German agricultural tractors were discontinued with the introduction of the load and draft control for the 3-point implement hitch system. Second, the Daimler-Benz MB-TRAC tractors improve the ride comfort but do not have load and draft control for the 3-point implement hitch. Third, the load and draft sensing 3-point implement hitch for the Deutz INTRAC 2006 is attached to the suspended chassis frame, so there must be minimal relative displacement of the suspension. The design of the chassis suspension for improved ride comfort for high-speed farming operations becomes an extremely complex problem when the control-sensing implement hitch is considered. Fourth, the federal German law requires a suspension system on any vehicle that is operated in excess of 25 km/h on the highway.

Huang et al. (1964) have developed an elastic wheel suspension system for reducing the terrain-induced vibration and improving the operator ride comfort. The suspension was comprised of a hub center containing six torsion springs and 24 links connecting the hub to the rim. The springs were supported by needle bearings while the linkage revolute joints contained ball bearings. The linkage was designed so that the loads transmitted by the applied wheel loads would cancel each other. Analytical and experimental studies were conducted for evaluating the suspension. The suspension was effective for improving the ride comfort and the vehicle's dynamic behavior.

DESIGN EVALUATION BY COMPUTER SIMULATION

At the design concept stage, analytical computer simulation techniques are especially useful for evaluating agricultural tractor ride characteristics. This design approach, illustrated in Figure 1, requires the quantitative description of the following ride analysis components: (1) description of a truly representative terrain or test track for exciting the tractor, (2) description of the tractor as a system of components, (3) calculation or computation of the tractor's dynamic response, and (4) conversion of the vibrational characteristics on the operator to a human stress parameter.

Terrain inputs and excitations

For a valid physical or subjective assessment of ride comfort, the tractor must travel over a surface that can be accurately specified and easily duplicated at numerous test sites. The disadvantage of using actual field surfaces is that the surfaces change with varying soil and climatic conditions and repeated travel over them. Therefore, a non-yielding test surface is more desirable providing it can generate similar vibrational characteristics on the tractor and the operator, as would the actual field surface.

Matthews (1964b and 1966) sought information on the intensity and characteristics of ride vibration produced during normal tractor usage. From this investigation, two test tracks have been developed to encompass the range of agricultural terrain profiles and the ride vibration frequency spectra. The rough track, which is 35 meters (m) in length, simulated a severely uneven, plowed field while the smooth track, which is 100 m in length, simulated a gently undulating, unpaved farm road. These two test tracks with their respective station-elevation coordinates have been included as part of the ISO 5007 standard.

Wendenborn (1966) investigated the influence of farm roads and fields as a vibrational source to the tractor and its operator. Representative farm terrains were selected, measured for obtaining their profile coordinates, and defined as spectral power density functions. These functions were then combined with the mathematical equations that defined a tractor for obtaining the average spectral acceleration as a function of time and frequency.

Techniques to model vehicle response

Simulation techniques must use viable mathematical representations of the tractor, tires, cab, and seat suspension for predicting the vehicle's dynamic response and for evaluating the operator ride response. The typical model will be comprised of a series of mathematical expressions which describe the dynamic response of the tractor traveling over a specified terrain.

The most common analytical technique is modeling the tractor as a two- or three-dimensional system of rigid masses and linear springs and dampers. The methods of Newtonian or Lagrangian dynamics are used for deriving the equations of motion which correspond to the DOF of the tractor. A computer program is then implemented for evaluating the equations and predicting the dynamic response of the model. Raney et al. (1961), and Matthews and Talmo (1965) were the earliest researchers to implement computer programs for predicting the dynamic behavior of tractors.

Smith (1977) has developed a 3-dimensional tractor simulation computer program for predicting the time response of the lateral, longitudinal, and vertical accelerations of any point on the tractor's chassis, cab, or seat. The program used the ISO 5007 test tracks for inducing the terrain motion to the tractor model. Spectral analysis of the time domain acceleration history provided the basis for evaluating the effect of design changes on the ride comfort. The program also computed the system's natural frequencies and corresponding mode shapes of the vibrational modes for gaining insight into the resonances indicated by the spectral analysis.

NIAE has implemeted a computer program for evaluating the effect of design changes on operator ride comfort (McCready and Stayner, 1975; and Stayner, 1975). The program predicted the dynamic response of tractor models with five, six, or ten DOF. The computer program used the equations of motion for obtaining the frequency response functions, natural and damped frequencies, and the mode shapes of the models. The program used the ISO 5007 test track profiles for inducing motion to the models at the tire contact points.

Other research studies on vehicle dynamics and ride comfort have been conducted with analytical computer simulation techniques. Wolken and Yoerger (1974) developed mathematical models of an agricultural tractor and combine for determining the effect of random terrain vibrational input on the response of the operator. Ponsonnet and Gandelot (1974) used simulation techniques for evaluating operator ride comfort. Smith (1967) implemented a simulation program for designing the suspension and evaluating the dynamic loads for axles and frames of agricultural equipment.

The finite element analysis and the system analysis via the building block approach have provided other important simulation techniques for predicting the dynamic response of a vehicle. The tractor is modeled as a system of interconnected components for obtaining the dynamic response. The technique assembles the dynamic equations of motion for the components and uses a linearized frequency response method for computing the system transfer functions. These transfer functions along with the power spectral density function of the terrain input are used for computing the power spectral density of the accelerations on the operator. Illustrative examples of this technique have been described by Howell (1974), McClelland and Klosterman (1974), Martz et al. (1974), and Shryock et al. (1977) for predicting the dynamic behavior of off-road vehicles and the operator ride comfort.

The generalized mechanical systems simulation programs have provided another method for predicting the dynamic response of tractors. These programs have been developed for automatically formulating the system equations of motion, the constraint equations, and the input functions, and simulating the dynamic response of any two- or three-dimensional, discrete system of constrained rigid bodies.

Automatic Dynamic Analysis of Mechanical Systems (ADAMS) has been developed for analyzing both two- and three-dimensional mechanical systems (Orlandea and Chace, 1977; Orlandea et al., 1978). These researchers used a problem-oriented language for defining and formulating the system configuration as either an open- or closed-loop kinematic chain mechanism as the following: (1) bodies by mass, moment of inertia, and initial estimates of the generalized coordinates, (2) joints by type, (3) linear and rotational springs and dampers, (4) linear and nonlinear force and motion functions. The program has been developed for providing the following design capabilities: (1) static equilibrium position analysis, (2) large displacement (nonlinear) analysis, (3) linearized small displacement analysis about any point in time, (4) transformation of a system variable in the time domain to the frequency domain with the Fast Fourier Transform algorithm, (5) plot of any system variable in either the time or frequency domain, and (6) graphic display of the mechanical system in the time domain.

Integrated Mechanisms Program (IMP) was developed for analyzing two- and three-dimensional, motion-constrained, rigid body, closed-loop kinematic chain mechanisms. The desired mechanism is formulated and analyzed by the use of a problem-oriented language consisting of definition, data, request, control, delete, and graphics statements. The program has the following design capabilities: (1) static equilibrium mode, (2) kinematic mode, and (3) linear and nonlinear dynamics modes.

The utility of the generalized mechanical systems simulation programs can be illustrated with the ADAMS program by modeling the dynamic response of an agricultural wheel tractor and plow system, as shown in Figure 2. The following assumptions were made:

1. The tractor-plow model was two-dimensional.
2. The tractor chassis had three DOF - bounce, pitch, and longitudinal modes.
3. The operator cab enclosure had three DOF - bounce, pitch, and longitudinal.
4. The tires and cab isomount pads were modeled as linear springs and dampers.
5. The three-point hitch was constrained to provide implement position control.
6. The moldboard plow was constrained at the implement hitch pin.
7. The plow draft forces were applied at the centers of reaction of the moldboards which were taken to be 27 percent of the way from the tip to the rear of the moldboard. The draft force that was applied at each moldboard was 6.4 kilonewtons. The plow had a total cutting width of 1365 mm and was operated at a depth of 200 mm.
8. The plow suction forces were applied at the centers of reaction of the moldboards which were taken to be 25 percent of the way up from the bottom of the moldboard. The suction force which was applied at each moldboard was 45 percent of the draft force.
9. The ISO 5007 smooth track profile was passed under the

tractor tires at a constant velocity of 12.5 km/h.

The longitudinal and vertical displacements and acceleration responses for the point seat are shown in Figures 3, 4, 5, and 6, respectively. The time response of the tractor-plow system is graphically illustrated in Figure 7.

Ride response

The acceleration responses either computed by an analytical simulation technique or measured in the laboratory and/or the field, are evaluated in terms of human stress and fatigue criteria for quantifying the tractor ride comfort. Stikeleather (1976) has presented a thorough review on ride evaluation for whole-body vibration. The three most accepted criteria are:

1. Overall weighted RMS acceleration (SAE, 1980).
2. Total integrated absorbed power over the frequency range of interest, which is based on the theory of Pradko and Lee (1968) and implemented by Lins and Dugoff (1972).
3. ISO Standard 2631 - Guide for the Evaluation of Human Exposure to Whole-Body Vibration (1972).

SUMMARY AND CONCLUSIONS

During the past 25 years, research investigations have been conducted for improving the operator ride comfort of off-road vehicles. The following systems have been used for reducing and attenuating terrain-induced vibrations on the operator: (1) seat suspensions, (2) cab suspensions, (3) front axle suspensions, and (4) front and rear axle chassis suspensions.

Analytical computer simulation techniques have proven to be valuable design tools for evaluating the improvements for these suspension systems. These techniques included: (1) specialized vehicle analysis simulation programs, (2) finite element analysis programs, and (3) generalized motion-constrained mechanical systems programs. These techniques have been used for formulating the mathematical model of the vehicle system and its input, and for predicting the dynamic response of the vehicle and the acceleration history on the operator. These acceleration responses have provided the basis for evaluating the effect of design changes on the improvement of operator ride comfort.

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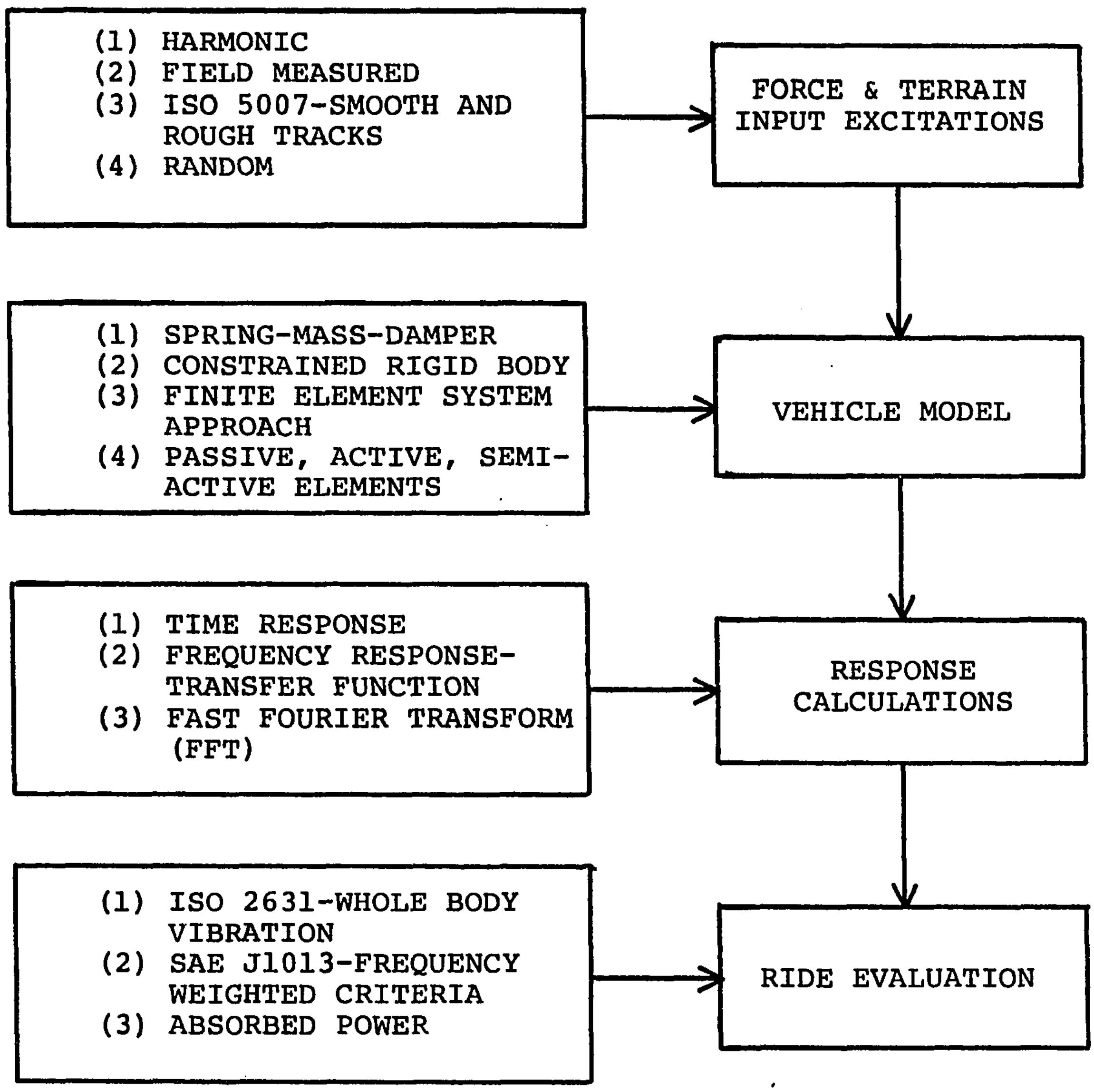

FIGURE 1. Flow diagram for the tractor ride simulation and evaluation

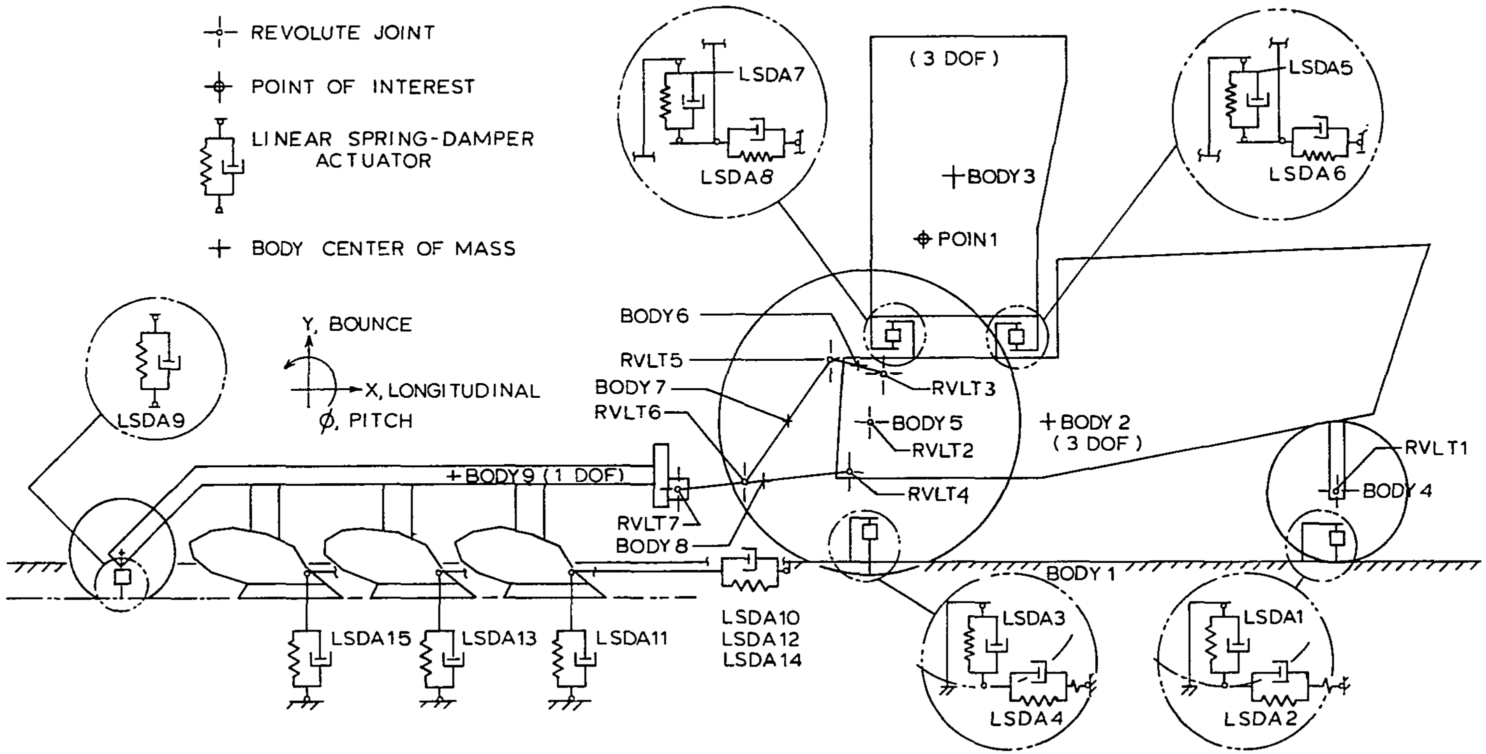

FIGURE 2. Two-dimensional ADAMS tractor-implement system model

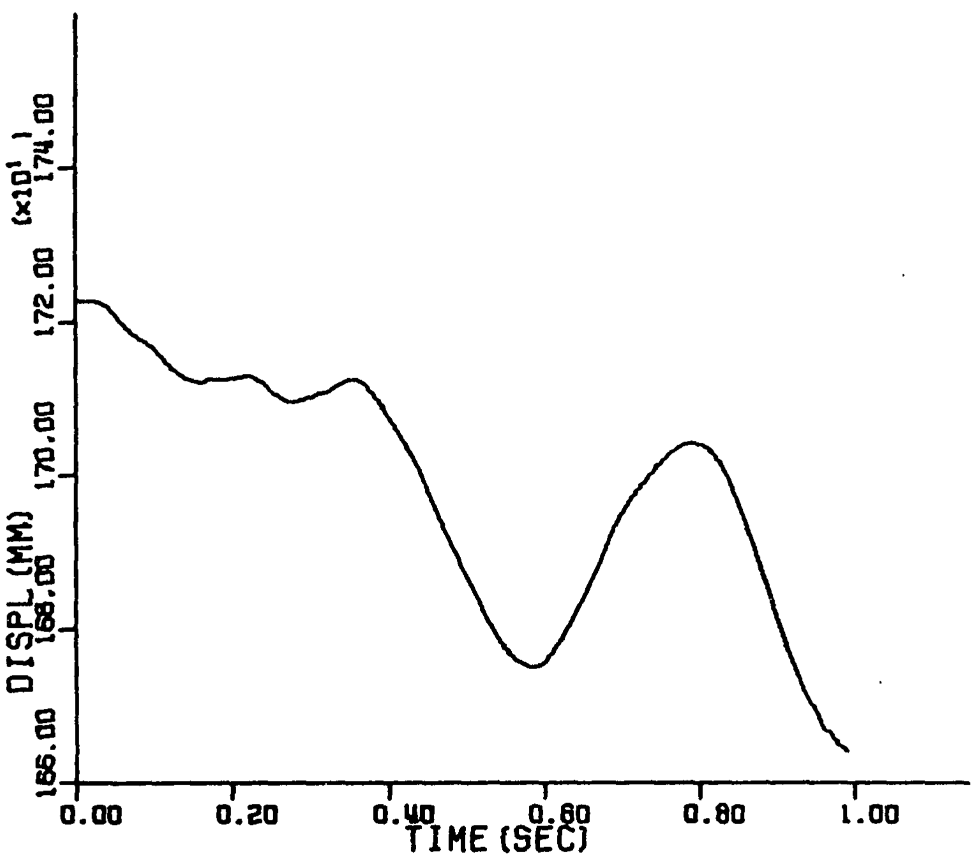

FIGURE 3. Vertical displacement of the tractor seat

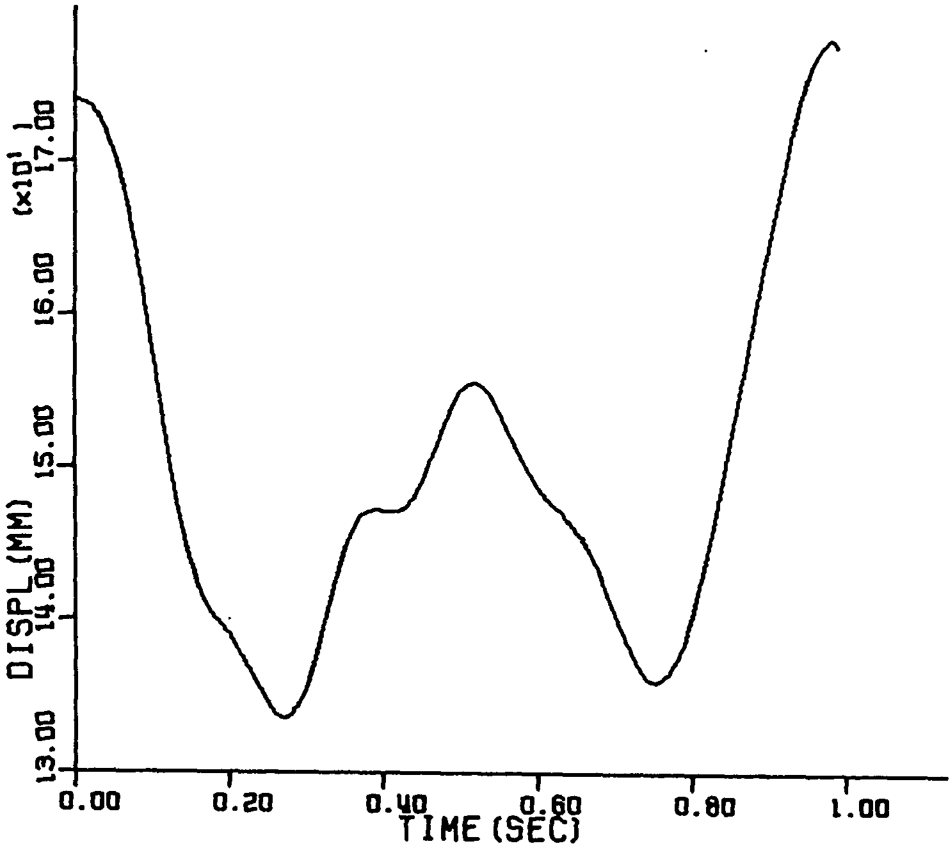

FIGURE 4. Longitudinal displacement of the tractor seat

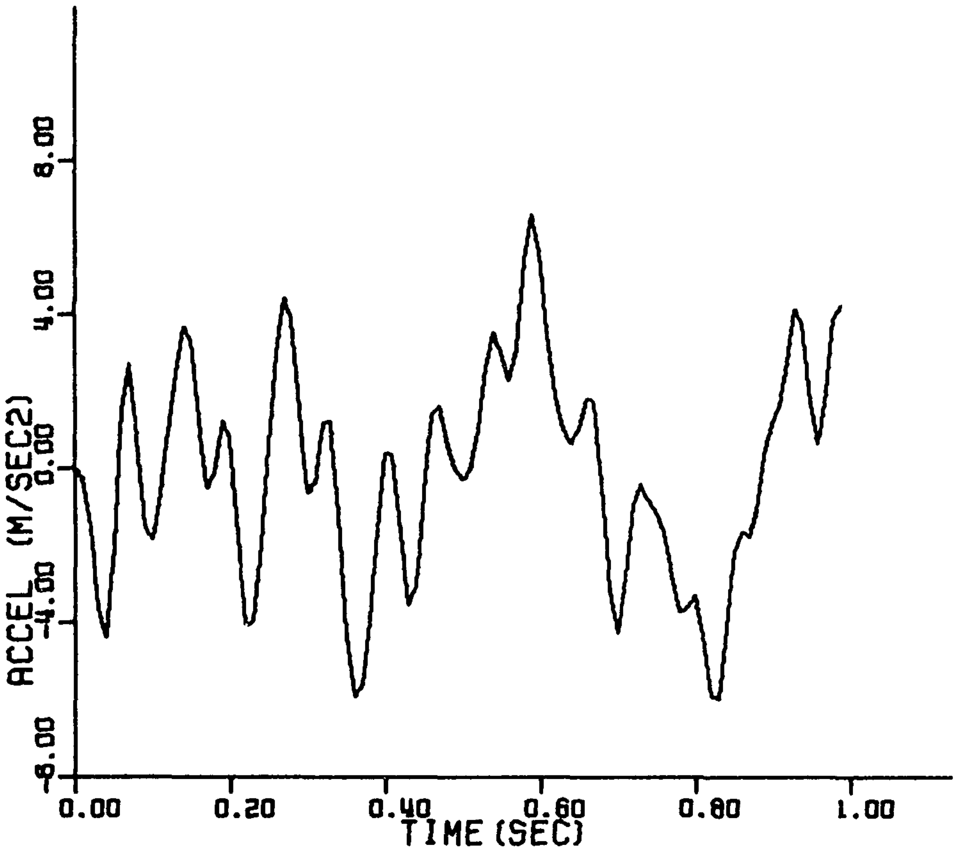

FIGURE 5. Vertical acceleration of the tractor seat

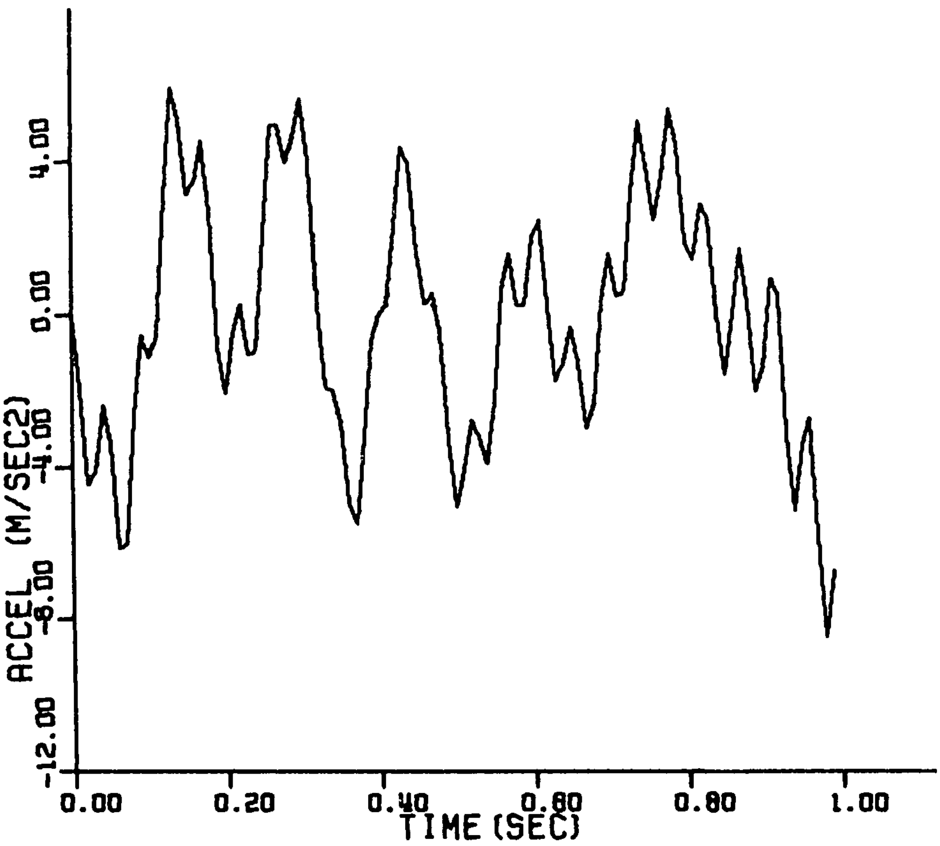

FIGURE 6. Longitudinal acceleration of the tractor seat

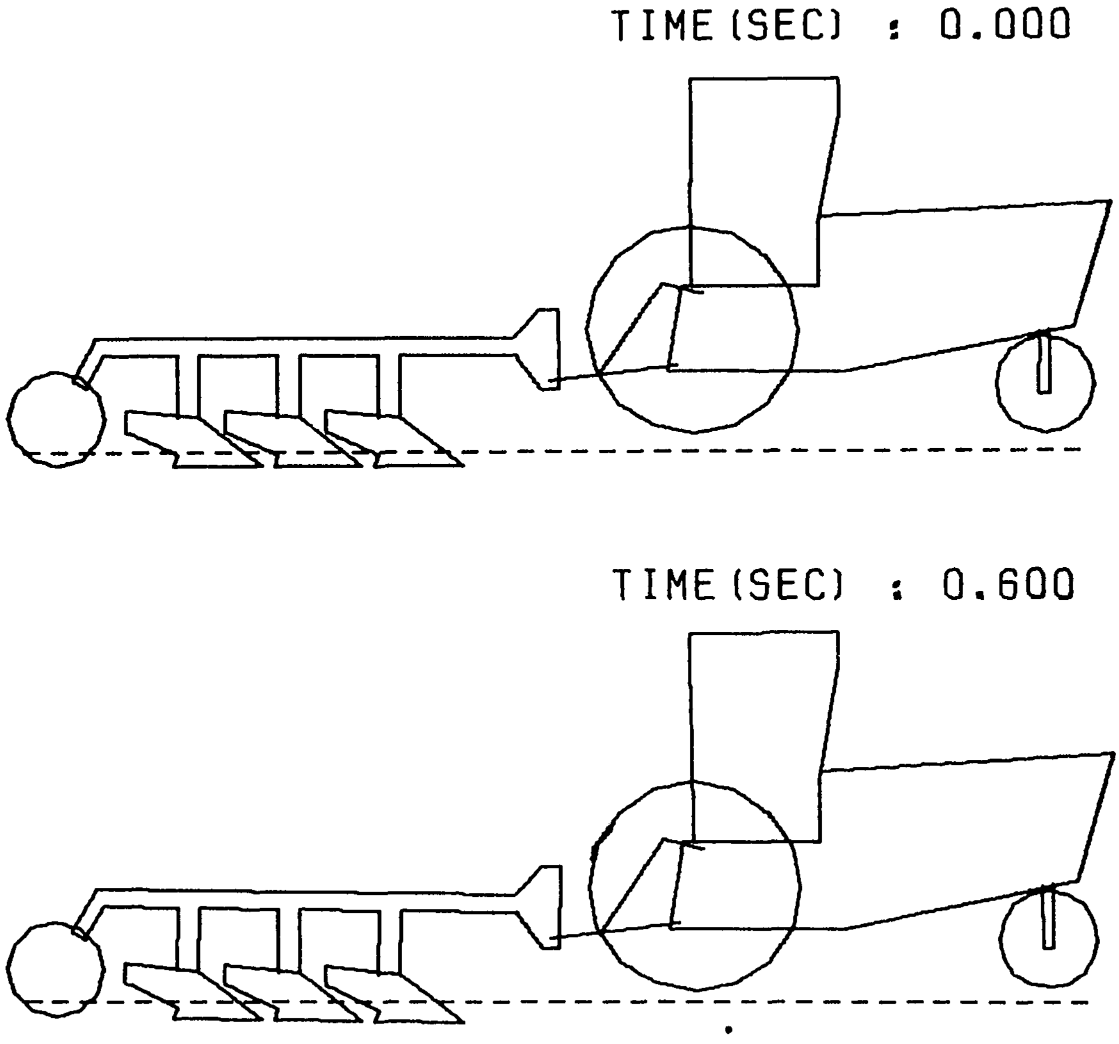

FIGURE 7. Graphical time response of the tractor-implement system model

SECTION II: A MATRIX-COMPUTER PROCEDURE FOR VIBRATION ANALYSIS OF ARTICULATED MECHANICAL SYSTEMS

INTRODUCTION

Computer-aided design (CAD) techniques are being used to increase the productivity of American industry (Bylinsky, 1981). In the past, the off-road equipment industry has expended large quantities of money and manpower to develop a machine for sales. Typically, several experimental prototypes were designed, fabricated, and evaluated to meet the specified marketing and engineering goals for function, reliability, and performance. With CAD techniques, many alternative design concepts can be evaluated in the initial design stages without fabricating hardware for evaluation. Thus, the optimum design concept can be selected for fabrication and evaluation. For example, finite element analysis and generalized mechanical systems simulation techniques can be used for analyzing the response of the machine and its components to any number of test conditions that were based on field and laboratory data.

In this section, analytical techniques for computing the system transfer functions, the linearized frequency response and the time response of a constrained mechanical system are described. Analytical methods were developed to formulate the linearized dynamic matrices for the vibrational behavior of the mechanical system and to compute the frequency-domain force and base-motion transfer functions. These analytical techniques were incorporated into the generalized computer design program INTEGRATED MECHANISMS PROGRAM (IMP) which simulates the behavior of rigid-body, multi-loop, multi-degree-of-freedom, planar and spatial mechanisms with associated constraint paths and loops. With the numerical results from the IMP transfer function procedure, the frequency response and the system mode shapes can be computed. With the Fast Fourier Transform (FFT) algorithm, the arbitrary force and/or base-motion histories were combined with the transfer functions to simulate the frequency-domain response of the system. These linearized dynamics modelling techniques are applicable to the vibrational analysis of articulated machinery and vehicle suspension systems. Specific applications of these techniques were illustrated by means of numerous examples.

THE IMP SYSTEM

With the advent of the high-speed digital computers, linear and nonlinear linkage-type (inertia-invariant) systems have been analyzed both kinematically and dynamically. Several general-purpose mechanical systems simulation programs have been developed (Kauffman, 1978). These programs have the following features:

1. Automatically generate the differential equations of motion and the equations for the constraints and the force and motion excitations.
2. Solve the system equations of motion by numerical integration techniques.
3. Solve for the internal reactions at the constraint joints of the mechanism (Paul, 1979).

IMP has been developed to analyze two- and three-dimensional, motion-constrained, rigid-body, closed-loop, kinematic chain mechanisms that have single or multiple degrees-of-freedom (Sheth and Uicker, 1972). The flow process for this program is shown in Figure 1. The rigid links for the mechanical system were interconnected by constraint joints or kinematic pairs, as shown in Figure 2. Also, force-producing elements, such as the springs and dampers shown in Figure 3, were included to restrain the systems.

The program was developed with the following simulation modes:

1. Kinematic Mode--The user specifies the range of motion and the step size for each of the specified generalized coordinates (SGC). Input displacements, velocities, and

accelerations, and applied forces may also be specified.

2. Static Mode--A minimization procedure, which seeks a minimum potential energy state for the mechanism and locates the stable static equilibrium configuration.
3. Dynamic Mode--The mechanism can be driven with externally applied forces and/or base-motions as a function of time. The Hamiltonian of the system is formulated, then followed by taking the derivatives of the Hamiltonian of the system with respect to the generalized coordinates to yield a set of first-order canonical dynamic equations of motion which are solved in the time- or frequency-domain.

Upon request, any or all of the following entities may be determined by IMP:

1. The positions, velocities, and accelerations of the joints and the points of interest on the links.
2. The static and dynamic constraint forces, as well as the cyclic variations in the driving source characteristics.
3. The forces acting in the springs and dampers.
4. The graphical display of the mechanism at specified time intervals.
5. The natural frequencies and the damping coefficients.

6. The transfer functions, which were incorporated for the dissertation research project, for the small steady-state oscillations of a mechanical system resulting from the sinusoidally-varying, base motion, force, and/or torque inputs.

IMP ALGORITHMIC APPROACH

The computational procedure taken by IMP is summarized in Figure 1. Briefly, the procedure consisted of the following steps:

1. A topological analysis was performed to recognize the number of links, the number and types of joints, the order in which the links and joints are arranged, the number and order of the kinematic loops, and other characteristics which are solely determined by the connectivity of the mechanism. The independent kinematic loops were selected so that the number of joints in each loop is the minimum possible.
2. The geometric dimensional data was used to describe the relative orientation and the location of the joints on the various links. With this description, the kinematic shape of a rigid link may be specified by the constant spatial relationships which exist between the strategically placed Cartesian reference frames at the ends of the links. The motion occurring within a joint may be specified as a variable relationship between the two coordinate systems attached to the two mating joint elements.
3. The total number of constraint variables C_t was determined from the various joint types in assembling the mechanism. From the constraint considerations, the independent variables were identified by using the constraint equations to eliminate the dependent variables. A matrix rank approach was used to identify the number of constraint variables that

acquire motion independent of the constraint conditions and to select the specific variables to act as the independent variables or generalized coordinates. The number of independent equations was equal to the rank R of the system matrix [N]. Hence, the total number of degrees-of-freedom (DOF) was

$$DOF = C_t - R \qquad [1]$$

where DOF and R were dependent on the positional geometry of the mechanism.

Some of the generalized coordinates or the degrees-of-freedom specified prescribed-motions for the mechanism. Because no additional conditions were requested for these coordinates, they were identified as specified generalized coordinates (SGC). Any additional generalized coordinates or the remaining degrees-of-freedom were free to assume motions dictated by the static or dynamic conditions of the system and were identified as the free generalized coordinates (FGC). Thus,

$$DOF = FGC + SGC \qquad [2]$$

4. With the geometric dimensional data, the requested simulation mode was performed by IMP.

LINEARIZED DYNAMIC BEHAVIOR OF CONSTRAINED MECHANICAL SYSTEMS

The linearized dynamics numerical technique is applicable for modelling the dynamic behavior of a mechanical system for several reasons. First, the computed natural frequencies and damping coefficients provide the designer a qualitative feel for the system. Second, the cost of computing these system parameters for a linearized system is significantly less than the costs for computing with a numerical integration procedure, the dynamic behavior of a nonlinear geometry-dependent system. Typically, the designer is interested in the small steady-state oscillations of a constrained mechanical system about its static equilibrium position. The system is restrained by springs and dampers, and is acted on by external forces and base motions. The external excitations are comprised of constant terms and small amplitude time- or frequency-dependent terms.

With the IMP linearized dynamics procedure, the static equilibrium position of the mechanical system was determined. For small steady-state oscillations about the static equilibrium position, the displacements of the N generalized coordinates f were expressed as

$$\{f\} = \{f\}_* + \{\eta\} \qquad [3]$$

where $\{f\}_*$ represented the generalized coordinates of the static equilibrium position and $\{\eta\}$ represented the "small" oscillations of the generalized coordinates about the static equilibrium position.

Typically, the system equations of motion were highly nonlinear because the equation coefficients were dependent on the changing system geometry. The nonlinear differential equations were linearized by assuming the "small" departures result in a set of N-coupled equations

$$[\ulcorner M \lrcorner]\ \{\ddot{\eta}\} + [\ulcorner C \lrcorner]\ \{\dot{\eta}\} + [\ulcorner K \lrcorner]\ \{\eta\} = \{F\} \qquad [4]$$

where $[\ulcorner M \lrcorner]$, $[\ulcorner C \lrcorner]$, and $[\ulcorner K \lrcorner]$ were N by N real symmetric mass, damping, and stiffness matrices, respectively. The real vector $\{F\}$ of N generalized coordinates correspond to the arbitrarily applied sinusoidally-varying force, torque, and/or base motion inputs.

Classical normal modes exist for undamped linear systems. For each normal mode, the system components vibrate at the same natural frequency. For damped linearized dynamic systems, classical normal modes do not exist except when the matrix $[\ulcorner C \lrcorner]$ in Equation [4] is proportional to either the matrix $[\ulcorner M \lrcorner]$ or $[\ulcorner K \lrcorner]$ so that the velocity coupling is eliminated.

To obtain the classical normal modes for a damped linearized dynamic system, the set of N by N second-order differential equations must be transformed into a set of 2N by 2N first-order differential equations. This normal mode method uses the generalized coordinate velocities as auxiliary variables in the following matrix identity

$$[\ulcorner M \lrcorner]\ \{\dot{\eta}\} = [\ulcorner M \lrcorner]\ \{\dot{\eta}\} \qquad [5]$$

Equations [4] and [5] were combined to yield a set of 2N by 2N first-order differential equations, which written in reduced form were

$$[A]\ \{\dot{q}\} + [B]\ \{q\} = \{P\} \qquad [6]$$

where

$$[A]_{2N*2N} = \left[\begin{array}{c|c} \lceil 0 \rfloor_{N*N} & \lceil M \rfloor_{N*N} \\ \hline \lceil M \rfloor_{N*N} & \lceil C \rfloor_{N*N} \end{array}\right]$$

$$[B]_{2N*2N} = \left[\begin{array}{c|c} \lceil M \rfloor_{N*N} & \lceil 0 \rfloor_{N*N} \\ \hline \lceil 0 \rfloor_{N*N} & \lceil K \rfloor_{N*N} \end{array}\right]$$

$$\{\dot{q}\}_{2N*1} = \left\{\begin{array}{c} \{\ddot{\eta}\}_N \\ \hline \{\dot{\eta}\}_N \end{array}\right\}$$

$$\{q\}_{2N*1} = \left\{\begin{array}{c} \{\dot{\eta}\}_N \\ \hline \{\eta\}_N \end{array}\right\}$$

$$\{P\}_{2N*1} = \left\{\begin{array}{c} \{0\}_N \\ \hline \{F\}_N \end{array}\right\}$$

The homogeneous solution to Equation [6] was obtained by setting the vector {P} equal to zero to yield

$$[A]\ \{\dot{q}\} + [B]\ \{q\} = \{0\} \qquad [7]$$

Equation [7] was premultiplied by $-[A]^{-1}$ to yield

$$-[I]\,\{\dot{q}\} + [G]\,\{q\} = \{0\} \qquad [8]$$

where

$[I] = [A]^{-1}[A]$, the identity matrix

$$[G] = -[A]^{-1}[B] = \left[\begin{array}{c|c} [\sim M \sim]^{-1}\,[\sim C \sim] & [\sim M \sim]^{-1}\,[\sim K \sim] \\ \hline [\sim I \sim] & [\sim 0 \sim] \end{array}\right]$$

The matrix [G] was not symmetric even though the matrices $[\sim M \sim]$, $[\sim C \sim]$, and $[\sim K \sim]$ were symmetric. Equation [8] was solved by assuming the trial solution was

$$\{q_i\} = \{\psi_i\}e^{\lambda_i t} \qquad [9]$$

where $\{\psi_i\}$ was the modal vector of length 2N which corresponded to the normal mode frequency λ_i. Equation [9] was substituted into Equation [8] to yield

$$\Big[[G] - \lambda_i[I]\Big]\{\psi_i\} = \{0\} \qquad [10]$$

where the exponential factors were cancelled out and a set of 2N homogeneous equations in terms of ψ was obtained.

If the trial solution in Equation [9] was fitted to the differential equations of motion, the eigenvalue equation must hold. For a nontrivial solution of Equation [10], the determinant must vanish that

$$\mathrm{Det}\left|[G] - \lambda_i[I]\right| = 0 \qquad [11]$$

Equation [11] was the characteristic equation of the matrix [G]. For undamped systems, these computed eigenvalues were negative real numbers and represented the natural frequencies for the system. For damped systems, the computed eigenvalues were conjugate sets of complex numbers. The negative real part of the complex numbers corresponded to the damped natural frequencies (Baethe and Wilson, 1976; Clough and Penzien, 1975; Meirovitch, 1975; Craig, 1981).

A characteristic vector of the modal amplitudes or an eigenvector was computed. The eigenvectors had the following orthogonality condition

$$\begin{aligned} \{\psi\}_R^T\,[A]\,\{\psi\}_S &= 0 \qquad R \neq S \\ &= \alpha_R \qquad R = S \end{aligned} \qquad [12]$$

and

$$\begin{aligned} \{\psi\}_R^T\,[B]\,\{\psi\}_S &= 0 \qquad R \neq S \\ &= \alpha_S \qquad R = S \end{aligned} \qquad [13]$$

where the superscript T denotes a transposed vector.

The transformation of the generalized coordinates yielded

$$\{y\} = [\psi]_{2N*2N}\,\{q\} \qquad [14]$$

and

$$[\psi] = \left[\{\psi\}_1\ \{\psi\}_2\ \cdots\ \{\psi\}_{2N}\right] \qquad [15]$$

The column vectors in Equation [15] of the eigenvector matrix represented the characteristic shape of the system for each mode of vibration. The displacement vector $\{q\}$ defined a new non-physical principal coordinate system. Equation [14] was substituted into Equation [6]. Then Equation [6] was premultiplied by $[\psi]^T$ to yield

$$[\psi]^T\,[A]\,[\psi]\,\{\dot{q}\} + [\psi]^T\,[B]\,[\psi]\,\{q\} = [\psi]^T\,\{P\} \tag{16}$$

Using the orthogonality conditions from Equations [12] and [13], Equation [16] was rewritten to yield

$$[\ulcorner \alpha \lrcorner]\,\{\dot{q}\} + [\ulcorner \beta \lrcorner]\,\{q\} = [\psi]^T\,\{P\} \tag{17}$$

where

$$[\ulcorner \alpha \lrcorner] = [\psi]^T\,[A]\,[\psi]$$

$$[\ulcorner \beta \lrcorner] = [\psi]^T\,[B]\,[\psi]$$

The vector $\{P\}$ consisted of the generalized forces that corresponded to an arbitrarily applied sinusoidal excitation Q

$$\{P\} = \left\{\begin{array}{c}\{0\}\\ \hline \{F\}_N\end{array}\right\} = \left\{\begin{array}{c}\{0\}_N\\ \hline \left\{\begin{array}{c}g_1\\ g_2\\ \vdots\\ g_N\end{array}\right\}\end{array}\right\} Q \tag{18}$$

where the vector elements g_1 , g_2 , ... , g_n were the first derivatives of the geometric factors for the links with respect to the generalized coordinates (Sheth, 1972).

The kth equation from Equation [17] had the following form

$$\alpha_k \dot{q}_k + \beta_k q_k = \left[\sum_{i=1}^{N} (\psi_{N+i})_k g_i \right] Q \qquad [19]$$

where the factors $(\psi_{n+1})_k$ were the elements from the kth eigenvector The value of the subscript variable, k, ranged from one to two times the N number of system FGCs while the value of the subscript variable, i, ranged from one to N number of system FGCs.

A trial solution was assumed such that

$$q_k = \{\psi\}_k e^{\lambda_i t} \qquad [20]$$

and was substituted into the homogeneous form of Equation [17] to yield

$$\beta_k = -\lambda_k \alpha_k \qquad [21]$$

Equation [21] was substituted into Equation [19] to yield

$$\alpha_k [\dot{q}_k - \lambda_k q_k] = \left[\sum_{i=1}^{N} (\psi_{N+i})_k g_i \right] Q \qquad [22]$$

Equation [22] was transformed to the frequency- or s-domain to yield

$$\alpha_k[s-\lambda_k]q_k(s) = \left[\sum_{i=1}^{N} (\psi_{N+i})_k g_i\right] Q(s) \tag{23}$$

Equation [23] was rearranged to yield

$$\frac{q_k(s)}{Q(s)} = \frac{\frac{1}{\alpha_k}\left[\sum_{i=1}^{N} (\psi_{N+i})_k g_i\right]}{(s - \lambda_k)} \tag{24}$$

which was then simplified to yield

$$\frac{q_k(s)}{Q(s)} = \frac{D_k}{s-\lambda_k} \tag{25}$$

where

$$D_k = \frac{1}{\alpha_k} \left[\sum_{i=1}^{N} (\psi_{N+i})_k g_i\right]$$

The mathematical variables D_k and λ_k were computed as pairs of complex conjugate numbers.

Equation [14] and the expression for the vector {q} from Equation [6] were substituted into Equation [25] to yield

$$\{y\} = \begin{Bmatrix} \{s\eta\} \\ \{\eta\} \end{Bmatrix} = [\psi] \left\{ \frac{D}{s-\lambda} \right\} Q(s) \qquad [26]$$

In the s-domain, Equation [26] was expanded for each generalized coordinate $\eta_i(s)$ to yield

$$\eta_i(s) = \left[(\psi_{N+i})_1 \left[\frac{D_1}{s-\lambda_1} \right] + (\psi_{N+i})_2 \left[\frac{D_2}{s-\lambda_2} \right] + \ldots + (\psi_{N+i})_{2N} \left[\frac{D_{2N}}{s-\lambda_{2N}} \right] \right] Q(s) \qquad [27]$$

The value of the subscript variable, i, ranged from one to the N number of system FGCs. Equation [27] was rearranged to relate the generalized coordinates to the input excitation Q(s) and to yield the system transfer functions

$$\frac{\eta_i(s)}{Q(s)} = \left[(\psi_{N+i})_1 \left[\frac{D_1}{s-\lambda_1} \right] + (\psi_{N+i})_2 \left[\frac{D_2}{s-\lambda_2} \right] + \ldots + (\psi_{N+i})_{2N} \left[\frac{D_{2N}}{s-\lambda_{2N}} \right] \right] \qquad [28]$$

The system transfer functions were defined as an algebraic expression in the frequency- or s-domain which relate the transformed steady-state oscillatory system response to the input excitation.

For the linearized dynamics method, IMP computed the eigenvalues and the eigenvectors from Equation [11] with the IMP subroutine EIGVCU.

The subroutine used the upper Hessenberg form and the QR transformation algorithms to compute the quantities to provide a complete representation of the linearized dynamic characteristics for a constrained mechanical system (Chua and Lin, 1975; Burden et al., 1978).

TRANSFER FUNCTION RATIOS

The transfer function for a constrained mechanical system is defined as the steady-state vibratory motion of an output variable to a sinusoidally-varying input excitation Q. With IMP, the system transfer function was computed for the motion of a point of interest on any link or a generalized coordinate for a single degree-of-freedom revolute or prismatic joint. The mechanical system was excited by force and/or torque input excitations, and/or base motion input excitations--accelerations, velocities, and/or displacements.

The vibratory motion of a single degree-of-freedom joint, ϕ_t, was expressed in equation form to yield

$$\phi_t = \phi_{t*} + \gamma_t \qquad [29]$$

where

ϕ_{t*} = the static equilibrium position of the joint

γ_t = the "small" vibratory displacement of the joint

$$\gamma_t = \left[\frac{\partial \phi_t}{\partial f_1} \eta_1 + \frac{\partial \phi_t}{\partial f_2} \eta_2 + \ldots + \frac{\partial \phi_t}{\partial f_N} \eta_N \right]$$

Substituting Equation [27] into the expression for the vibratory displacement γ_t for the joint with respect to the input excitation was

$$\frac{\gamma_t(s)}{Q(s)} = \left\{ \begin{aligned} & \frac{\partial\phi_t}{\partial f_1} \left[(\psi_{N+1})_1 \frac{D_1}{s-\lambda_1} + \ldots + (\psi_{N+1})_{2N} \frac{D_{2N}}{s-\lambda_{2N}} \right] \\ + & \frac{\partial\phi_t}{\partial f_2} \left[(\psi_{N+2})_1 \frac{D_1}{s-\lambda_1} + \ldots + (\psi_{N+2})_{2N} \frac{D_{2N}}{s-\lambda_{2N}} \right] \\ + & \ldots \\ & \vdots \\ + & \frac{\partial\phi_t}{\partial f_N} \left[(\psi_{N+N})_1 \frac{D_1}{s-\lambda_1} + \ldots + (\psi_{N+N})_{2N} \frac{D_{2N}}{s-\lambda_{2N}} \right] \end{aligned} \right\} \qquad [30]$$

Equation [30] was rewritten in concise form to yield

$$\frac{\gamma_t(s)}{Q(s)} = \sum_{k=1}^{2N} \left[\sum_{i=1}^{N} (\psi_{N+i})_k \left\{ \frac{\partial\phi_t}{\partial f_i} \right\} \right] \frac{D_k}{s-\lambda_k} \qquad [31]$$

The term $(\partial\phi_t/\partial f_i)$ was the real first derivative of the geometric constraint variable for a single degree-of-freedom joint with respect to the ith independent generalized coordinate. These partial derivatives were evaluated at the system reference position for vibratory motion. If the joint of interest was a dependent constraint variable, these first partial derivatives were available in columnar form from the matrix [H] which was computed during the iterative position analysis procedure (Sheth, 1972). However, if the joint of interest was one of the independent generalized coordinates, the first partial derivative $(\partial\phi_t/\partial f_i)$ was equal to zero except for the derivative term of the independent constraint variable, ϕ_t, with respect to itself or f_i which was equal to one.

For a point of interest on a vibrating link of the mechanism, three sets of transfer functions were computed for the input excitation, which represents the X, Y, and Z coordinate motions as measured in the global coordinate frame. The absolute or global coordinates for a point were written in vector form to yield

$$\{R\} = \begin{Bmatrix} 1 \\ X \\ Y \\ Z \end{Bmatrix} = \begin{Bmatrix} 1 \\ x \\ y \\ z \end{Bmatrix}_* + \begin{Bmatrix} o \\ u \\ v \\ w \end{Bmatrix} \qquad [32]$$

where the vector $\{\ \}_*$ was the reference position of the mechanical system. The vibratory displacements from the reference position were written in equation form as

$$\begin{Bmatrix} o \\ u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial R}{\partial f_1} \end{Bmatrix} \eta_1 + \begin{Bmatrix} \dfrac{\partial R}{\partial f_2} \end{Bmatrix} \eta_2 + \ldots + \begin{Bmatrix} \dfrac{\partial R}{\partial f_N} \end{Bmatrix} \eta_N \qquad [33]$$

Equation [33] was written in an alternate form as

$$\begin{Bmatrix} o \\ u \\ v \\ w \end{Bmatrix} = \left[[\bar{W}_1]\{R\}\eta_1 + [\bar{W}_2]\{R\}\eta_2 + \ldots + [\bar{W}_N]\{R\}\eta_N \right] \qquad [34]$$

where

$$[W_i]\ \{R_i\} = (\partial R/\partial f_i)$$

The matrix $[W_i]$ was a 4 by 4 derivative matrix for the relative joint coordinate system on the link to which the point was located with respect to the independent generalized coordinate. Subsituting Equation [27] into Equation [34], the vibratory displacement $\gamma_j(s)$ for the point of interest was

$$\frac{\gamma_j(s)}{Q(s)} = \sum_{k=1}^{2N} \left[\sum_{i=1}^{N} (\psi_{N+i})_k \left\{ \frac{\partial R_j}{\partial f_i} \right\} \right] \frac{D_k}{s-\lambda_k} \qquad [35]$$

The value of the subscript variable, j, was equal to one, two, and three, and corresponded to the x, y, and z coordinate motion, respectively.

The quantities on the right-hand side of Equations [31] and [35] were the transfer functions for the mechanism. Equations [31] and [35] were written in concise form to express the vibratory motion in the s-domain to yield

$$\frac{\gamma_j(s)}{Q(s)} = \sum_{k=1}^{2N} \frac{N_k}{s-\lambda_k} \qquad [36]$$

where for a joint

$$N_k = \left[\sum_{i=1}^{N} (\psi_{N+i})_k \left\{ \frac{\partial \phi_t}{\partial f_i} \right\} \right] D_k$$

where for a point of interest

$$N_k = \left[\sum_{i=1}^{N} (\psi_{N+i})_k \left\{ \frac{\partial R_j}{\partial f_i} \right\} \right] D_k$$

The variable, N_k, was the kth complex modal numerator while the variable, ω_k, was the kth system eigenvalue. For a one degree-of-freedom joint, the value of the subscript variable, j, was equal to one. For a point, the value of the subscript variable, j, was equal to one, two, and three which corresponded to the x, y, and z global coordinate motion, respectively. The velocity of a design variable in the s-domain was obtained by multiplying Equation [36] by s to yield the equation

$$\frac{s\gamma(s)}{Q(s)} = \sum_{k=1}^{2N} s \left[\frac{N_k}{s-\lambda_k} \right] \qquad [37]$$

The acceleration of a design variable in the s-domain was obtained by multiplying Equation [36] by s^2 to yield the equation

$$\frac{s^2\gamma\ (s)}{Q_{(s)}} = \sum_{k=1}^{2N} s^2 \left[\frac{N_k}{s-\lambda_k} \right] \qquad [38]$$

The complete derivation of the transfer function relationships for the motion of a single degree-of-freedom joint and a point of interest resulting from a force and/or torque input excitations is presented in

Appendix A. The complete derivation of the transfer function relationships for the motion of a single degree-of-freedom joint and a point of interest resulting from the base-motion input excitations is presented in Appendix B.

IMP TRANSFER FUNCTION ANALYSIS ENTITY

The equations for the IMP transfer function procedure described in the last section were coded into the subroutine TFREQR which was incorporated into IMP. The N terms from Equation [36] and the equations from Appendices A and B were coded into the subroutine to compute the complex numerators for the joints and points of interest. The system eigenvalues and eigenvectors were computed from Equation [11] with the IMP subroutine EIGCVU.

The IMP transfer function procedure was developed to print out the system eigenvalues and eigenvectors, and the modal numerators for the requested joints and points of interest. The necessary IMP language input statement to request the transfer function entity is described in Appendix C.

FREQUENCY-DOMAIN SYSTEM RESPONSE

The response of a mechanical system to a steady-state driving excitation was composed of the sum of the responses for all of the modes of vibration which was expressed in Equations [31] and [35] or [36]. It was convenient to compute the magnitude and phase angle values for the steady-state response. This frequency response analysis was useful to evaluate the effect of design parameter changes. One application of this analysis was for computer-aided vehicle dynamics - e.g. the design of suspension systems.

The transfer function ratios computed by IMP were used to compute the frequency response magnitude and phase angle shift for each requested joint and point of interest as a function of the frequency and each input excitation.

The system frequency response magnitude was

$$\frac{\Upsilon(s_i)}{Q(s_i)} = \sum_{m=1}^{2N} \frac{N_m}{s_i-\lambda_m} \qquad [39]$$

The value of the subscript variable, i, ranged from one to k discrete frequency values while the value of the subscript variable, j, ranged from one to two times the N number of system FGCs. The term $(\gamma(s_i)/Q(s_i))$ was a complex number and was expressed in partial fraction form

$$\left(\frac{\gamma(s_i)}{Q(s_i)}\right) = \left(\frac{\gamma_{Re}(s_i)}{Q_{Re}(s_i)} + j\frac{\gamma_{Im}(s_i)}{Q_{Im}(s_i)}\right) \qquad [40]$$

where the subscript Re is the real component of the complex number and the subscript Im is the imaginary component of the complex number.

For graphical display purposes, the steady-state response was computed as the absolute value

$$\frac{\gamma(s_i)}{Q(s_i)}_M = \sqrt{\left|\frac{\gamma_{Re}(s_i)}{Q_{Re}(s_i)}\right|^2 + \left|\frac{\gamma_{Im}(s_i)}{Q_{Im}(s_i)}\right|^2} \qquad [41]$$

where the subscript M denotes the absolute frequency response magnitude at the ith discrete frequency value ω_i.

The steady-state frequency response phase angle shift was computed from Equation [40] and was written in equation form to yield

$$\theta(s_i) = \mathrm{Tan}^{-1}\left(\mathrm{Im}\left(\gamma(s_i)/Q(s_i)\right) \Big/ \mathrm{Re}\left(\gamma(s_i)/Q(s_i)\right)\right) \qquad [42]$$

where

Re = the real component of the frequency response magnitude;

Im = the imaginary component of the frequency response magnitude.

The steady-state frequency response magnitude and phase angle values for the output variable x as a function of the frequency for the sinusoidal input excitation F are shown in Figure 4 (Walgrave and Ehlbeck, 1978). The peak frequencies represented the resonant frequencies for the system. The system response at or near a resonant frequency was dominated by the response to that particular mode. For the system shown in Figure 4, the system frequency response exhibited three local maximum peaks at the frequencies ω_1, ω_2, and ω_3. With the plots of the system frequency response magnitude and phase angle, the excitation frequencies and modes of vibration were determined and modified by changes to the system design parameters.

The steady-state motions for the joints and points of interest were the sum of the responses for all modes of vibration due to the input excitations. The steady-state motion for the generalized coordinate was expressed in terms of Equation [39] to yield

$$\gamma(s_i) = \sum_{jj=1}^{m} \sum_{j=i}^{2N} \frac{N_j}{s_i-\lambda_j} Q_{jj}(s_i) \qquad [43]$$

The value of the subscript variable, i, ranged from one to k discrete frequency values; the value of the subscript variable, j, ranged from one to two times the N number of system FGCs; the value of the subscript variable, jj, ranged from one to m input excitations.

With the IMP transfer function analysis entity, the mechanism was excited with force, torque, and/or base-motion input excitations. These input excitations were any impulse, harmonic, periodic, or arbitrary time-domain history. For the frequency response analysis procedure, the input excitations were transformed from the time-domain to the frequency-domain with the FFT algorithm (Bendat and Piersol, 1971; Enochson and Piersol, 1967; Ramirez, 1975).

With the frequency response analysis procedure, the corresponding system mode shapes of vibration were graphically displayed. Each system mode shape corresponded to a particular resonant natural frequency and a FGC variable. The eigenvalues and eigenvectors from the IMP transfer function analysis entity were used for the direct mode superposition method to compute the system mode shapes. The small steady-state oscillation of a generalized coordinate about the static equilibrium position of the system was

$$n_i(t) = \sum_{m=1}^{2N} \left|\psi_{N+i}\right|_j e^{\lambda_{Rej} t} \cos(\lambda_{Imj} t + \phi_j) \qquad [44]$$

where

$\left|\psi_{N+i}\right|_j$ = the complex absolute value of the eigenvector $(\psi_{N+i})_j$;

λ_{Rej} = the real component of the complex eigenvalue λ_j;

λ_{Imj} = the imaginary component of the complex eigenvalue λ_j.

$$\phi_j = \mathrm{Tan}^{-1}\left(\frac{\mathrm{Im}(\psi_{N+i})_j}{\mathrm{Re}(\psi_{N+i})_j}\right)$$

To graphically display the system mode shapes, the coordinates of the points and line segments describing the geometric outline for each system link were defined and stored in an array. The first partial derivative matrices with respect to the independent generalized coordinates for the relative joint coordinate system on each link to which each point was associated were also used to compute the system mode shapes. The modal displacement for each point on each system link was computed for each corresponding mode shape by means of Equation [44]. The x, y, and z modal coordinate motions as measured in the global coordinate frame were in equation form

$$x_{ijn} = \sum_{k=1}^{2N} \left\{ \frac{\partial R_i}{\partial f_n} \right\} \left| \psi_{N+p} \right|_k \; e^{\lambda_{Rek} t} \; \cos(\lambda_{Imk} t + \phi_k) \qquad [45]$$

The value of the subscript variable, i, was equal to one, two, and three which corresponded to the x, y, and z modal displacements in the x, y, and z global coordinate frame, respectively; the subscript variable, j, ranged from one to NPT number of vertices which describe the shape or outline of the mechanical system; the subscript variables, n and p, were equal from one to N number of system FGCs; the term $(\partial R/\partial f_n)$ is a 4 by 4 derivative shape matrix for each point.

Since the modal displacement motion is sinusoidal, the system oscillated between a minimum and a maximum amplitude during a specific period of time. The system modal motion was displayed at specific time increments. At the initial time zero, the system had its position of maximum oscillation. Thus, Equation [45] was rewritten to yield

$$x_{ijn} = \sum_{k=1}^{2N} \left\{ \frac{\partial R_i}{\partial f_n} \right\} \left| \psi_{N+p} \right|_k \cos\phi_k \qquad [46]$$

For each mode shape, the static equilibrium position of the system was drawn in a solid line format while the modally-displaced system was superimposed on it in a dashed line format.

TIME-DOMAIN SYSTEM RESPONSE

The steady-state motion in the frequency-domain for a single degree-of-freedom joint or a point of interest was computed with Equation [43]. This system response was then transformed to the time-domain with the FFT algorithm.

Mathematically, the steady-state motion, γ, in the frequency-domain was

$$\gamma(k\Delta\omega) = \Delta t \cdot \sum_{l=0}^{M-1} \gamma(l\Delta t) \quad e^{-j2\pi k\Delta\omega l\Delta t} \qquad [47]$$

where

$\gamma(k\Delta\omega)$ = the set of Fourier coefficients for the steady-state motion which were determined by the FFT algorithm;

$\gamma(l\Delta t)$ = the set of discrete samples of the steady-state motion in the time-domain;

Δt = time interval or the sampling interval;

$\Delta\omega = 1/M\Delta t$, the sampling interval in the frequency-domain;

$M\Delta t$ = the length of the time record;

e = the base of the natural logarithm;

j = the symbol of complex notation.

The value of the subscript variable, l, ranged from zero to M-1 number of samples while the variable, k, was the index for the computed set of discrete frequency components. This expression was mathematically manipulated to yield the inverse expression

$$\gamma(1\Delta t) = \Delta\omega \cdot \sum_{1=0}^{M-1} \gamma(k\Delta\omega)e^{j2\pi k\Delta\omega 1\Delta t} \qquad [48]$$

This expression allowed a series of frequency-domain samples to be transformed to a series of time-domain samples.

IMP POST-PROCESSOR PROGRAM

A separate post-processor program was developed to compute and graphically display the frequency response magnitude and phase angle values of the system transfer functions. The flow process for this program is shown in Figure 5. Equations [39], [41], and [42] were coded into the program. The program included the ability to compute and graphically display the steady-state motion in the frequency-domain for the revolute and prismatic joints and points of interest from Equations [36]-[39] and [43], and the equations from Appendix C. The program included the capability to compute the time-domain response for revolute and prismatic joints and points of interest from the frequency-domain response by means of the FFT algorithm and to graphically display the steady-state motion. Finally, the program included the capability to compute the system modal displacements from Equations [45] and [46], and graphically display the system mode shapes. As previously mentioned, the program superimposed the system mode shapes in a dashed line format over the static equilibrium system configuration in a solid line format.

The program was interfaced with the numerical results from the IMP transfer function analysis entity.

ILLUSTRATIVE EXAMPLES

Numerical examples of numerous mechanical systems were presented to demonstrate the IMP transfer function analysis entity. Examples of both simple and complex mechanical systems were used to demonstrate the frequency- and time-domain response techniques to compute the dynamic system response. The numerical values for all mechanial system parameters in the selected examples were reported in their common English units of measurement because the IMP source code was developed with English units of measurement.

Example 1

A single degree-of-freedom system subjected to a general-forcing function is shown in Figure 6. The system equation of motion was

$$m\ddot{x} + c\dot{x} + kx = F(t) \tag{49}$$

Using the Laplace transform method and algebraic equation manipulation as outlined by Meirovitch (1975) and Craig (1981), the system transfer function was

$$\frac{x(s)}{F(s)} = \frac{1}{ms^2 + cs + k} \tag{50}$$

The right-hand side of Equation [50] or the system transfer function was expressed as a series of ratios or partial fractions

$$\frac{1}{ms^2 + cs + k} = \frac{N_1}{s-D_1} + \frac{N_2}{s-D_2} \qquad [51]$$

where

$$D_1 = \frac{-c}{2m} + j\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}\ ;$$

$$D_2 = \frac{-c}{2m} - j\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}\ ;$$

$$N_1 = \frac{-j}{2\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}}\ ;$$

$$N_2 = \frac{j}{2\sqrt{\frac{k}{m} - \frac{c^2}{4m^2}}}\ .$$

The IMP transfer function procedure computed the system transfer function denominators and numerators as respective pairs of complex conjugate numbers. The number of ratios was equal to two times the system degrees-of-freedom, or a pair of complex conjugate denominators and numerators existed for each system degree-of-freedom.

The theoretically exact expressions for the system displacement in the time-domain were

$$x = x_{st} \left[1 - \frac{e^{\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \cos\,(\omega_d t-\phi) \right] \qquad [52]$$

where

$0.0 \leq t \leq 2.0;$

$x_{st} = F/k;$

$\omega_n = \quad k/m;$

$\omega_d = \omega_n \sqrt{1-\zeta^2};$

$\zeta = c/c_c = c/2m\omega_n;$

$\phi = \sin^{-1} \zeta.$

$$x = \frac{x_{st}}{\sqrt{1-\zeta^2}}\, e^{-\zeta\omega_n t}\, [e^{\zeta\omega_n t} \cos\,(\omega_d t_1) -1] \cos\,(\omega_d t-\phi) \qquad [53]$$

$$+ [e^{\zeta\omega_n t_1} \sin(\omega_d t_1)] \sin\,(\omega_d t-\phi)\ ,$$

where

$t_1 > 2.0$

The theoretical expressions for the system acceleration in the time-domain were

$$\ddot{x} = x_{st}\left[\frac{-\zeta\omega_n^{2}\ e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos\ (\omega_d t-\phi) - \right. \tag{54}$$

$$\frac{2\omega_d\zeta\omega_n e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\ \sin(\omega_d t-\phi) +$$

$$\left.\frac{\omega_d^{2} e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\qquad \cos(\omega_d t-\phi)\right],$$

where

$0.0 \le t \le 2.0$

$$\ddot{x} = \frac{(\zeta\omega_n)^2-\omega_d^{2}}{\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t_1}\ x_{st}\ \left[[e^{\zeta\omega_n t_1}\ \cos(\omega_d t_1)-1]\ \cdot\right.$$

$$\left.\cos(\omega_d t-\phi) + e^{\zeta\omega_n t_1}\ \sin(\omega_d t_1)\ \cos(\omega_d t-\phi)\right] \tag{55}$$

$$-\frac{2\zeta\omega_n x_{st}}{\sqrt{1-\zeta^2}}\ e^{-\zeta\omega_n t_1}\ \omega_d x_{st}\ \cdot$$

$$\left[-[e^{\zeta\omega_n t_1}\ \cos(\omega_d t_1)-1]\ \sin(\omega_d t-\phi)\right.$$

$$\left.+\ e^{\zeta\omega_n t_1}\ \sin(\omega_d t_1)\ \cos(\omega_d t-\phi)\right]$$

where

$t > 2.0$

The expression for the system displacement in the frequency-domain was

$$x(s) = \left(\frac{N_1}{s-D_1} + \frac{N_2}{s-D_2} \right) F(s) \qquad [56]$$

while the expression for the system acceleration in the frequency-domain was

$$s^2 x(s) = s^2 \left(\frac{N_1}{s-D_1} + \frac{N_2}{s-D_2} \right) F(s) \qquad [57]$$

For small oscillations about the static equilibrium position of the linearized system, it was demonstrated that the system response from the frequency-domain analysis procedure was equivalent to the system response from the time-domain procedure. Thus, a computer program was developed to compare the frequency-domain system response with the time-domain response. For this example, Equations [50] - [57] and the numerical values for the system parameters were incorporated into a FORTRAN IV computer program, listed in Appendix D. The frequency-domain system displacement and acceleration responses were transformed by the program to the linearized time-domain responses by the FFT algorithm. The force excitation in both the frequency- and the time-domains, and the theoretical and transformed system displacement and acceleration responses were printed out and graphically displayed by the program.

With the frequency-domain response technique, the force excitation history in the time-domain was represented as a set of discrete digitized points and then was transformed to the frequency-domain with the FFT algorithm. The number of data points, NPT, required for the input to the FFT subroutine was an integer power (PWR) of 2. This relationship in equation form was

$$NPT = 2^{PWR} \tag{58}$$

The problem with digitizing an input excitation history was to determine the appropriate sampling interval Δt. On the one hand, sampling at points which are too close together yielded correlated and highly redundant data and increased the cost of computation. On the other hand, sampling at points which are too far apart lead to confusion between the high and low frequency components in the data. If the discrete sampling representation was not accurate, potential error or aliasing arose.

The proper sampling rate must be at least twice the frequency of the highest frequency component in the excitation history being sampled and is referred to as the Nyquist sampling criteria (Bendat and Piersol, 1971). The highest frequency component which can be defined at a rate of $(1/\Delta t)$ samples per second is given by $f = 1/(2\Delta t)$ Hertz (Hz) and is called the Nyquist frequency f_c. Since it takes two points per cycle to uniquely define a sinusoid of given amplitude and frequency, any frequency components below or at the Nyquist frequency are correctly defined. In turn, this implied that the time-domain response is obtained from the transformation of the frequency-domain response since the time increment is the reciprocal of the frequency increment

$$\Delta t = 1/\Delta f \qquad [59]$$

The system natural frequency was 10 Hz. Thus, the Nyquist frequency was equal to or greater than 10 Hz so that the time-domain response was computed correctly from the frequency-domain analysis. For this particular set of system parameters, the appropriate sampling interval was 0.05 sec. The force excitation input history was represented as 160 points. From Equation [58], the variable PWR was equal to 7.322 which was not an integer. Thus, to meet the Nyquist sampling criteria, the variable PWR was equal to 8 or greater.

To illustrate the Nyquist sampling criteria, five values for the variable PWR were selected. The time-domain force excitation history, shown in Figure 6, was digitized at each respective sampling rate and transformed into the frequency-domain. With the transformed force history, the system displacement and acceleration response were computed from Equations [56] and [57], respectively, which were then transformed to the time-domain responses. The transformed system displacement and acceleration results were computed from equations [52] - [55]. This comparison was made by graphical display in which the transformed results were superimposed on the theoretical results.

For the first test case, PWR was equal to four, and 16 points represented the force excitation history. The Nyquist sampling interval rate, computed by the program, was 0.5 sec, and the Nyquist frequency was equal to 1.0 Hz. The Nyquist sampling criteria were not meet because the Nyquist frequency was less than the system natural frequency.

The program digitized the force excitation history and computed the Fourier transform of the digitized history with the FFT subroutine. The program then computed the Inverse Fourier transform on the frequency-domain digitized history as a check. The frequency-domain representation of the excitation history is shown in Figure 7 while the transformed frequency-domain representation of the excitation history is shown in Figure 8.

The program computed the frequency-domain system displacement response by multiplying the system transfer function by the frequency-domain representation of the force excitation history, as expressed in Equation [56]. The frequency-domain system acceleration response was computed by multiplying the frequency-domain system displacement response by s^2 or $(j\omega)^2$, as expressed in Equation [57]. The Inverse Fourier transform of the frequency-domain displacement and acceleration responses yielded the time-domain system displacement and acceleration responses, respectively. The displacement of the system mass obtained from the frequency-domain response method was superimposed on the theoretically exact displacement of the system mass, as shown in Figure

9. The acceleration of the system mass obtained from the frequency-domain response method was superimposed on the theoretically exact acceleration of the system mass, as shown in Figure 10. It was observed from Figures 9 and 10 that the Nyquist sampling criteria were not satisfied.

For the second test case, PWR was equal to six, and the force excitation history was represented by 64 points. The Nyquist sampling interval rate was 0.125 sec, and the Nyquist frequency was 4.0 hz. Again, the Nyquist sampling criteria were not satisfied because the Nyquist frequency was less than the system natural frequency.

The frequency-domain representation of the force excitation history is shown in Figure 11 while the transformed frequency representation of the force excitation history to the time-domain is shown in Figure 12. The displacement of the system as a function of time obtained by the frequency response method and the theoretically exact displacement of the system mass defined by Equations [52]-[53] are shown in Figure 13. The acceleration of the system as a function of time obtained by the frequency response method and the theoretically exact acceleration of the system mass defined by Equations [54] - [55] are shown in Figure 14. Again, the Nyquist sampling criteria were not satisfied for the frequency response method.

For the third case, PWR was equal to seven, and 128 points were used to digitize the force excitation history. The Nyquist sampling interval rate was equal to 0.0625 sec, and the Nyquist frequency was equal to 8.0 Hz. Again, the Nyquist sampling rate criteria were not satisfied because the Nyquist frequency was less than the system natural frequency.

The frequency-domain representation of the force excitation history is shown in Figure 15 while the transformed frequency representation of the force excitation history to the time-domain is shown in Figure 16. The displacement of the system mass as a function of time obtained by the frequency response method and the theoretically exact displacement of the system mass defined by Equations [52] - [53] are shown in Figure 17. The acceleration of the system mass as a function of time obtained by the frequency response method and the theoretically exact acceleration of the system mass defined by Equations [54] - [55] are shown in Figure 18. The results from the frequency response method and the theoretically exact system equations were approaching each other. The Nyquist sampling criteria were not satisfied.

For the fourth case, PWR was equal to eight, and 256 points were used to digitize the force excitation history. The Nyquist sampling interval rate was 0.0312 sec, and the Nyquist frequency was equal to 16.0 Hz. The Nyquist sampling rate criteria were satisfied.

The frequency-domain representation of the force excitation history is shown in Figure 19 while the transformed frequency-domain representation of the force excitation history to the time-domain is shown in Figure 20. The displacement of the system mass as a function of time obtained by the frequency response method and the theoretically exact displacement of the system mass defined by Equations [52] - [53] are shown in Figure 21. The acceleration of the system mass as a function of time obtained by the frequency response method and the theoretically exact acceleration of the system mass defined by Equations [54]-[55] are shown in Figure 22. It was observed that the results from the frequency-response method and the theoretically exact system equations correlated with each other.

The displacement results from the frequency-response method and the theoretically exact system equations are listed as the variables X and DISP, respectively in Appendix D. The acceleration results from the frequency-response method and the theoretically exact system equations are also listed as the variables XDD and ACCL, respectively. The numerical results for the transformed frequency-domain and the digitized time-domain representations of the force excitation history are listed as the variables FM and FT, respectively.

For the fifth and final case, PWR was equal to nine, and 512 points were used to digitize the force excitation history. The Nyquist sampling interval rate was equal to 0.0156 sec, and the Nyquist frequency was equal to 32.0 Hz. Again, the Nyquist sampling criteria were satisfied.

The frequency-domain representation of the force excitation history is shown in Figure 23 while the transformed frequency-domain representation of the force excitation history to the time-domain is shown in Figure 24. The displacement of the system mass obtained by the frequency-response method and the theoretically exact displacement of the system mass defined by Equations [52]-[53] are shown in Figure 25. The acceleration of the system mass obtained by the frequency-response method and the theoretically exact acceleration of the system mass defined by Equations [54]-[55] are shown in Figure 26. Again, the results from the frequency-response method and the theoretically exact system equations correleated with each other. Thus, the Nyquist sampling criteria were satisfied.

<u>Example 2</u>

A two degree-of-freedom system subjected to a general-forcing function is shown in Figure 27. The system equations of motion were

$$m\ddot{y}_1 + c(\dot{y}_1 - \dot{y}_2) + 2ky_1 - ky_2 = F(t) \qquad [60]$$

$$m\ddot{y}_2 + c(\dot{y}_2 - \dot{y}_1) - 2ky_2 - ky_1 = 0 \qquad [61]$$

Using the Laplace transform method, the s-domain transformation of Equations [60]-[61] was in matrix form

$$\left[\begin{array}{c|c} (ms^2 + cs + 2k) & -(cs + k) \\ \hline -(cs + k) & (ms^2 + cs + 2k) \end{array}\right] \left\{\begin{array}{c} y_1(s) \\ \hline y_2(s) \end{array}\right\} = \left\{\begin{array}{c} F(s) \\ \hline 0 \end{array}\right\} \qquad [62]$$

Applying Cramer's rule to Equation [62], the system transfer functions were

$$\frac{y_1(s)}{F(s)} = \frac{(ms^2 + cs + 2k)}{\text{Det}} \qquad [63]$$

$$\frac{y_2(s)}{F(s)} = \frac{-(cs + k)}{\text{Det}} \qquad [64]$$

where

$$Det = (ms^2 + cs + 2k)^2 - (cs + k)^2$$

For this example, the numerical values for the system parameters, the listing of the IMP statements for the system, and the results of the IMP analyses are found in Appendix E (Uicker, 1974). The IMP listing included: (1) the statements to define the system topology, the force excitation, and the points of interest; (2) the numerical data statements to define the link coordinate frames, the weight and inertias for each link, the spring and damper parameters, the location for each point of interest, and the magnitude of the force excitation; and (3) the statements to request the static equilibrium position, the system natural frequencies, the resultant forces acting within each constraint joint, and the system transfer function denominators and numerators for an input force excitation.

The results for the requested IMP analyses were printed immediately after the IMP statement listing in the following format:

1. The first part of the output was the title providing general information about the system and the type of analysis performed.
2. The next information was the system degrees-of-freedom which was equal to two. The FGC variables selected by IMP were the joints PRM2 and PRM3. The next information was the system natural and damped frequencies. The natural frequencies were 10.000 and 17.321 rad/sec, respectively while the damped frequencies were 10.000 and 16.851 rad/sec, respectively. The first mode of vibration corresponded to the motion at

joint PRM2 while the second mode of vibration corresponded to the motion at joint PRM3.

3. The next information was the location of the constraint joints and the points of interest at the system static equilibrium position.

4. The next information was the resultant forces and torques acting within the constraint joints at the system static equilibrium position.

5. The final information was the system transfer function ratios for the steady-state vibratory displacement of the points PNTA and PNTC, and the joints PRM1, PRM2, and PRM3, resulting from the input force excitation EXCT. The eigenvectors, the eigenvalues or complex common denominators, and the complex modal numerators for the desired points and joints were listed, respectively.

For example, the set of position transfer functions for the design variable PNTA expressed in the form of Equation [35] was

$$\frac{X(s)}{EXCT(s)} = \frac{0.0}{s-(-4.0 + j16.85)} + \frac{0.0}{s-(-4.0 - j16.85)} + \frac{0.0}{s-(0.0 + j10.0)} + \frac{0.0}{s-(0.0 - j10.0)} \qquad [65]$$

$$\frac{Y(s)}{EXCT(s)} = \frac{(-j0.0297)}{s-(-4.0 + j16.85)} + \frac{(j0.0297)}{s-(-4.0 - j16.85)} + \frac{(-j0.05)}{s-(0.0 + j10.0)} + \frac{(j0.05)}{s-(0.0 - j10.0)} \qquad [66]$$

$$\frac{Z(s)}{EXCT(s)} = \frac{0.0}{s-(-4.0 + j16.85)} + \frac{0.0}{s-(-4.0 - j16.85)} + \frac{0.0}{s-(0.0 + j10.0)} + \frac{0.0}{s-(0.0 - j10.0)} \qquad [67]$$

The vibration attenuation occurred only in the global y-direction. Equation [66] written in concise polynomial form was

$$\frac{Y(s)}{EXCT(s)} = \frac{0.5s^2 + 2s + 100}{0.25s^4 + 2s^3 + 100s^2 + 200s + 7500} \qquad [68]$$

For the design variable PNTC, the vibration attenuation occurred only in the global y-direction. The transfer function for the steady-state vibratory displacement for PNTC in concise polynomial form was

$$\frac{Y(s)}{EXCT(s)} = \frac{-(2s + 50)}{0.25s^4 + 2s^3 + 100s^2 + 200s + 7500} \qquad [69]$$

The position transfer functions in concise polynomial form for the vibratory displacement of prismatic joints PRM1, PRM2, PRM3 were

$$\frac{Y(s)_{PRM1}}{EXCT(s)} = \frac{0.5s^2 + 2s + 100}{0.25s^4 + 2s^3 + 100s^2 + 200s + 7500} \qquad [70]$$

$$\frac{\gamma(s)_{PRM2}}{EXCT(s)} = \frac{0.5s^2 + 2s + 100}{0.25s^2 + 2s^3 + 100s^2 + 200s + 7500} \qquad [71]$$

$$\frac{\gamma(s)_{PRM3}}{EXCT(s)} = \frac{-(2s + 50)}{0.25s^2 + 2s^3 + 100s^2 + 200s + 7500} \qquad [72]$$

The IMP transfer function procedure was verifed by comparing the computed transfer functions with the theoretical transfer functions. The numerical values for the system parameters in Appendix E were substituted into the expressions in Equations [63]-[64]. It was found that the transfer functions calculated from Equation [63] checked with the algebraic expressions in Equations [68], [70], and [71] while the transfer function calculated from Equation [64] checked with the algebraic expressions in Equations [69] and [72].

Example 3

This example was the same system described in Example 2 except that the system was subjected to two general-forcing functions. The first force excitation EXCT acted at a point on link LNK1 while the second force excitation PRM3 acted within the prismatic joint PRM3. This example illustrated that the system may be subjected to more than one input force excitation.

The system equations of motion were

$$m\ddot{y}_1 + c(\dot{y}_1 - \dot{y}_2) + 2ky_1 - ky_2 = F_1(t) \quad [73]$$

$$m\ddot{y}_2 + c(\dot{y}_2 - \dot{y}_1) + 2ky_2 - ky_1 = F_2(t) \quad [74]$$

Using the Laplace transform method, the s-domain transformation of Equations [73]-[74] was in matrix form

$$\left[\begin{array}{c|c} (ms^2 + cs + 2k) & -(cs + k) \\ \hline -(cs + k) & (ms^2 + cs + 2k) \end{array}\right] \left\{\begin{array}{c} y_1(s) \\ \hline y_2(s) \end{array}\right\} = \left\{\begin{array}{c} F_1(s) \\ \hline F_2(s) \end{array}\right\} \quad [75]$$

where F_1 corresponded to F_{EXCT} and F_2 corresponded to F_{PRM3}. Applying Cramer's rule to Equation [75], the system transfer functions were

$$\frac{y_1(s)}{F_1(s)} = \frac{(ms^2 + cs + 2k)}{\mathrm{Det}} \quad [76]$$

$$\frac{y_2(s)}{F_1(s)} = \frac{-(cs + k)}{\mathrm{Det}} \quad [77]$$

$$\frac{y_1(s)}{F_2(s)} = \frac{-(cs + k)}{\mathrm{Det}} \quad [78]$$

$$\frac{y_2(s)}{F_2(s)} = \frac{(ms^2 + cs + 2k)}{Det} \qquad [79]$$

where

$$Det = (ms^2 + cs + 2k)^2 - (cs + k)^2$$

The results of the requested IMP analyses followed the same format outlined in Example 2. In this example, the system transfer functions were requested for the steady-state vibratory displacement of joints PRM1, PRM2 and PRM3, and points PNTA and PNTC. The eigenvectors and eigenvalues for the system were equal to the ones in Example 2.

The IMP computed complex modal numerators for the desired joints and points with repect to the force excitation EXCT were the same numerators as in Example 2. Thus, the position transfer functions for joints PRM1, PRM2, and PRM3, and points PNTA and PNTC were the same algebraic expressions as in Equations [68] - [72], respectively.

The position transfer functions for the vibratory displacement of points PNTA and PNTC, and joints PRM1, PRM2, and PRM3 in concise polynomial form were

$$\frac{V(s)_{PNTA}}{PRM3(s)} = \frac{-(2.0s + 50)}{0.25s^4 + 2.0s^3 + 100.0s^2 + 200.0s + 7500.0} \qquad [80]$$

$$\frac{V(s)_{PNTC}}{PRM3(s)} = \frac{0.5s^2 + 2.0s + 100}{0.25s^4 + 2.0s^3 + 100.0s^2 + 200.0s + 7500.0} \qquad [81]$$

$$\frac{V(s)_{PRM1}}{PRM3(s)} = \frac{-(2.0s + 50)}{0.25s^4 + 2.0s^3 + 100.0s^2 + 200.0s + 7500.0} \qquad [82]$$

$$\frac{V(s)_{PRM2}}{PRM3(s)} = \frac{0.5s^2 + 2.0s + 100.0}{0.25s^4 + 2.0s^3 + 100.0s^2 + 200.0s + 7500.0} \qquad [83]$$

$$\frac{V(s)_{PRM3}}{PRM3(s)} = \frac{0.5s^2 + 2.0s + 100.0}{0.25s^4 + 2.0s^3 + 100.0s^2 + 200.0s + 7500.0} \qquad [84]$$

The IMP transfer function procedure was verified by comparing the computed transfer functions with the theoretical transfer function expressions. The numerical values for the system parameters from Appendix F were substituted into Equations [76] - [79]. It was found:

1. The transfer function calculated from Equation [76] checked with the algebraic expressions in Equations [68], [70], and [71];
2. The transfer function calculated from Equation [77] checked with the algebraic expressions in Equations [69] and [72];
3. The transfer function calculated from Equation [78] checked with the algebraic expressions in Equations [80] and [82];

and

4. The transfer function calculated from Equation [79] checked with the algebraic expressions in Equations [81], [83], and [84].

Example 4

A simple pendulum system subjected to a torque excitation acting at joint RLT1 is shown in Figure 28. For this example, two fictitious or dummy links were added to form a four-bar mechanism because IMP can only handle closed-loop mechanisms. The linearized system equation of motion was

$$I\theta + mgl/2\theta = T(t) \qquad [85]$$

where

$$I = \frac{mL^2}{12} + \frac{mL^2}{4} = \frac{mL^2}{3}$$

Using the Laplace transform method, the s-domain transformation of Equation [85] was

$$Is^2\theta(s) + mgl/2\theta(s) = T(s) \qquad [86]$$

The system transfer function was

$$\frac{\theta(s)}{T(s)} = \frac{1}{Is^2 + mg\frac{L}{2}} \qquad [87]$$

The results of the requested IMP analyses followed the same format as outlined in EXAMPLE 2. The FGC variables selected by IMP was joint RLT3. The natural frequency derived from Equation [85] was

$$\omega = \sqrt{\frac{3g}{2L}} \qquad [88]$$

The numerical values from Appendix G were substituted into Equation [88]. It was found that the natural frequency computed by IMP checked with the theoretical value which was 7.609 rad/sec.

The system transfer functions were requested for the steady-state vibratory angular rotation of joints RLT1, RLT2, RLT3, and RLT4, and the displacement of point PNTA. The transfer functions in concise polynomial form were

$$\frac{\theta(s)_{RLT1}}{T(s)} = \frac{-1.0}{0.863s^2 + 50.0} \qquad [89]$$

$$\frac{u(s)_{PNTA}}{T(s)} = \frac{-5.0}{0.863s^2 + 50.0} \qquad [90]$$

The transfer functions for joints RLT2, RLT3, and RLT4 were the same as the algebraic expression in Equation [89].

The IMP transfer function procedure was verified by comparing the computed transfer functions with the theoretical transfer functions. The numerical values for the system parameters in Appendix G were substituted into Equation [87]. It was found that the transfer function calculated from Equation [87] checked with the algebraic expression in Equation [89].

<u>Example 5</u>

A compound pendulum system subjected to a torque excitation acting within the joint RLT1 is shown in Figure 29. The two pendulums were connected by a flexible coupler containing a translational spring and damper. The system equations of motion were

$$\frac{m_1 L^2}{3}\ddot{\theta}_1 + \frac{cL^2}{4}(\dot{\theta}_1 - \dot{\theta}_2) + m_1 g\frac{L}{2}\theta_1 + \frac{kL^2}{4}(\theta_1 - \theta_2) = T(t) \qquad [91]$$

$$\frac{m_2 L^2}{3}\ddot{\theta}_2 + \frac{cL^2}{4}(\dot{\theta}_2 - \dot{\theta}_1) + m_2 g\frac{L}{2}\theta_2 + \frac{kL^2}{4}(\theta_2 - \theta_1) = 0 \qquad [92]$$

Using the Laplace transform method, the s-domain transformation of Equations [91] and [92] was in matrix form

$$\left[\begin{array}{c|c} m\frac{L^2}{3}s^2 + \frac{cL^2 s}{4} + mg\frac{L}{2} + \frac{kL^2}{4} & \frac{cL^2 s}{4} + \frac{kL^2}{4} \\ \hline -\frac{cL^2 s}{4} + \frac{kL^2}{4} & \frac{mL^2 s^2}{3} + \frac{cL^2}{4} + \frac{mgL}{2} + \frac{kL^2}{4} \end{array}\right] \left\{\begin{array}{c} \theta_1(s) \\ \hline \theta_2(s) \end{array}\right\} = \left\{\begin{array}{c} T(s) \\ \hline 0 \end{array}\right\} \qquad [93]$$

Applying Cramer's rule to Equation [93], the system transfer functions were

$$\frac{\Theta_1(s)}{T(s)} = \frac{\frac{mL^2}{3} s^2 + \frac{cL^2}{4} s + \frac{mgL}{2} + \frac{kL^2}{4}}{Det} \qquad [94]$$

$$\frac{\Theta_2(s)}{T(s)} = \frac{-\frac{cL^2 s}{4} + \frac{kL^2}{4}}{Det} \qquad [95]$$

where

$$Det = \left(\frac{mL^2}{3} s^2 + \frac{cL^2}{4} s + \frac{mgL}{2} + \frac{kL^2}{4}\right)^2 - \left(\frac{cL^2}{4} + \frac{kL^2}{4}\right)^2$$

For this example, the numerical values for the system parameters, the listing of the IMP statements for the system, and the results for the IMP analyses are found in Appendix H.

The results of the requested IMP analyses followed the format outlined in Example 2. The FGC variables selected by IMP were joints RLT3 and PRM1. The system natural frequencies computed by IMP were 7.609 and 54.341 rad/sec. For this pendulum system, the expressions for the natural frequencies derived from Equations [91] and [92] were

$$\omega_{1,2} = \pm j\sqrt{\frac{3g}{4L}} \qquad [96]$$

$$\omega_{3,4} = \pm j\sqrt{\frac{3g}{4L} + \frac{3k}{2m}} \qquad [97]$$

The numerical values for the system parameters in Appendix H were substituted into Equations [96] and [97]. The system natural frequencies were

$$\omega_{1,2} = \pm\ j(7.609)\ \text{rad/sec} \qquad [98]$$

$$\omega_{3,4} = \pm\ j(54.341)\ \text{rad/sec} \qquad [99]$$

The natural frequencies computed by IMP checked with the natural frequencies in Equations [98] and [99] calculated from Equations [96] and [97].

The system transfer functions were computed for the steady-state vibratory angular rotation of joints RLT1 and RLT4. These transfer functions in concise polynomial form were

$$\frac{\Theta(s)_{RLT1}}{T(s)} = \frac{0.863s^2 + 2.500s + 1300.000}{0.745s^4 + 4.316s^3 + 2244.750s^2 + 250.000s + 127505.398} \qquad [100]$$

$$\frac{\Theta(s)_{RLT4}}{T(s)} = \frac{-2.500s + 1250.000}{0.745s^4 + 4.316s^3 + 2244.75s^2 + 250.000s + 127505.398} \qquad [101]$$

The IMP transfer function procedure was verified against the transfer functions derived from the system equations of motion. The numerical values for the system parameters in Appendix H were substituted into Equations [94] and [95]. It was found that the transfer functions calculated from Equations [94] and [95] checked with the algebraic expressions in Equations [100] and [101].

Example 6

A single degree-of-freedom system excited by three translational base acceleration inputs along the global X, Y, and Z axes and three rotational base acceleration inputs about the global X, Y, and Z axes is shown in Figure 30.

In terms of the global coordinate system, the base motion inputs were: (1) the inertial accelerations $\ddot{X}$, $\ddot{Y}$, $\ddot{Z}$, $\ddot{\theta}_x$, $\ddot{\theta}_y$, and $\ddot{\theta}_z$; (2) the damping velocities $\dot{X}$, $\dot{Y}$, $\dot{Z}$, $\dot{\theta}_x$, $\dot{\theta}_y$, and $\dot{\theta}_z$; (3) the stiffness displacements X, Y, Z, θ_x, θ_y, and θ_z. The absolute displacement of the points of interest on the links were U, V, and W along the global X, Y, and Z axes, respectively.

The system equation of motion was

$$m(\ddot{y} + \ddot{y}_B) + c\dot{y} + ky = 0 \tag{102}$$

Using the Laplace transform method, the s-domain transformation of Equation [102] was

$$ms^2y(s) + csy(s) + ky(s) = - ms^2y_B(s) \tag{103}$$

while the system transfer function was

$$\frac{y(s)}{y_B(s)} = \frac{-ms^2}{ms^2 + cs + k} \tag{104}$$

The results for the requested IMP analyses followed the format outlined in Example 2. The FGC variable selected by IMP was the joint $\dot{\text{PM}}$ while the SGC variables specified by the user were joints P1, R1, P2, R2, P3, and R3.

The position transfer function for the relative displacement for the design variable SEAT was

$$\frac{V(s)}{s^2 Y(s)} = \left[\frac{(j0.05)}{s-(-0.05 + j9.9875)} + \frac{(-j0.05)}{s-(-0.05 - j9.9975)}\right] \qquad [105]$$

which can be expressed in concise polynomial form as

$$\frac{V(s)}{Y(s)} = s^2 \left[\frac{-1.0}{1.0s^2 + 1.0s + 100.0}\right] \qquad [106]$$

The vibration attenuation was only in the global Y-direction because the vibration isolation was only in the global Y-direction. The tangential accelerations for the six acceleration components were accounted for and were expressed in Equations [79] and [81].

The absolute displacements for the design variable SEAT were expressed as

$$u(s) = x(s) - 5.0\theta_z(s) \qquad [107]$$

$$v(s) = \left[\frac{-1.0}{1.0s^2 + 1.05 + 100.0}\right] s^2 Y(s) + 1.0\ Y(s) \qquad [108]$$

$$w(s) = 1.0Z(s) + 5.0\theta_x(s) \qquad [109]$$

The absolute accelerations for the design variable (point) SEAT were obtained by multiplying the left- and right-hand sides of Equations [107] - [109] by s^2 and were expressed as

$$s^2u(s) = s^2\ (X(s) - 5.0\theta_z(s)) \tag{110}$$

$$s^2v(s) = s^2\left(\left[\frac{-1.0}{1.0s^2 + 1.0s + 100.0}\right] s^2Y(s) + 1.0\ Y(s)\right) \tag{111}$$

$$s^2w(s) = s^2\ (Z(s) + 5.0\theta_x(s)) \tag{112}$$

The position transfer functions for the absolute displacement for the design variables (joints) PM and P3 were expressed in concise polynomial form as

$$\frac{\gamma(s)_{PM}}{Y(s)} = s^2\left[\frac{-1.0}{1.0s^2 + 1.0s + 100.0}\right] \tag{113}$$

$$\frac{\gamma(s)_{P3}}{Y(s)} = s^2\left[\frac{-1.0}{1.0s^2 + 1.0s + 100.0}\right] \tag{114}$$

respectively. The absolute displacements of joints PM and P3 included the base displacement input component Y(s).

The IMP transfer function procedure was verified against the transfer functions derived from the system equations of motion. The numerical values for the system parameters from Appendix I were substituted into Equation [104]. It was found that the transfer functions calculated from Equation [104] checked with the algebraic expressions in Equations [106], [113], and [114].

This same system was offset along the global X-axis, as shown in Figure 31.

The absolute displacements for the design variable SEAT were expressed as

$$u(s) = X(s) - 5.0\theta_z(s) \qquad [115]$$

$$v(s) = s^2\left[\frac{-1.0}{1.0s^2 + 1.0s + 100.0}\right] Y(s) + s^2\left[\frac{-10.0}{1.0s^2 + 1.0s + 100.0}\right] \theta_z(s) \quad + 1.0\ Y(s) + 10.0\ \theta_z(s) \qquad [116]$$

$$w(s) = Z(s) + 5.0\theta_x(s) - 10.0\theta_y(s) \qquad [117]$$

Equations [115] - [117] were essentially the same as Equations [107] - [109] except for the terms $s^2\theta_z(s)$ and $\theta_z(s)$.

Example 7

A simple pendulum which is constrained by a translational spring and damper is shown in Figure 32. The system was excited by three translational base acceleration inputs along the global X, Y, and Z axes, and three rotational base acceleration inputs about the global X, Y, and Z axes.

The results of the requested IMP analyses followed the format outlined in EXAMPLE 2. The FGC variable selected by IMP was joint PM while the SGC variables specified by the user were joints P1, R1, P2, R2, P3, and R3.

The absolute displacements for the design variable SEAT was expressed as

$$u(s) = X(s) - 5.0\theta_z(s) \qquad [118]$$

$$v(s) = \frac{-20.0s^2\theta_z(s) - 5.6s\,\theta_z(s) - 1440.0\,\theta_z(s) - s^2Y(s) - 0.4sY(s) - 80.0\,Y(s)}{1.0s^2 + 0.165 + 64.0}$$

$$+ 1.0\,Y(s) + 20.0\,\theta_z(s) \qquad [119]$$

$$w(s) = Z(s) - 5.0\theta_x(s) - 20.0\theta_z(s) \qquad [120]$$

By hand calculations, the IMP transfer function results were verified against the transfer functions derived from the system equations of motion.

When only the effect of a base acceleration input , $-\ddot{Y}$, was considered, the system equation of motion was

$$ml^2(\ddot{y} + \ddot{V}) + cl_1{}^2\dot{V} + kl_2V = 0 \qquad [121]$$

The numerical values for the system parameters from Appendix K were substituted into Equation [121] to yield

$$100.0\ddot{V} + 16.0\dot{V} + 6400.0V = -100.0y \qquad [122]$$

or

$$\ddot{V} + 0.16\dot{V} + 64.0V = -\ddot{y} \qquad [123]$$

Using the Laplace transform method, the s-domain transformation of Equation [123] was

$$\frac{V(s)}{Y(s)} = s^2 \left[\frac{-1.0}{1.0s^2 + 0.16s + 64.0} \right] \qquad [124]$$

Equation [124] was equivalent to the term, s Y(s), in Equation [119].

When only the effect of a tangential base acceleration input, $-\Theta_z$ was considered, the system equation of motion was

$$ml^2(\ddot{V} + 20.0\ddot{\theta}_z) + cl_1{}^2\dot{V} + kl_2{}^2V = 0 \qquad [125]$$

The numerical values for the system parameters from Appendix K were substituted into Equation [125]. This equation in simplified form was

$$\ddot{V} + 0.16\dot{V} + 64.0V = -20.0\ddot{\theta}_z \qquad [126]$$

Using the Laplace transform method, the s-domain transformation of Equation [126] was

$$\frac{V(s)}{\theta_z(s)} = s^2 \left[\frac{-20.0}{1.0s^2 + 0.16s + 64.0} \right] \qquad [127]$$

Equation [127] was equivalent to the term, $-20.0s^2\theta_z(s)$, in Equation [119].

The remainder of the numerator terms in Equation [119] were verified in a similar manner.

Example 8

A conventional agricultural wheel tractor, excited by four terrain input excitations, is shown in Figure 33. This example illustrated the IMP transfer function analysis entity, and the capabilities of the post-processor programs. The operator's cab included the seat suspension and the operator platform, and was modelled with five degrees-of-freedom. The tractor chassis included the oscillating front axle and was modelled with five degrees-of-freedom.

For this example, the numerical values for the system parameters, the listing of the IMP statements for the system, and the results of the IMP analyses are found in Appendix L.

The results of the requested IMP analyses followed the same format as outlined in Example 2. The prismatic joints RRSMP, LRSMP, RFSMP, and LFSMP were selected as the SGC variables by the program user. IMP selected ten joints for the FGC variables to correspond to the ten degrees-of-freedom or modes of vibration.

The system transfer function ratio sets were requested for the steady-state acceleration for the point SEAT on the link CAB. The X-, Y-, and Z-component transfer function ratio sets were

$$\frac{s^2 u(s)_{SEAT}}{y_j(s)} = s^2 \left[\sum_{i=1}^{20} \left[\frac{\omega^2 NM_i + \omega NC_i + NK_i}{s-D_i} \right]_X + \sum_{j=1}^{4} BX_j \right] \qquad [128]$$

$$\frac{s^2 v(s)_{SEAT}}{y_j(s)} = s^2 \left[\sum_{i=1}^{20} \left[\frac{\omega^2 NM_i + \omega NC_i + NK_i}{s-D_i} \right]_y + \sum_{j=1}^{4} BY_j \right] \qquad [129]$$

$$\frac{s^2 w(s)_{SEAT}}{y_j(s)} = s^2 \left[\sum_{i=1}^{20} \left[\frac{\omega^2 NM_i + \omega NC_i + NK_i}{s-D_i} \right]_z + \sum_{j=1}^{4} BZ_j \right] \qquad [130]$$

where

$y_1(s) = y_{RRSMP}(s)$;

$y_2(s) = y_{LRSMP}(s)$;

$y_3(s) = y_{RFSMP}(s)$;

$y_4(s) = y_{LFSMP}(s)$.

The mode shapes corresponding to the natural frequencies of the tractor are shown in Figure 34. The listing for the geometric shape data for the tractor and the results from the mode shape computations from Equation [46] are found in Appendix L.

For this example, the frequency response magnitude and phase angle results for the X-, Y-, and Z-components for the point SEAT were computed with respect to the base motion inputs RRSMP, LRSMP, RFSMP, and LFSMP, respectively. The set of magnitude and phase angle results for the four wheel input excitations are shown in Figures 35, 36, 37, and 38, respectively.

For the point SEAT, the system response in the frequency-domain was computed from Equations [C15] - [C17] in Appendix C. The response results for the X-, Y-, and Z-components were computed and are shown in Figure 39. The system response results were transformed from the frequency-domain to the time-domain with the FFT algorithm. The results of the time-domain transformation are shown in Figure 39.

CONCLUSIONS

A methodology was developed for computing the frequency response transfer function ratio sets for an articulated mechanical system. The linearized dynamics technique was applicable to the vibrational analysis of a mechanical system having "small" oscillatory steady-state motion about its static equilibrium position. The analytical technique was incorporated into the generalized mechanical systems simulation program IMP. Coded computer algorithms were developed to use the results from the IMP transfer function entity to provide the system response both in the frequency- and time-domain for selected design variables and the system mode shapes.

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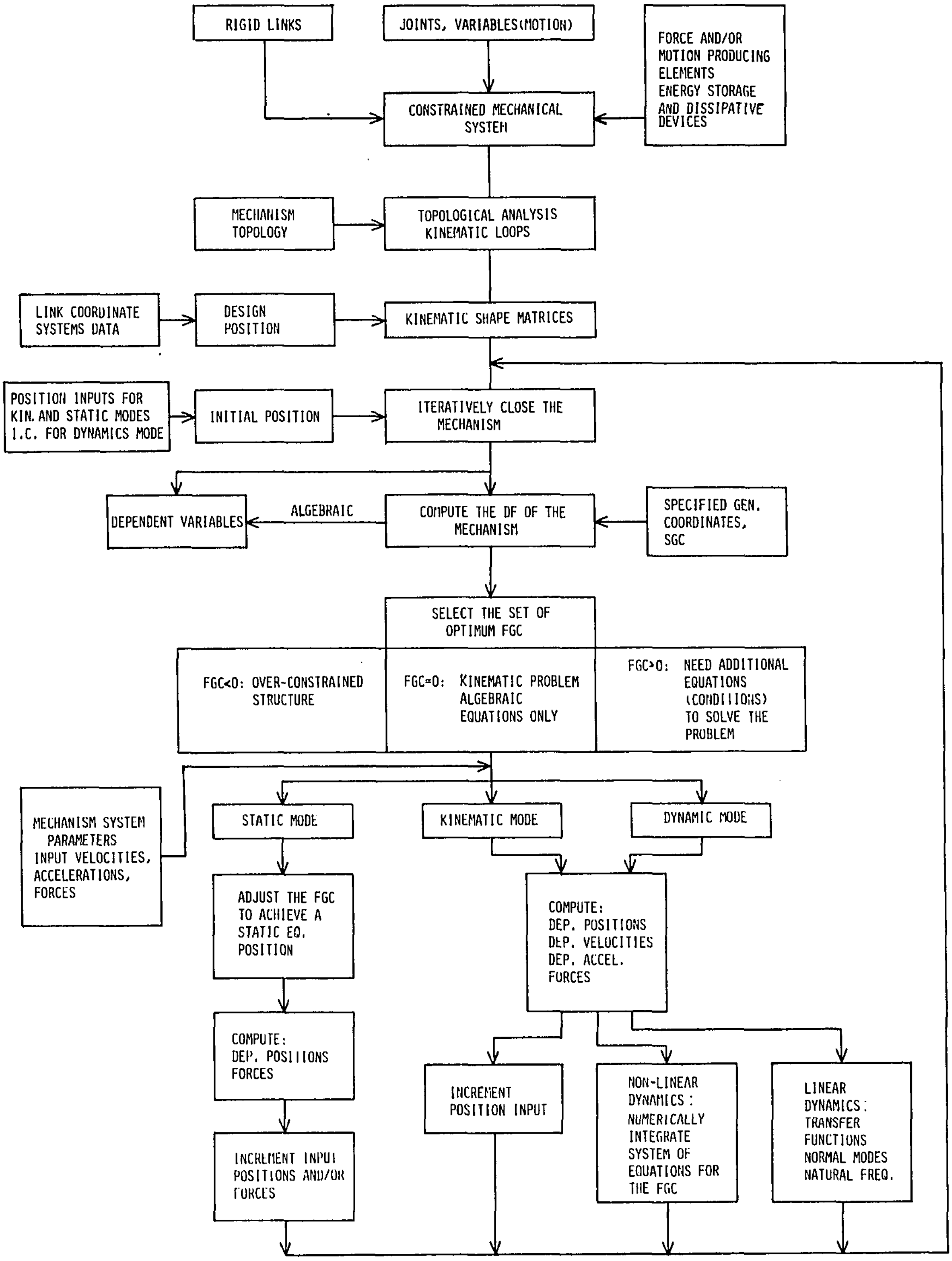


FIGURE 1. IMP system flow process

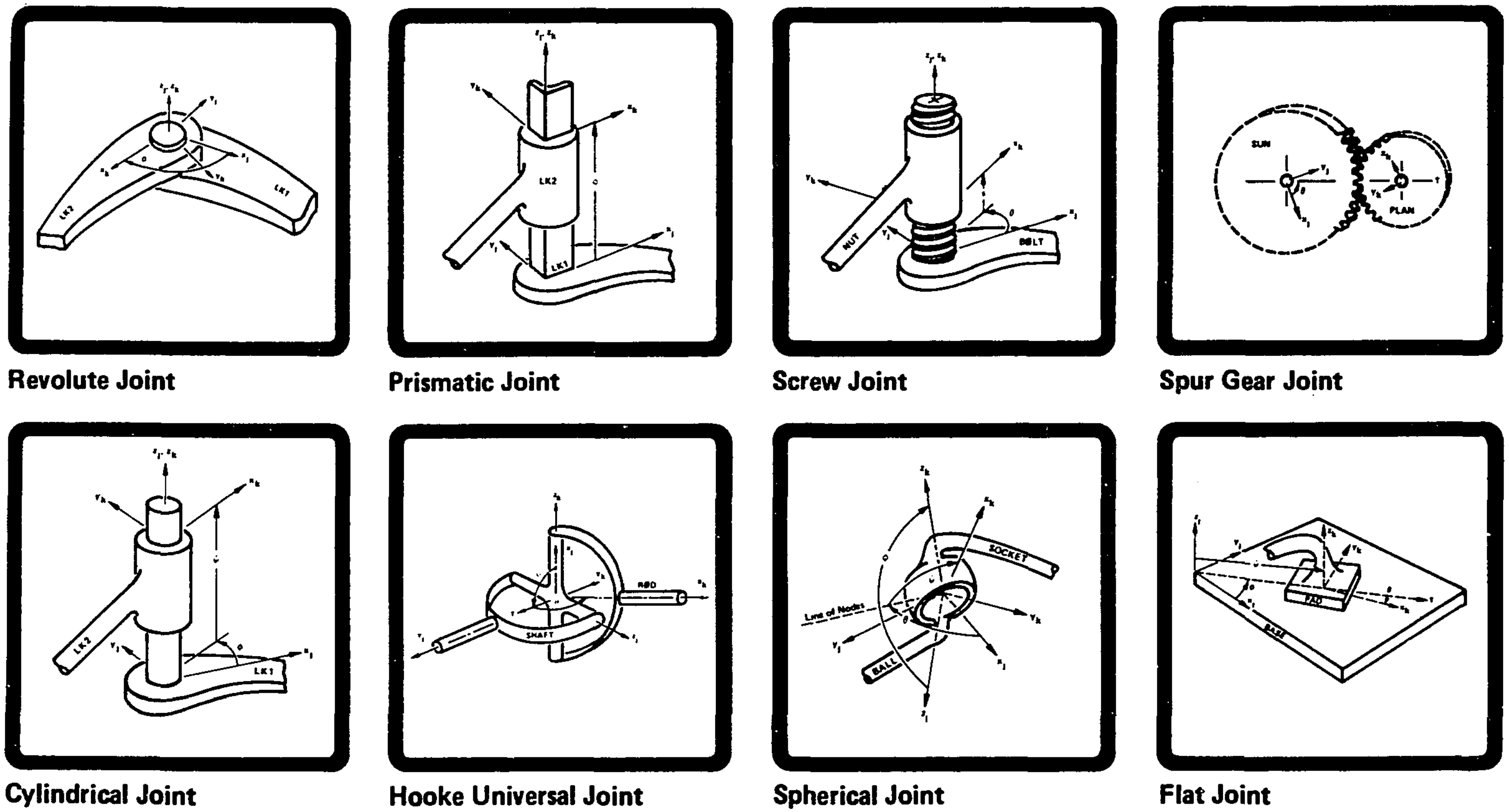


FIGURE 2. IMP system constraint joints

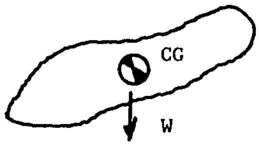
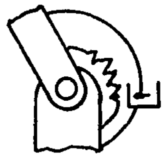
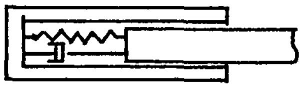
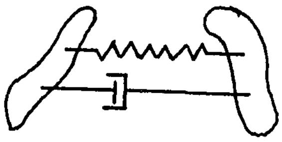
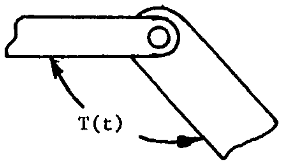
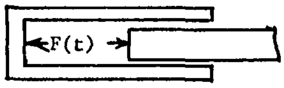
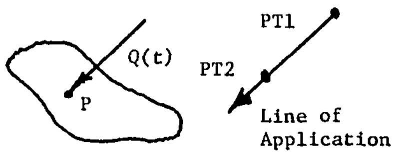
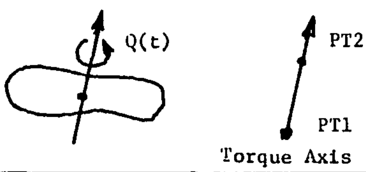
GRAVITY EFFECT	
SPRING AND/OR DAMPING IN A REVOLUTE JOINT	
SPRING AND/OR DAMPING IN A PRISMATIC JOINT	
SPRING AND/OR DAMPING BETWEEN TWO LINKS	
TORQUE WIHTIN A REVOLUTE JOINT	
FORCE WITHIN A PRISMATIC JOINT	
FORCE ON A LINK	
TORQUE ON A LINK	

FIGURE 3. IMP system force-producing elements

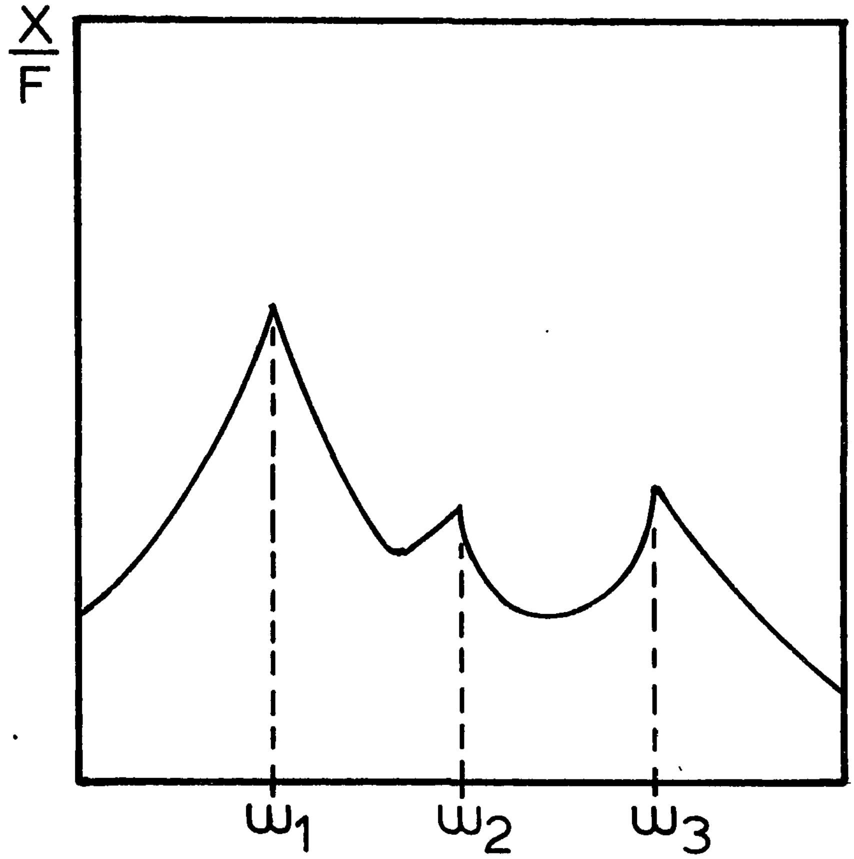


FIGURE 4. Frequency response of a system with three modes of vibration

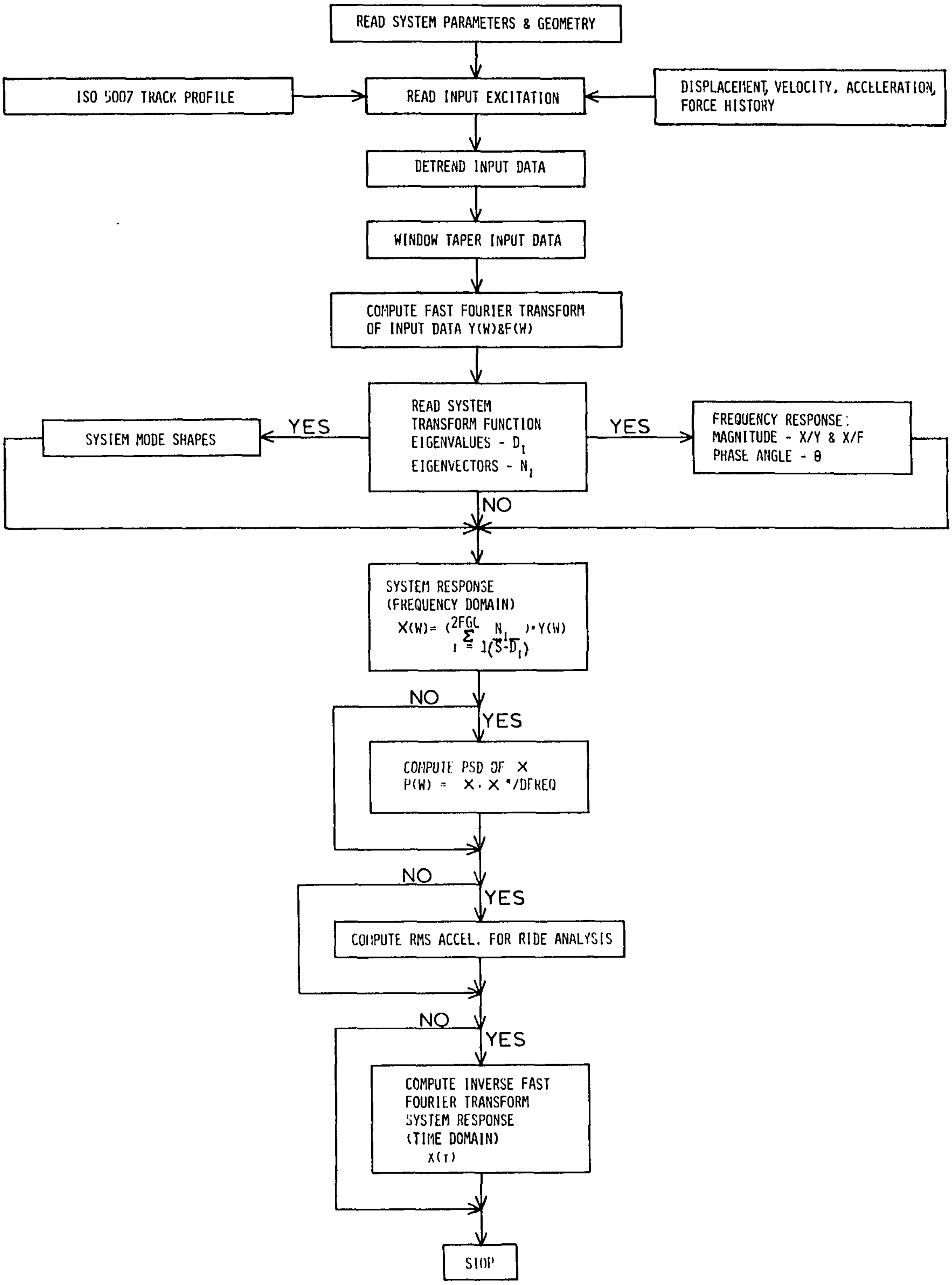


FIGURE 5. IMP system post-processor program flow process

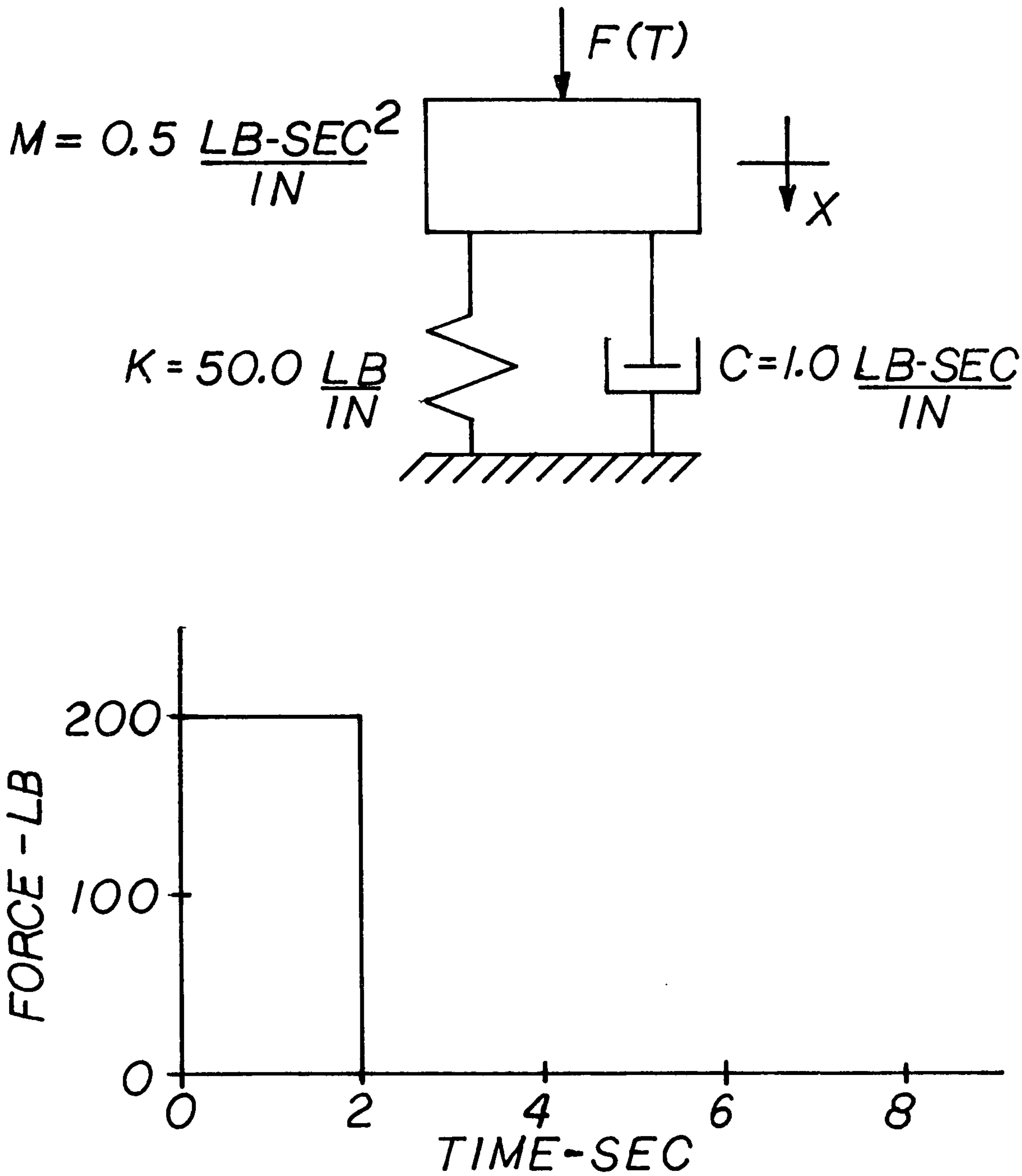


FIGURE 6. Single degree-of-freedom system with step force-input excitation history

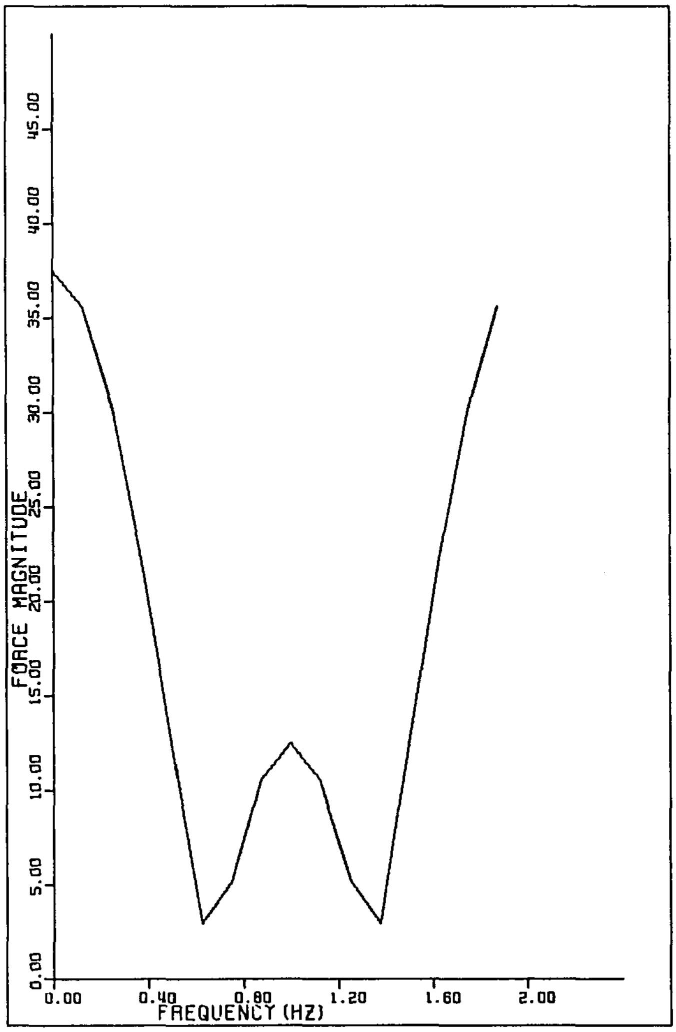


FIGURE 7. Frequency components of the force excitation history (16 points)

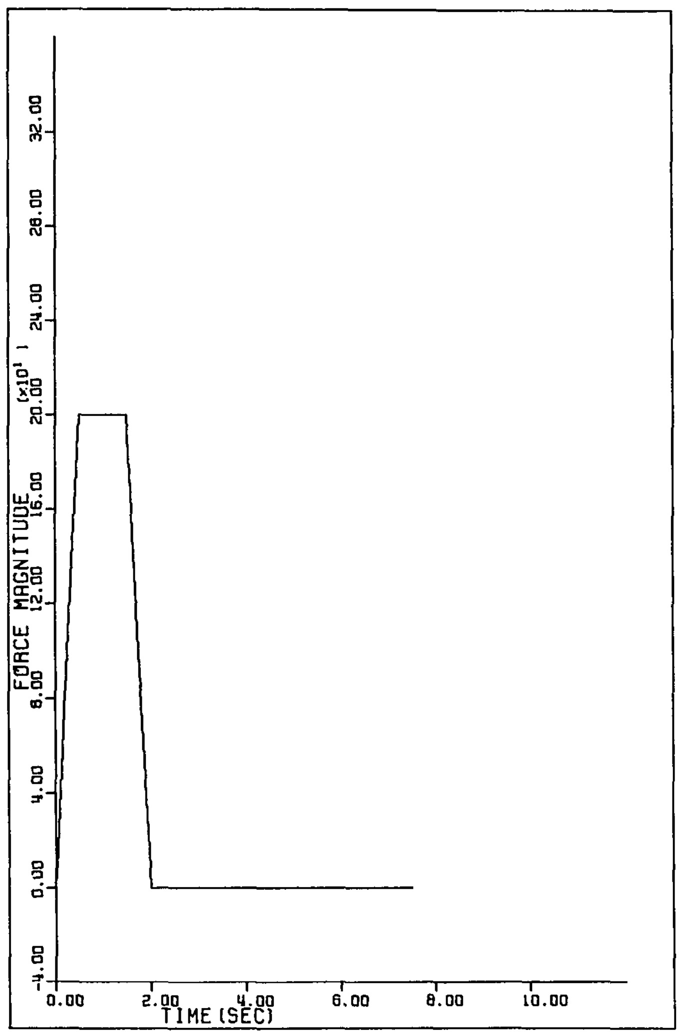


FIGURE 8. Discrete step force excitation history (16 points)

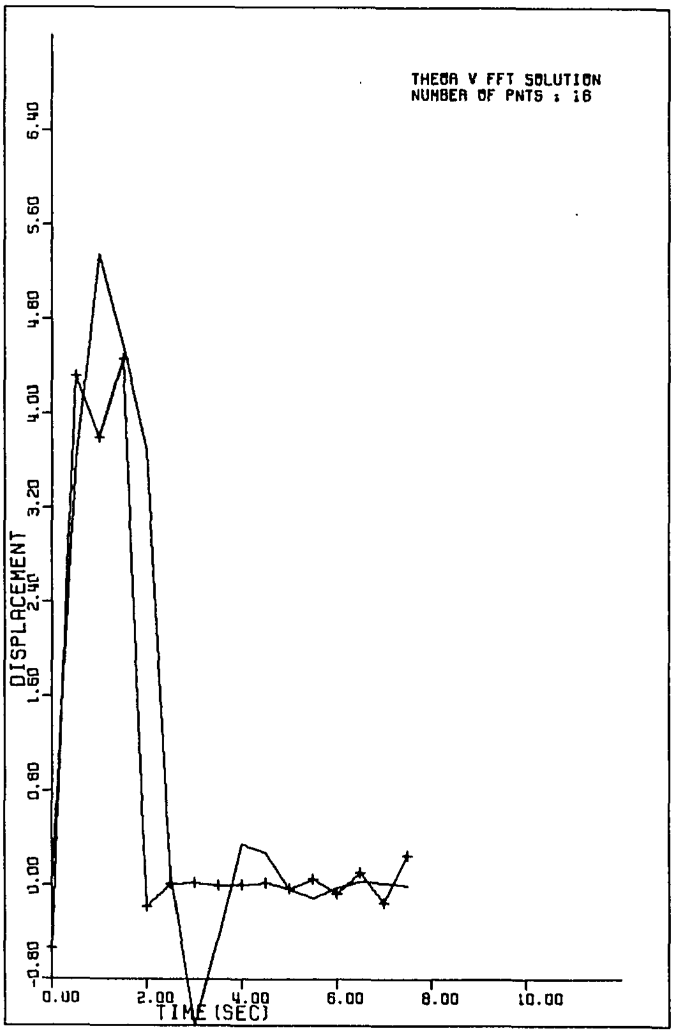


FIGURE 9. System displacement response - theoretical versus transfer function solution (16 points)

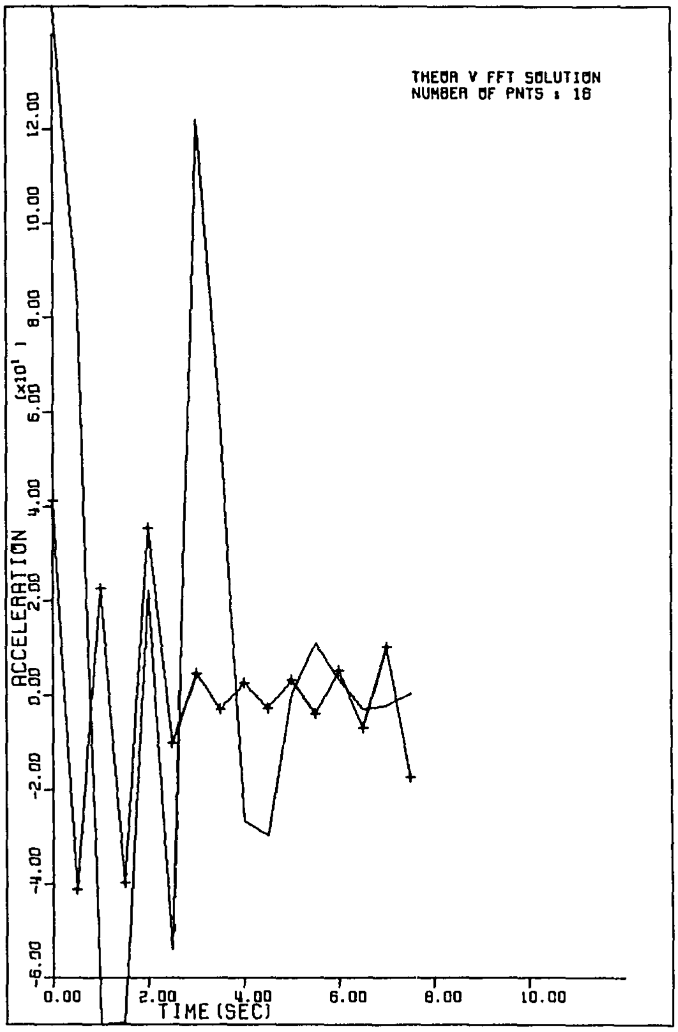


FIGURE 10. System acceleration response - theoretical versus transfer function solution (16 points)

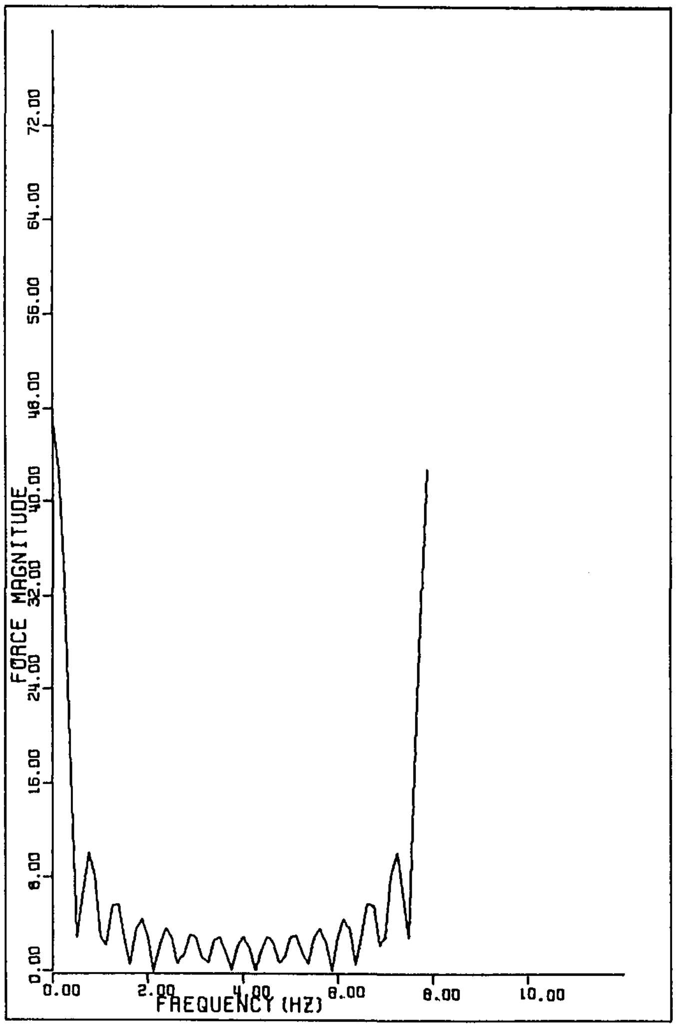


FIGURE 11. Frequency components of the force excitation history (64 points)

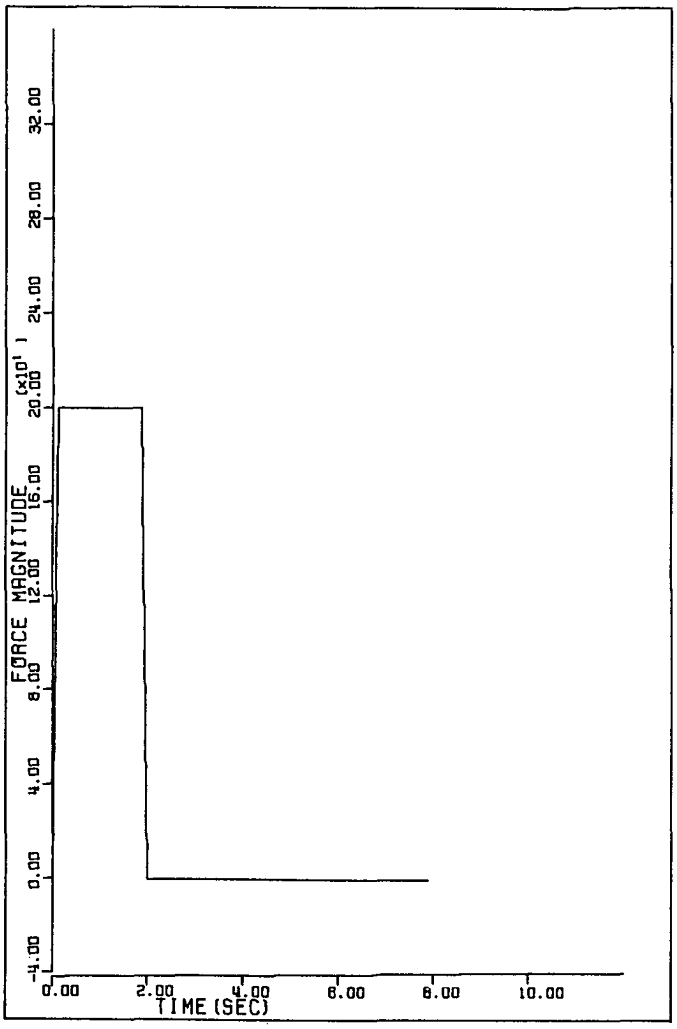


FIGURE 12. Discrete step force excitation history (64 points)

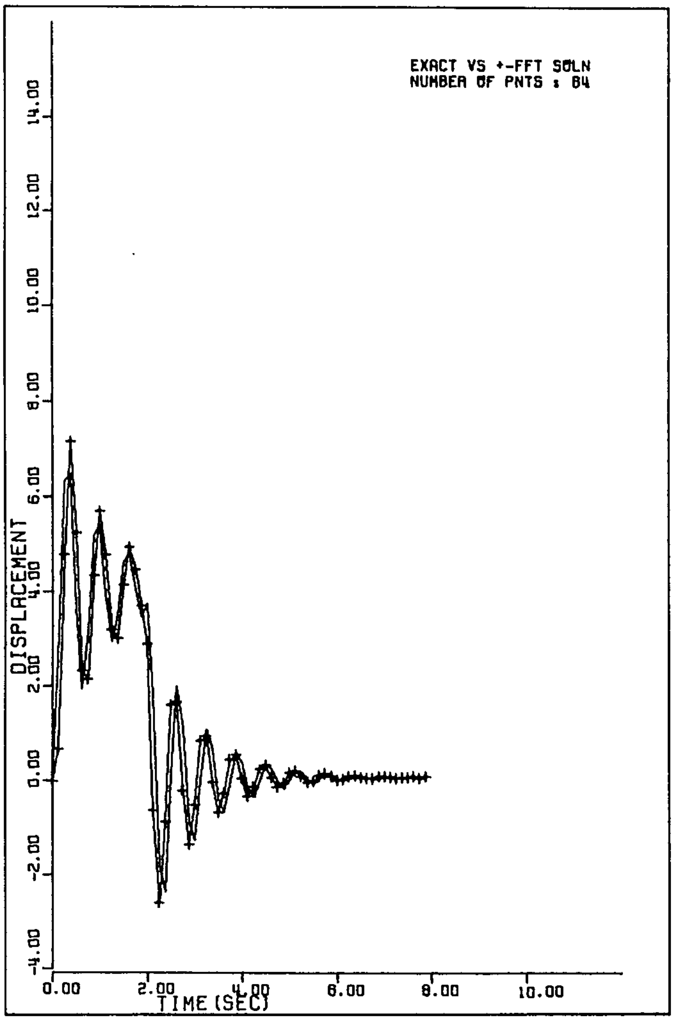


FIGURE 13. System displacement response - theoretical versus transfer funtion solution (64 points)

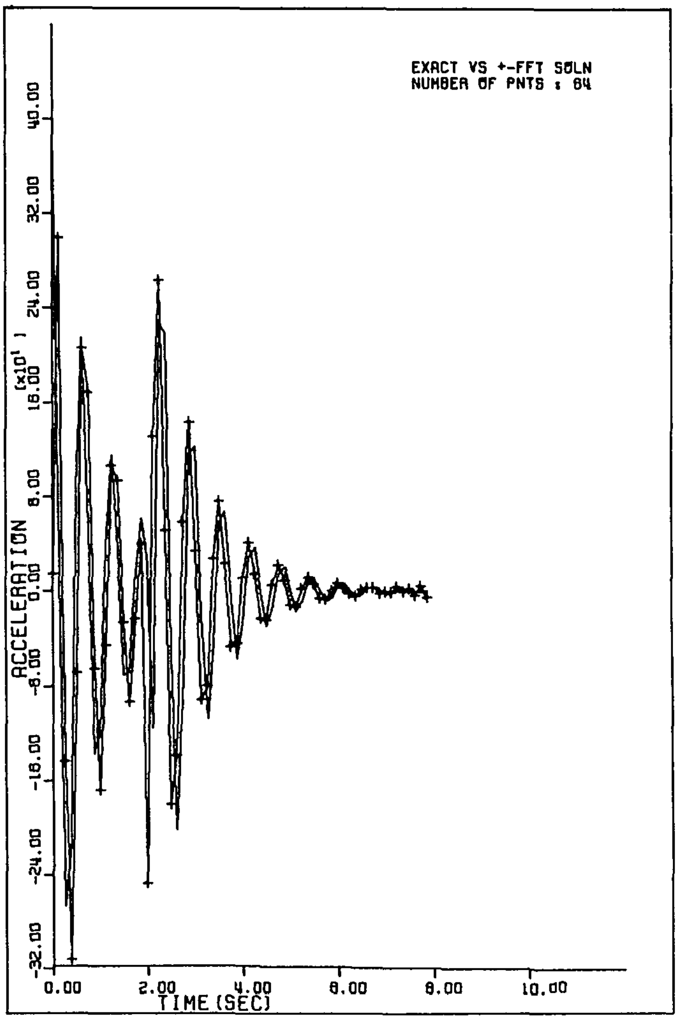


FIGURE 14. System acceleration response - theoretical versus transfer function solution (64 points)

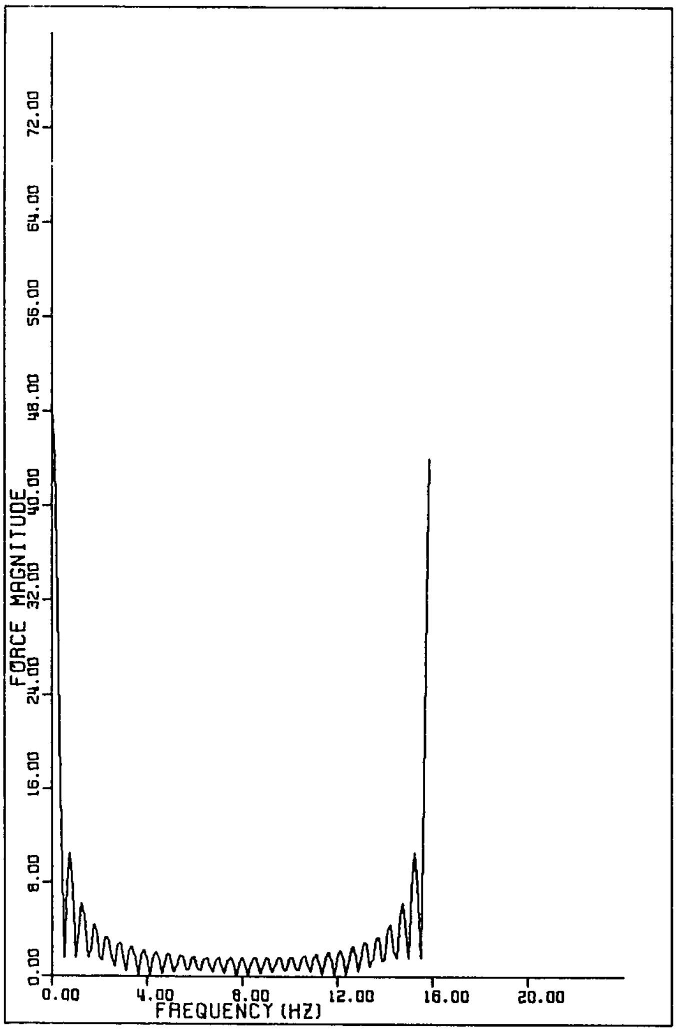


FIGURE 15. Frequency components of the force excitation history (128 points)

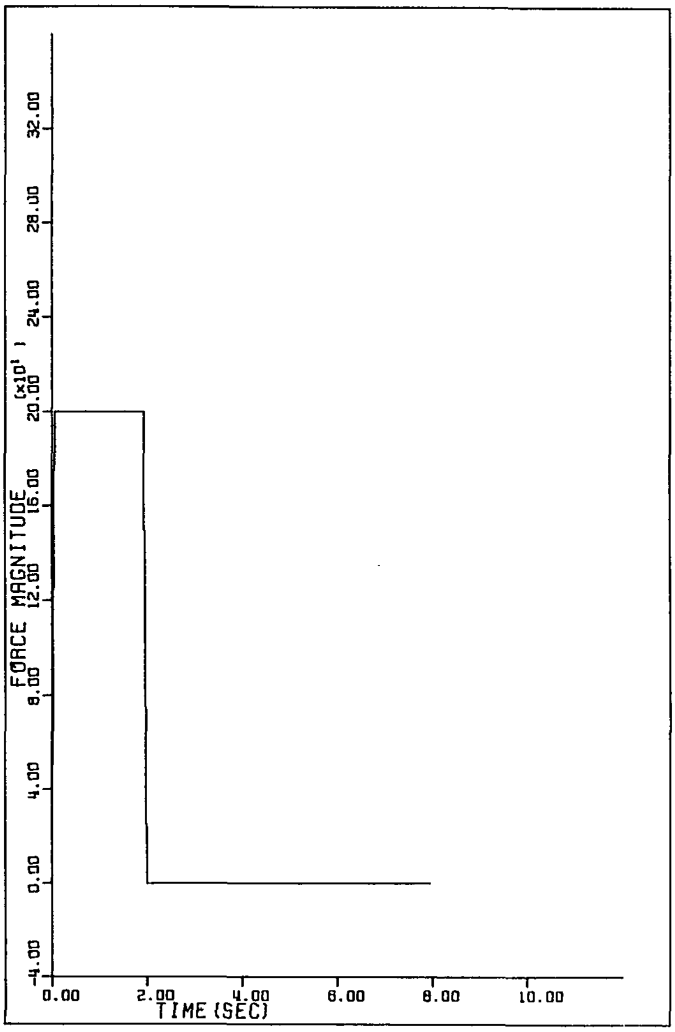


FIGURE 16. Discrete step force excitation history (128 points)

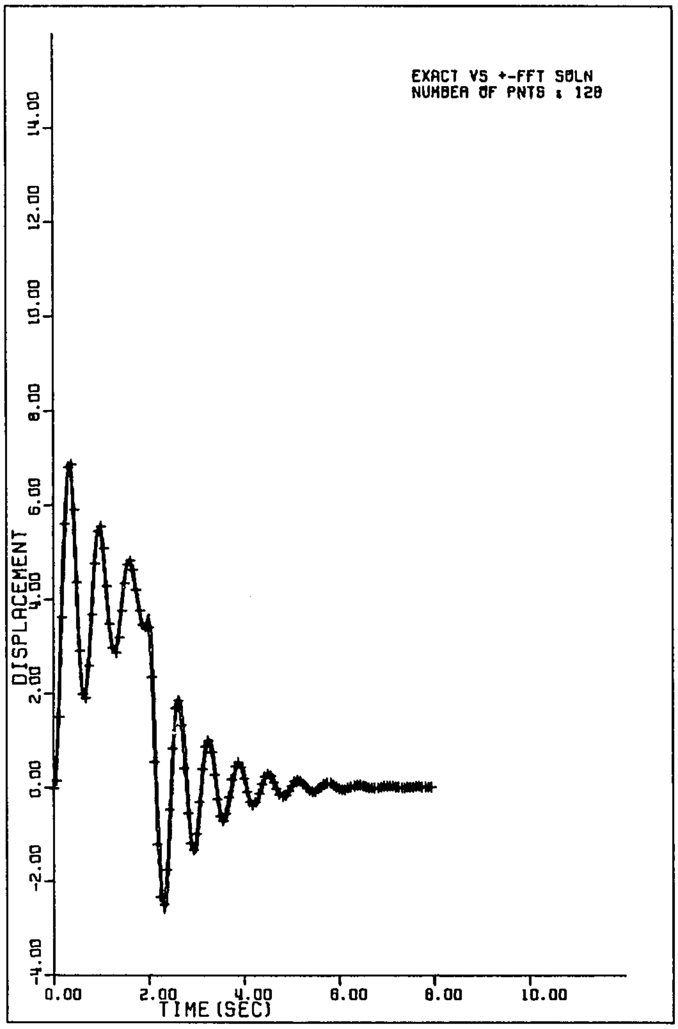


FIGURE 17. System displacement response - theoretical versus transfer function solution (128 points)

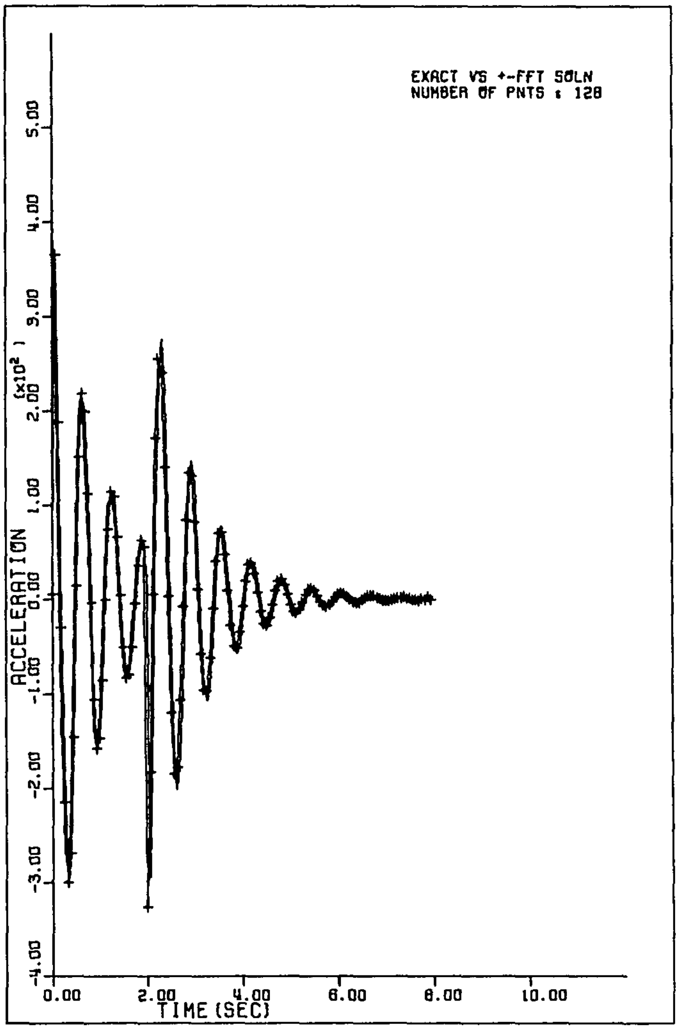


FIGURE 18. System acceleration response - theoretical versus transfer function solution (128 points)

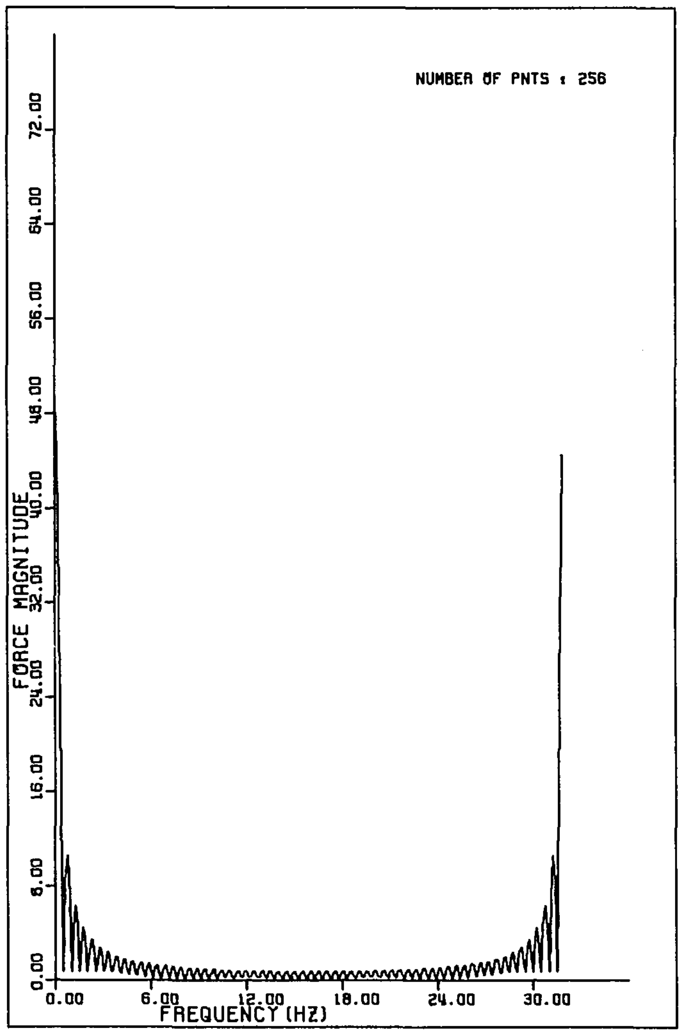


FIGURE 19. Frequency components of the force excitation history (256 points)

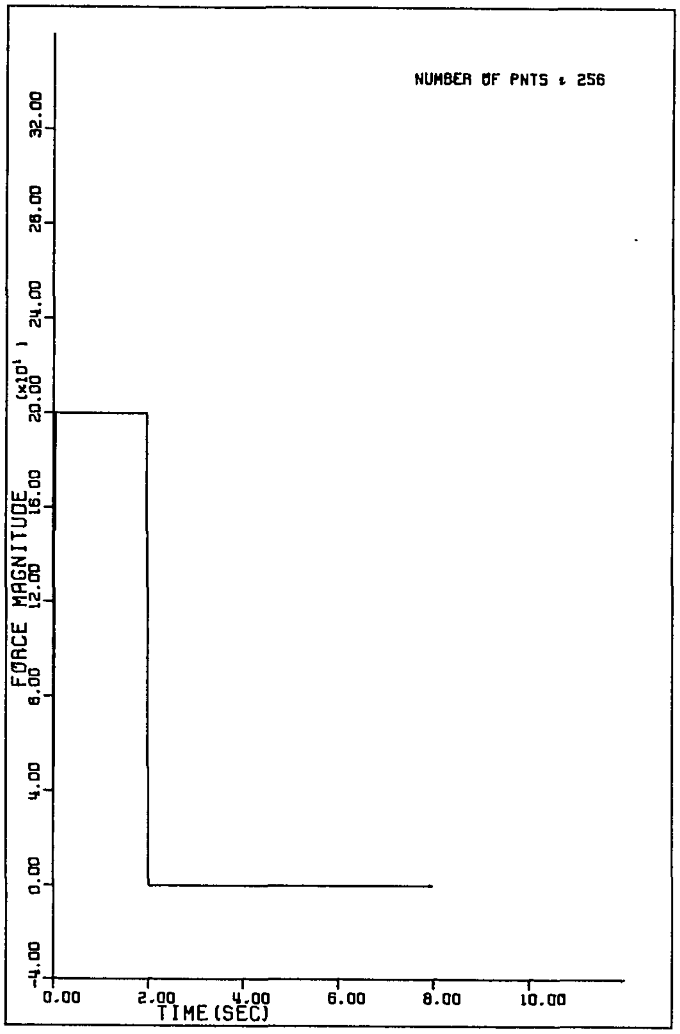


FIGURE 20. Discrete step force excitation history (256 points)

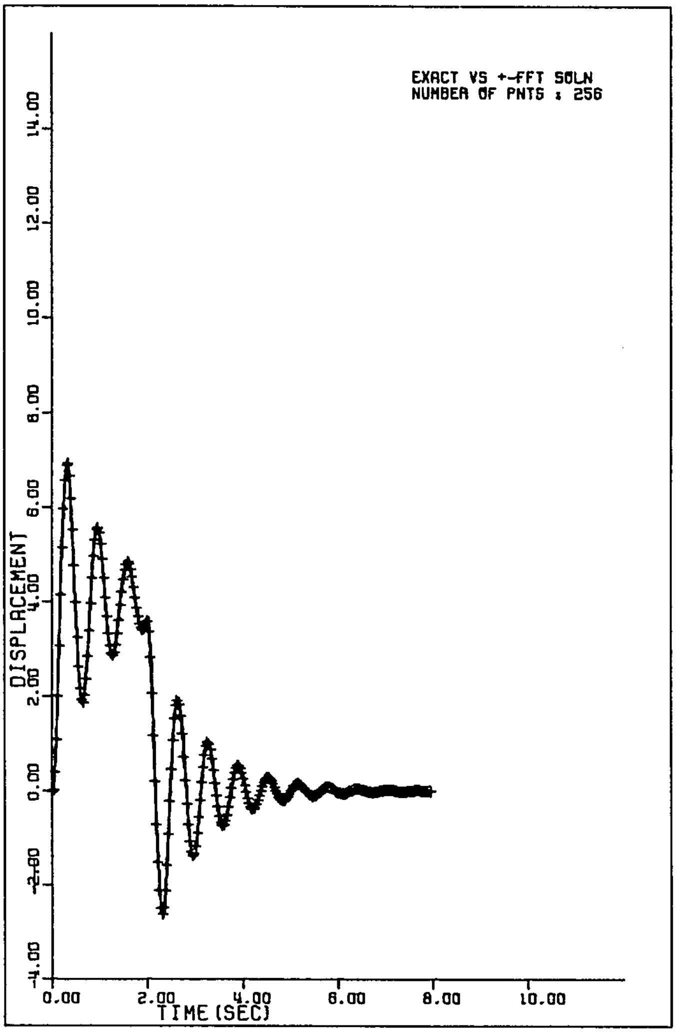


FIGURE 21. System displacement response - theoretical versus transfer function solution (256 points)

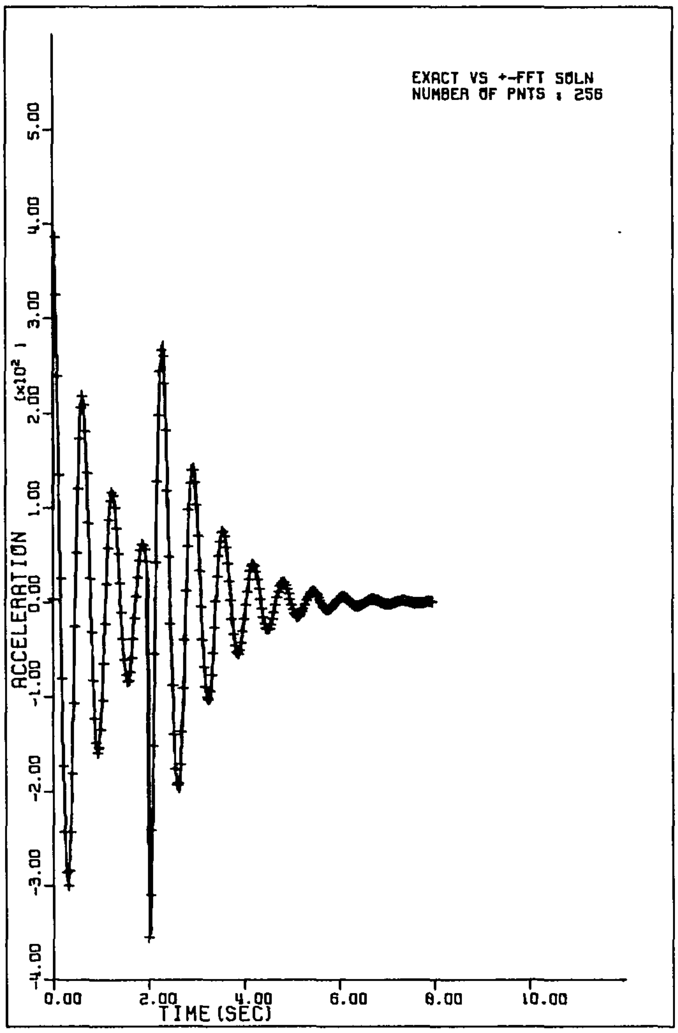


FIGURE 22. System acceleration response - theoretical versus transfer function solution (256 points)

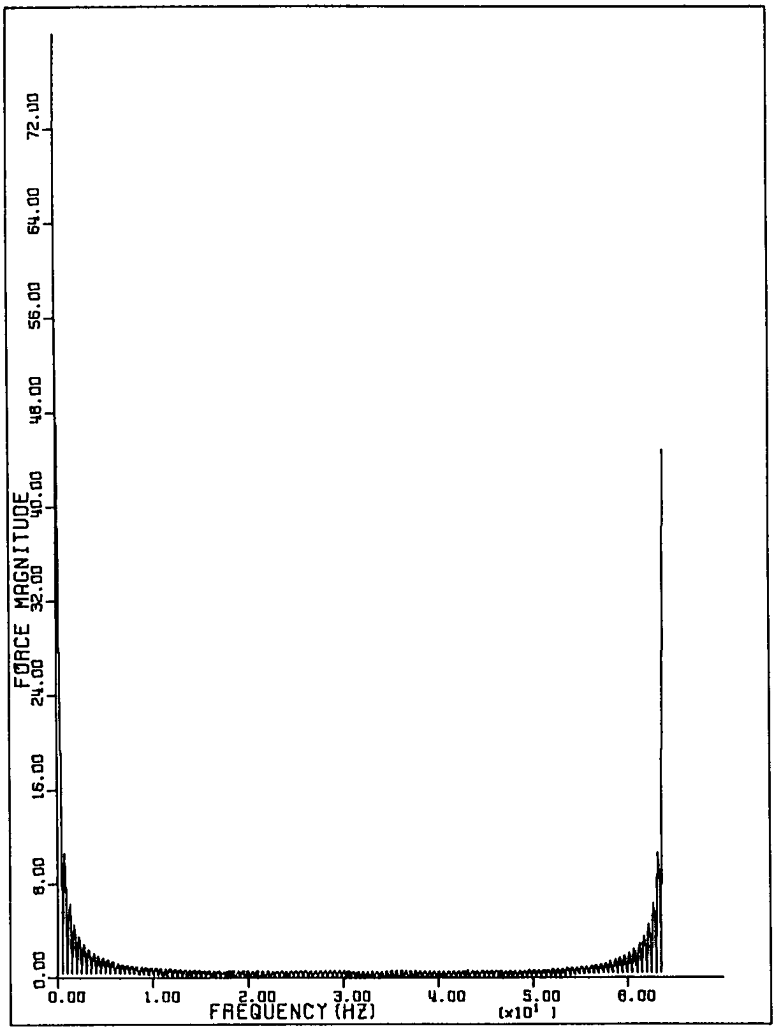


FIGURE 23. Frequency components of the force excitation history (512 points)

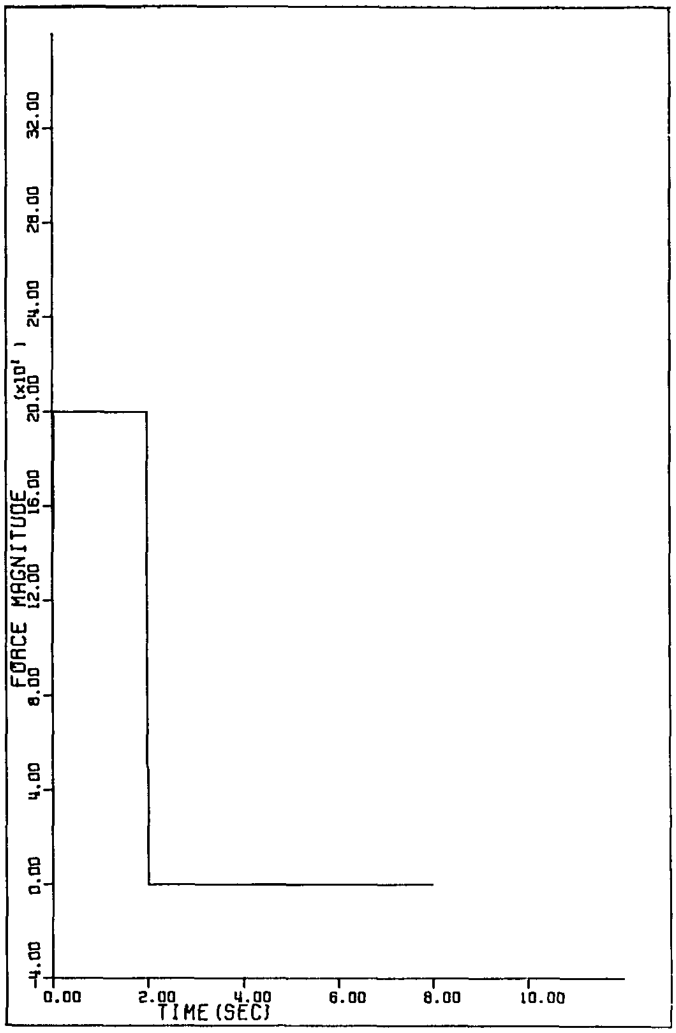


FIGURE 24. Discrete step force excitation history (512 points)

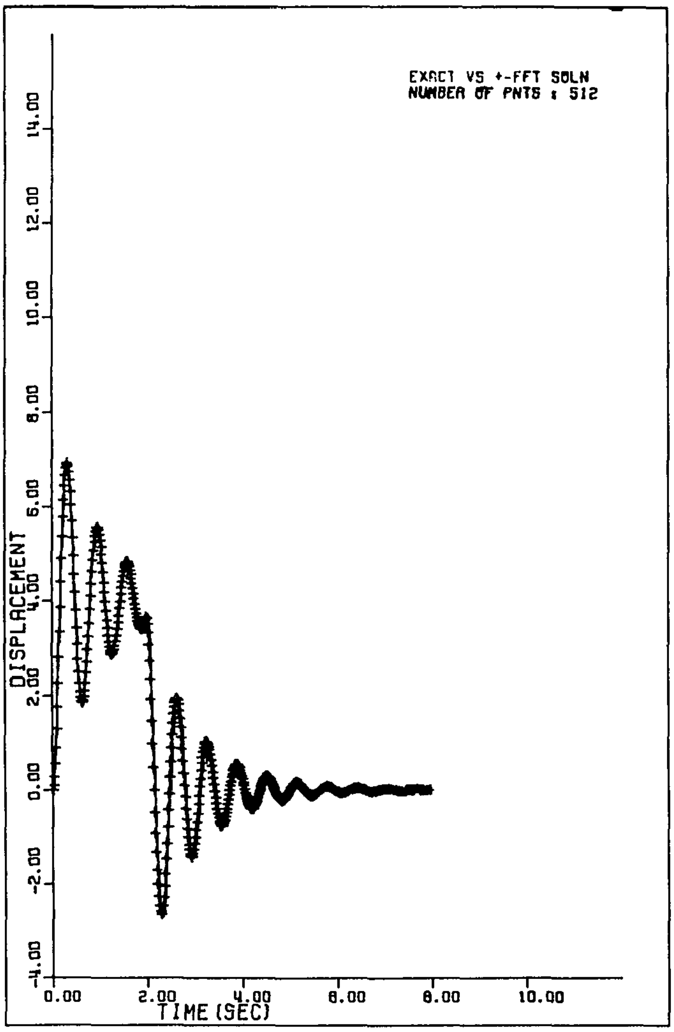


FIGURE 25. System displacement response - theoretical versus transfer function solution (512 points)

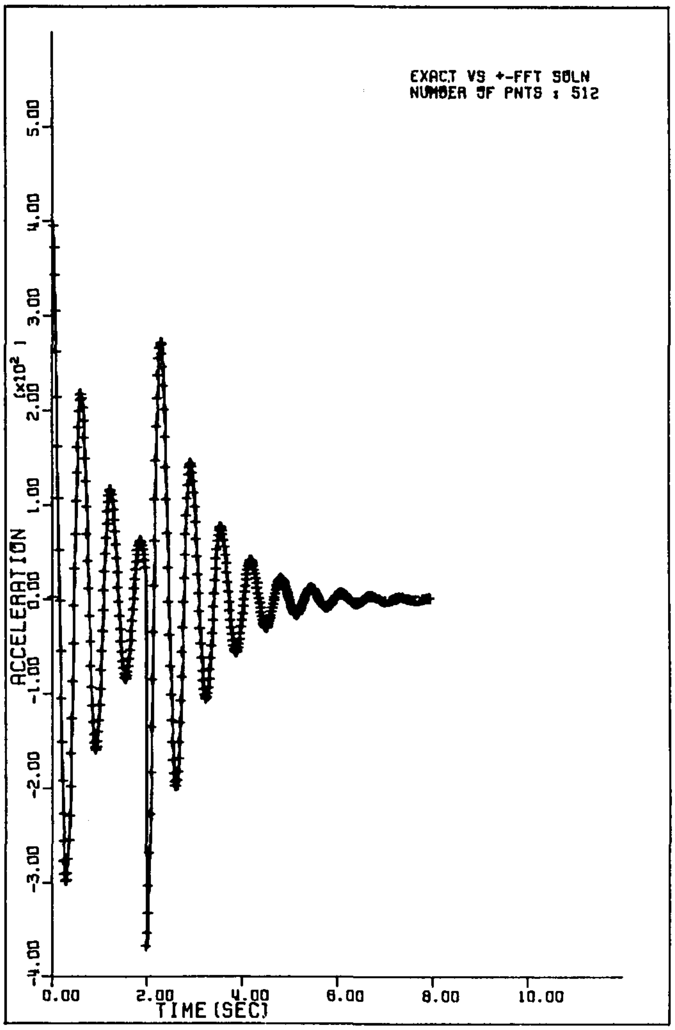


FIGURE 26. System acceleration response - theoretical versus transfer function solution (512 points)

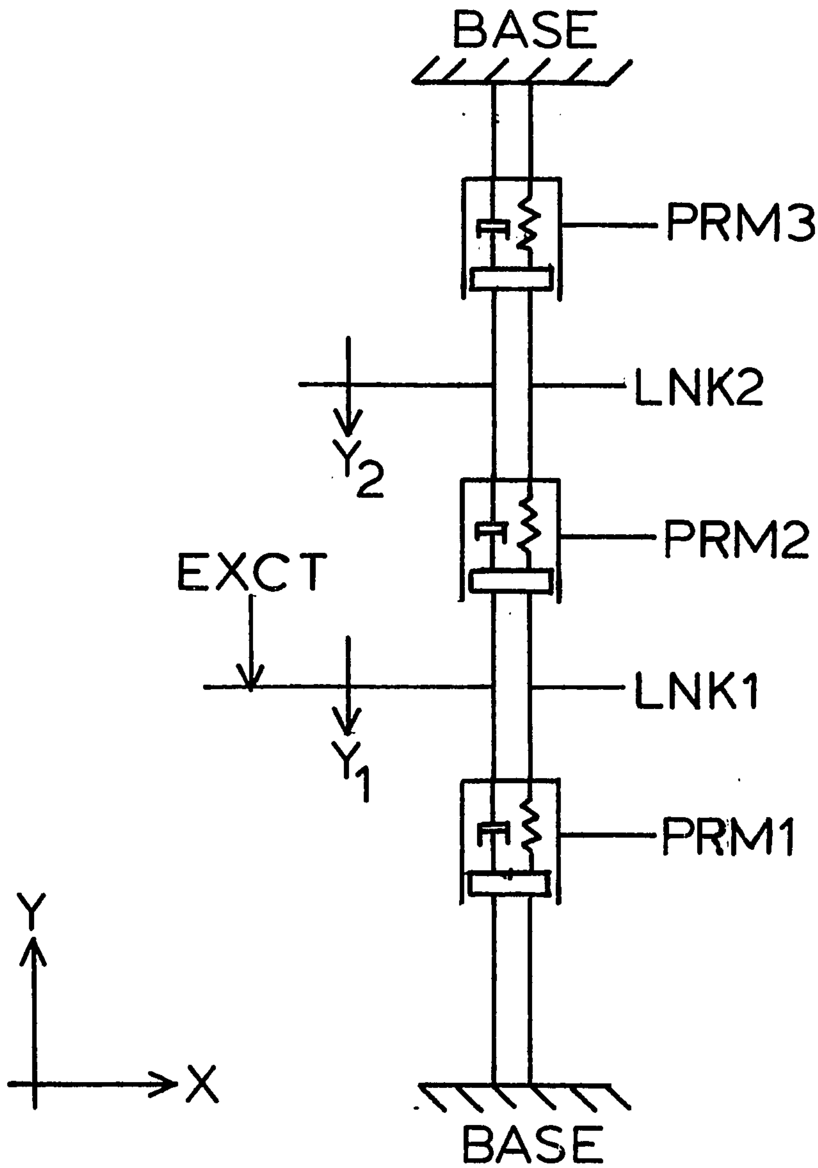


FIGURE 27. Two degree-of-freedom system with a single force excitation

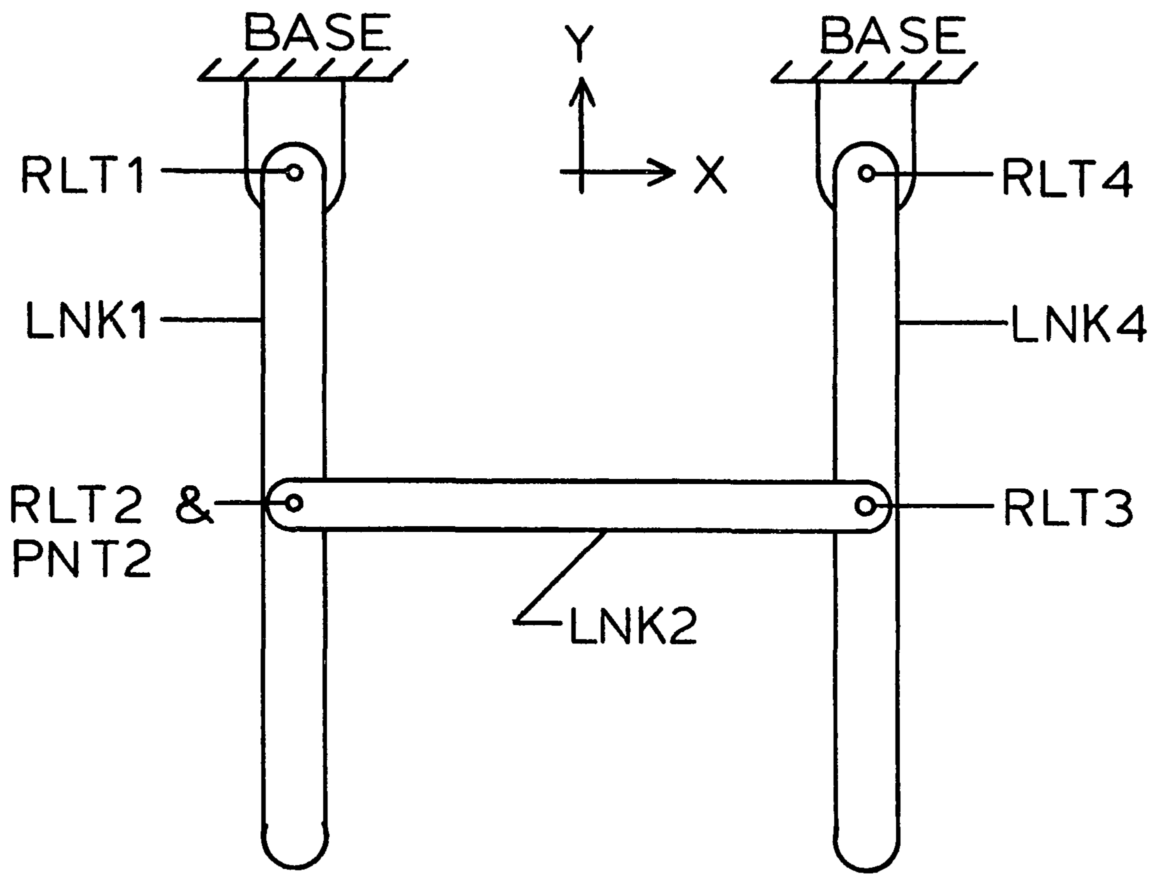


FIGURE 28. A simple pendulum system

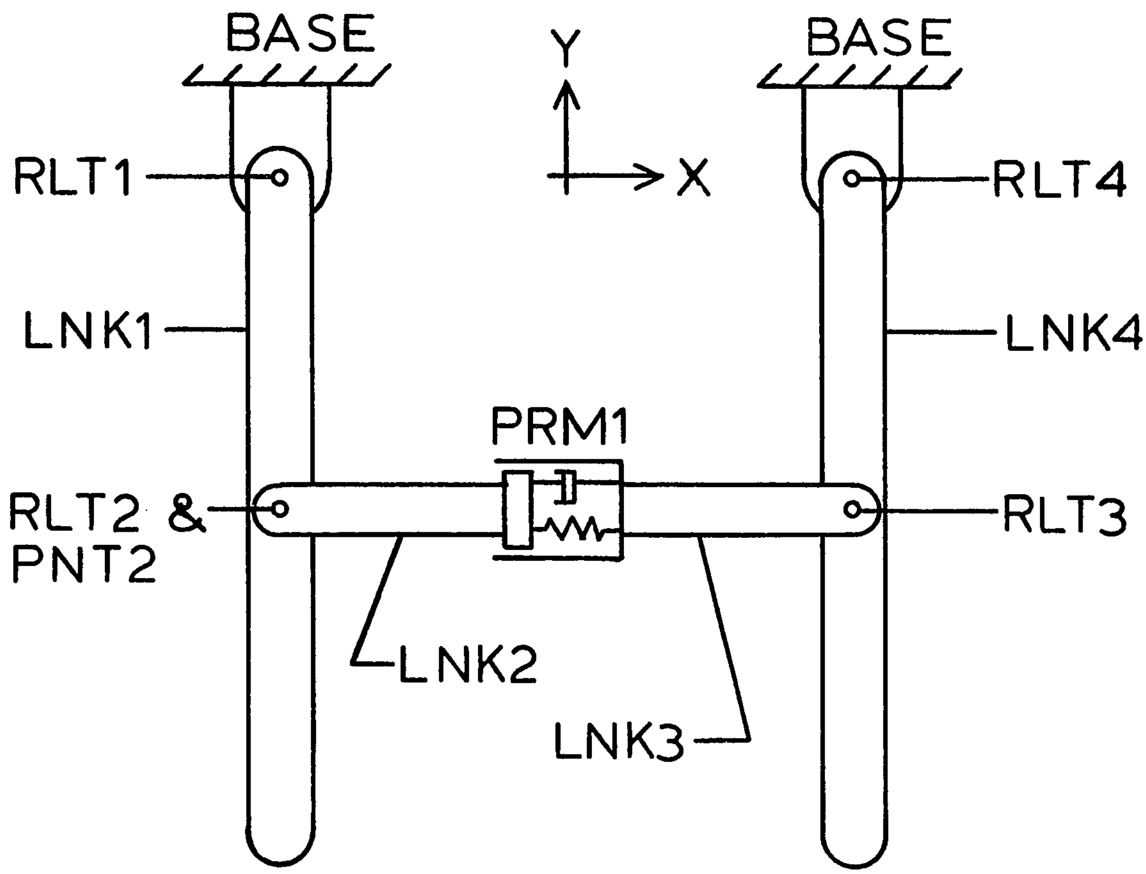


FIGURE 29. Compound pendulum system connected by a flexible coupler

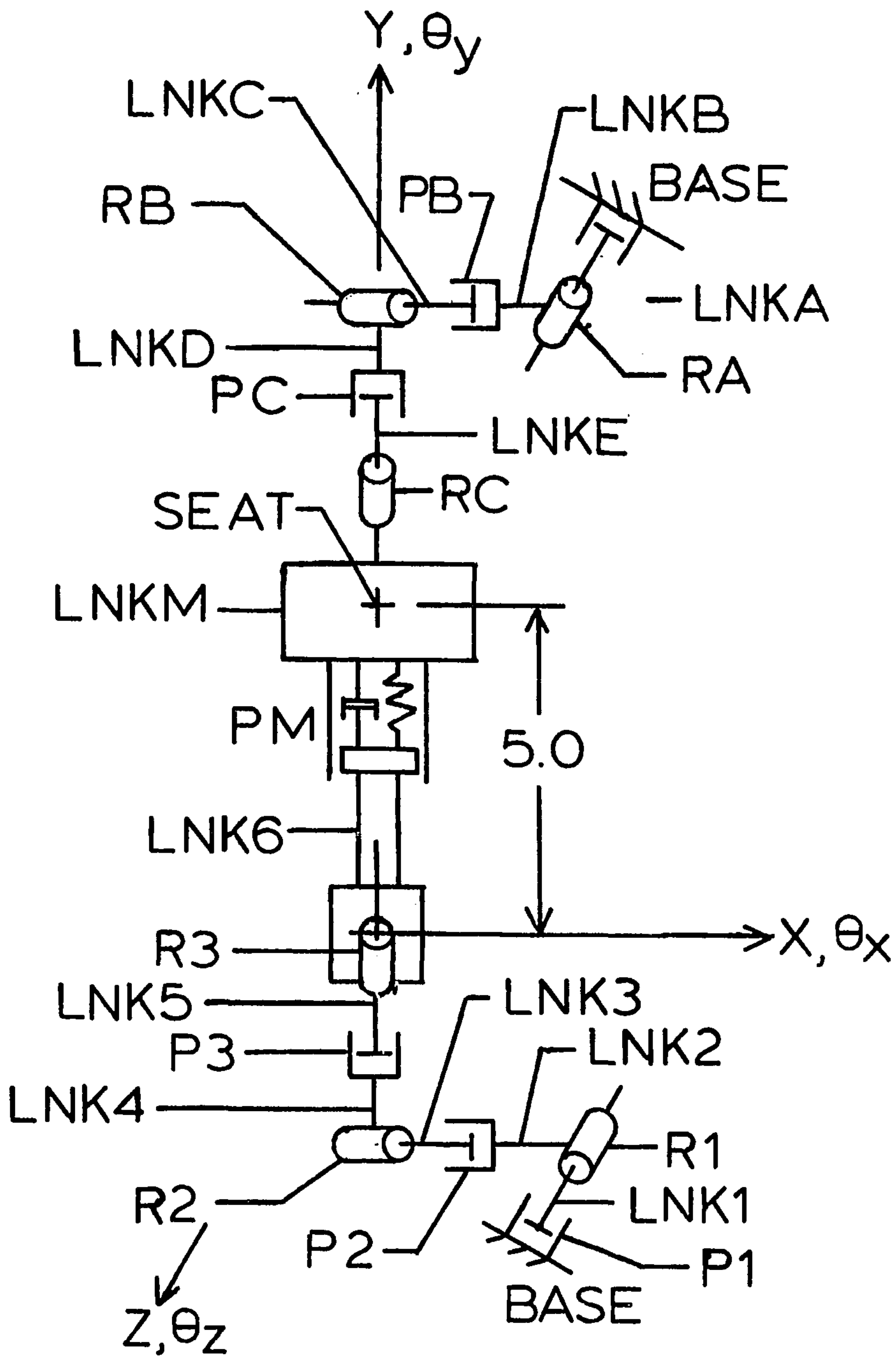


FIGURE 30. Single degree-of-freedom system with base motion input system

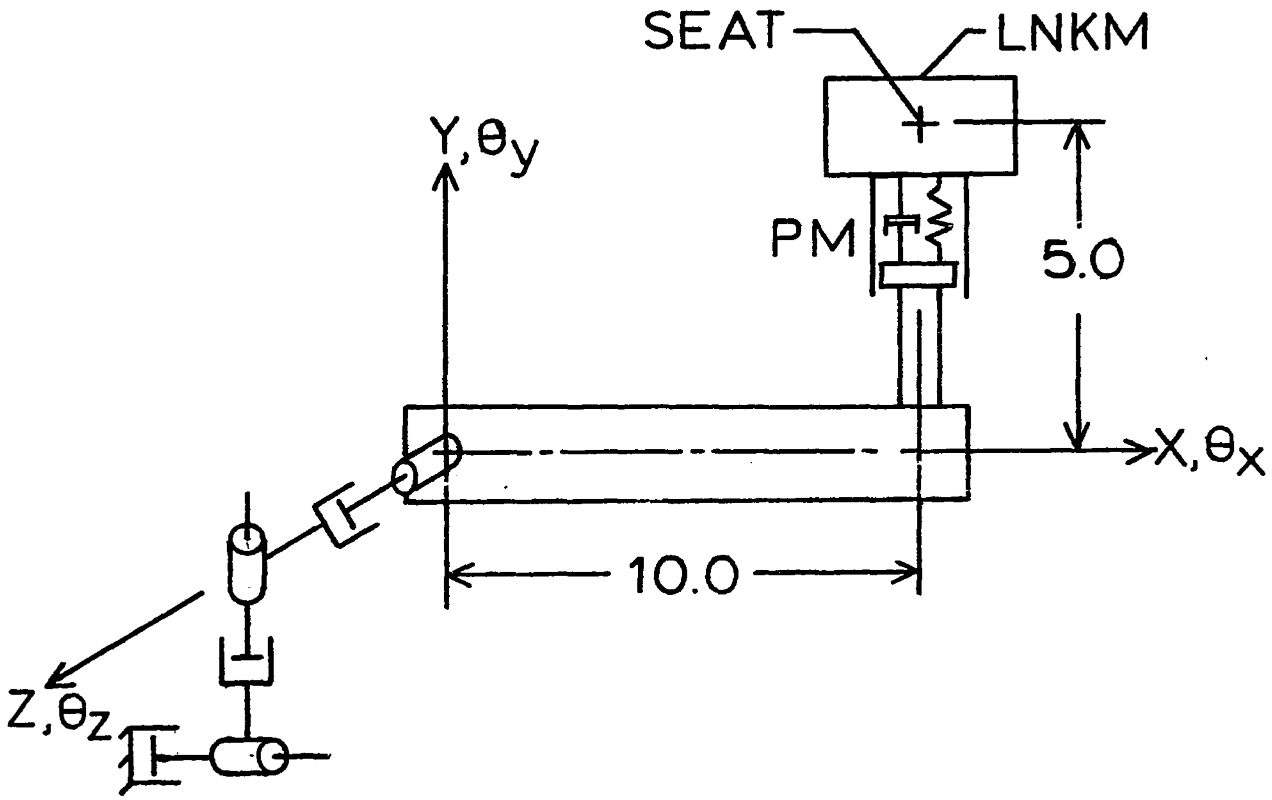


FIGURE 31. An offset single degree-of-freedom system with base motion input system

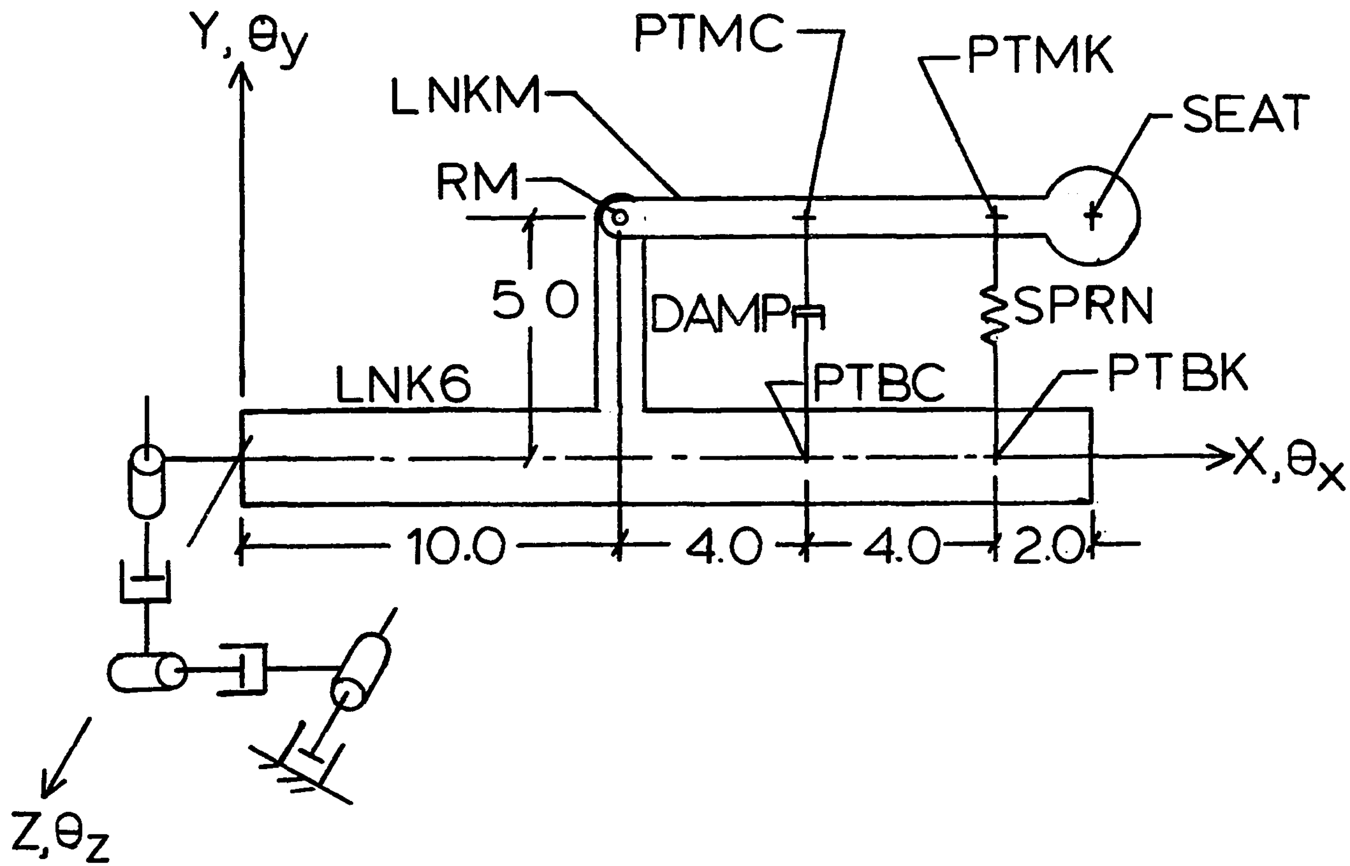


FIGURE 32. Pendulum system with base-input motions

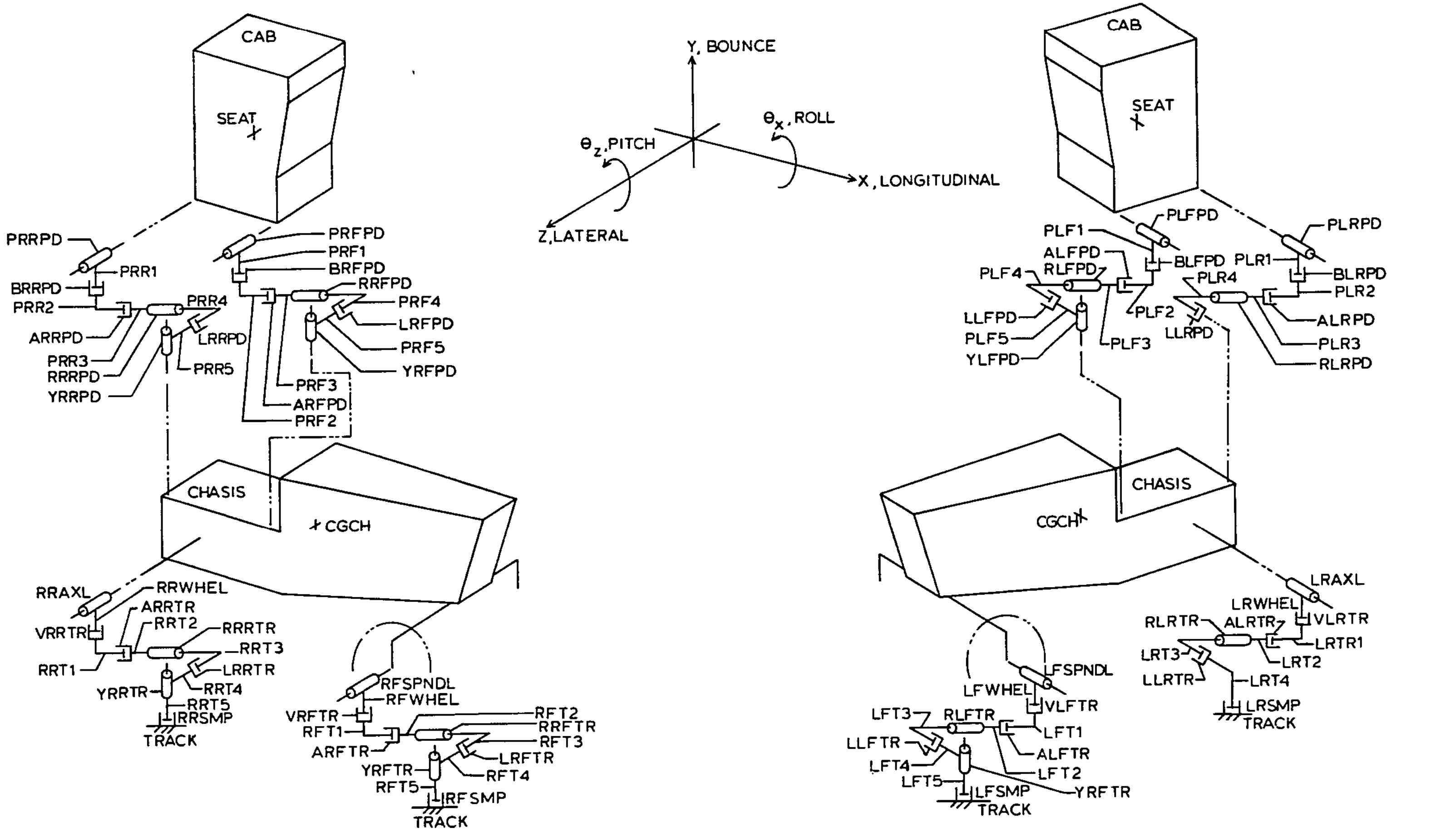


FIGURE 33. Three-dimensional agricultural wheel tractor IMP system model

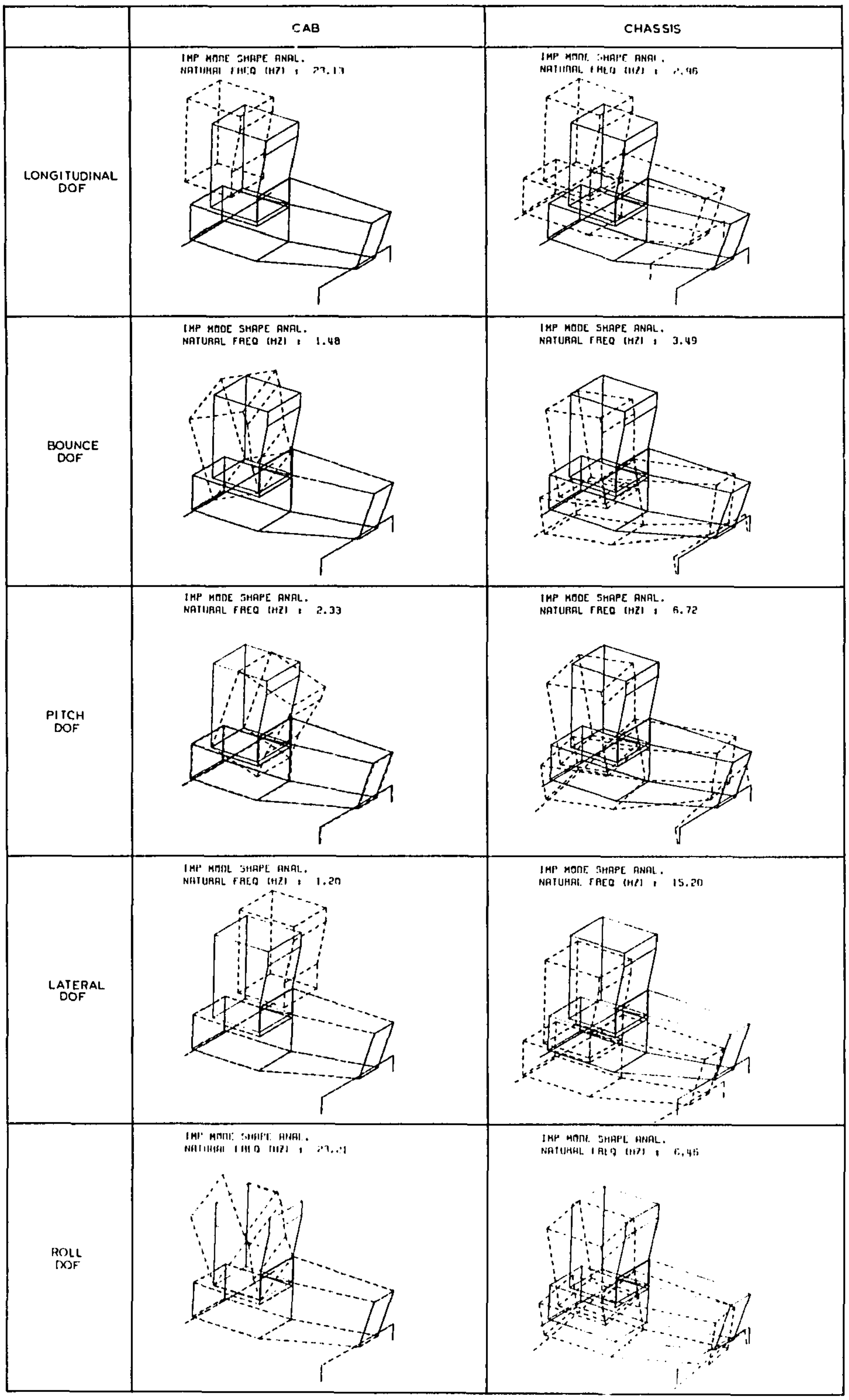


FIGURE 34. Mode shapes of vibration for an agricultural tractor

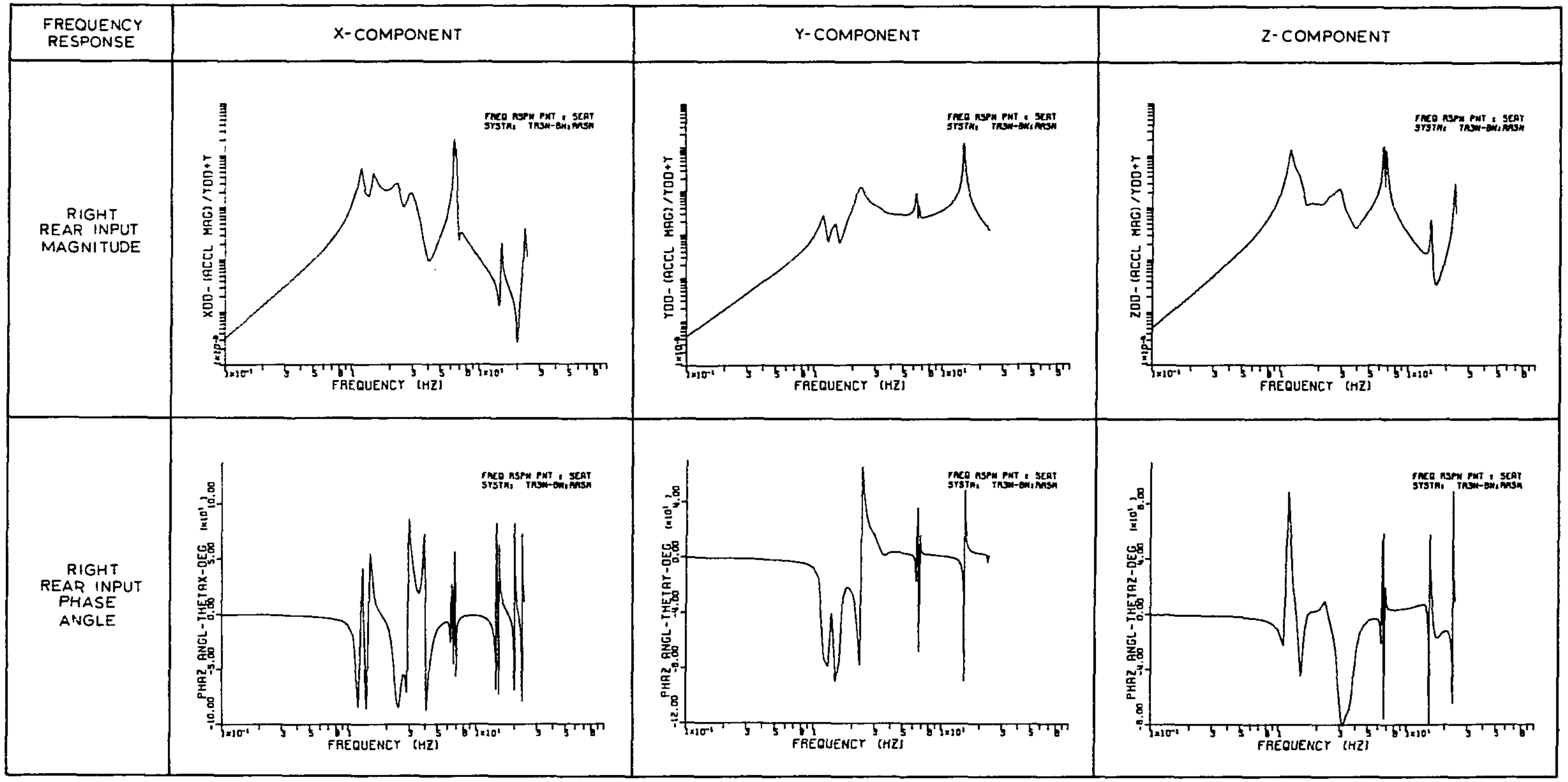


FIGURE 35. Frequency response for the tractor seat point with respect to the right rear wheel input excitation

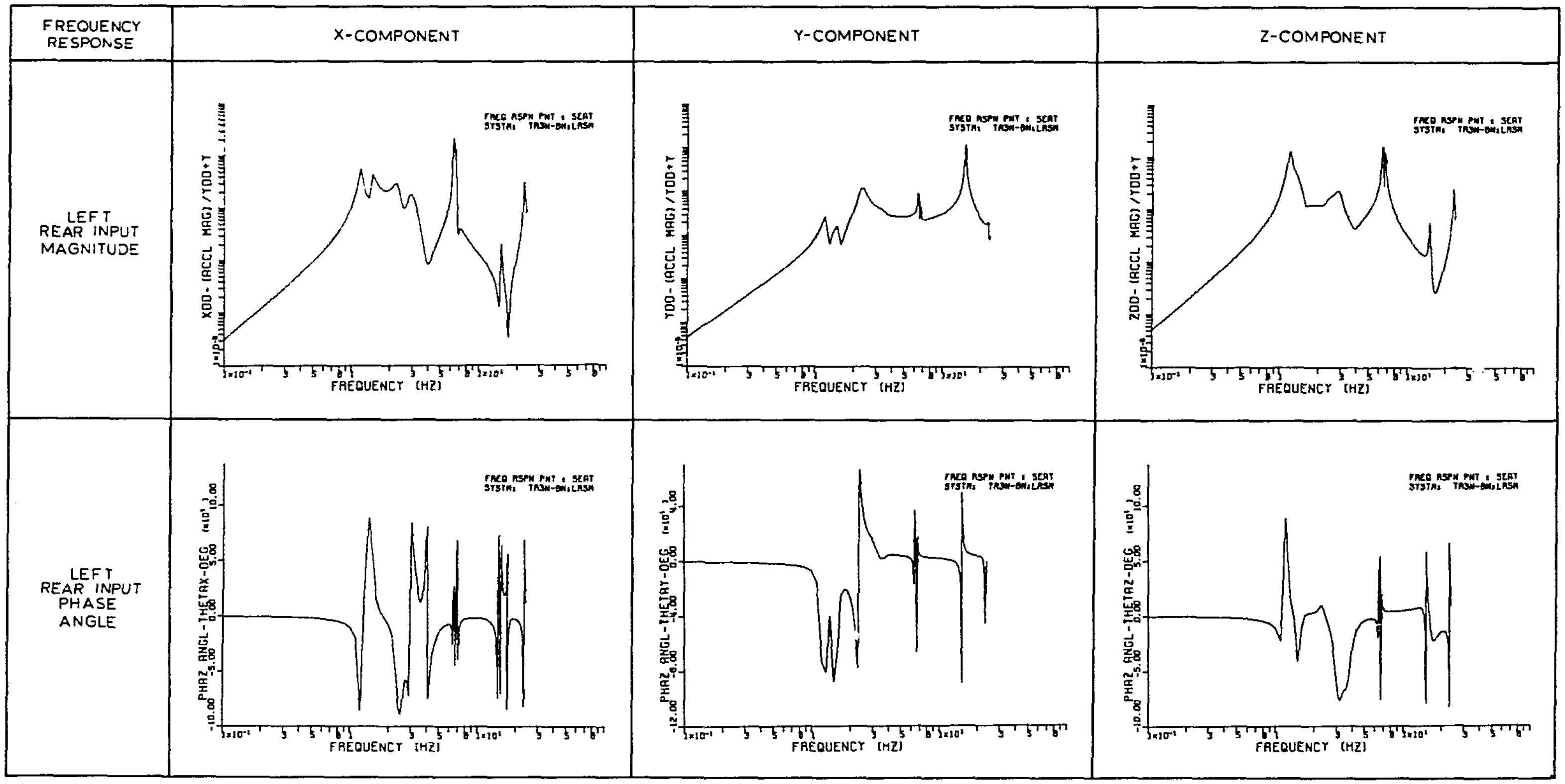


FIGURE 36. Frequency response for the tractor seat point with respect to the left rear wheel input excitation

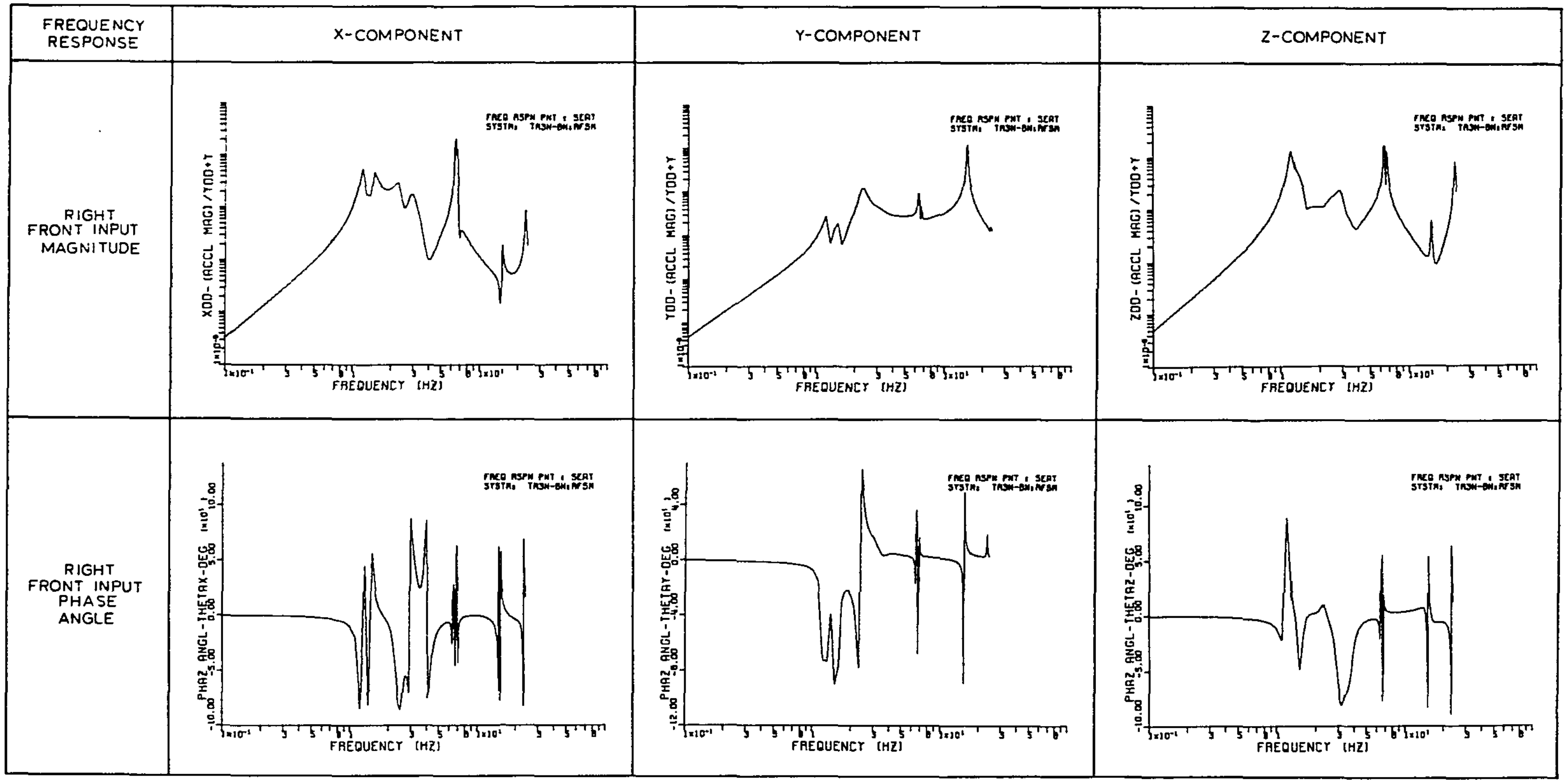


FIGURE 37. Frequency response for the tractor seat point with respect to the right front wheel input excitation

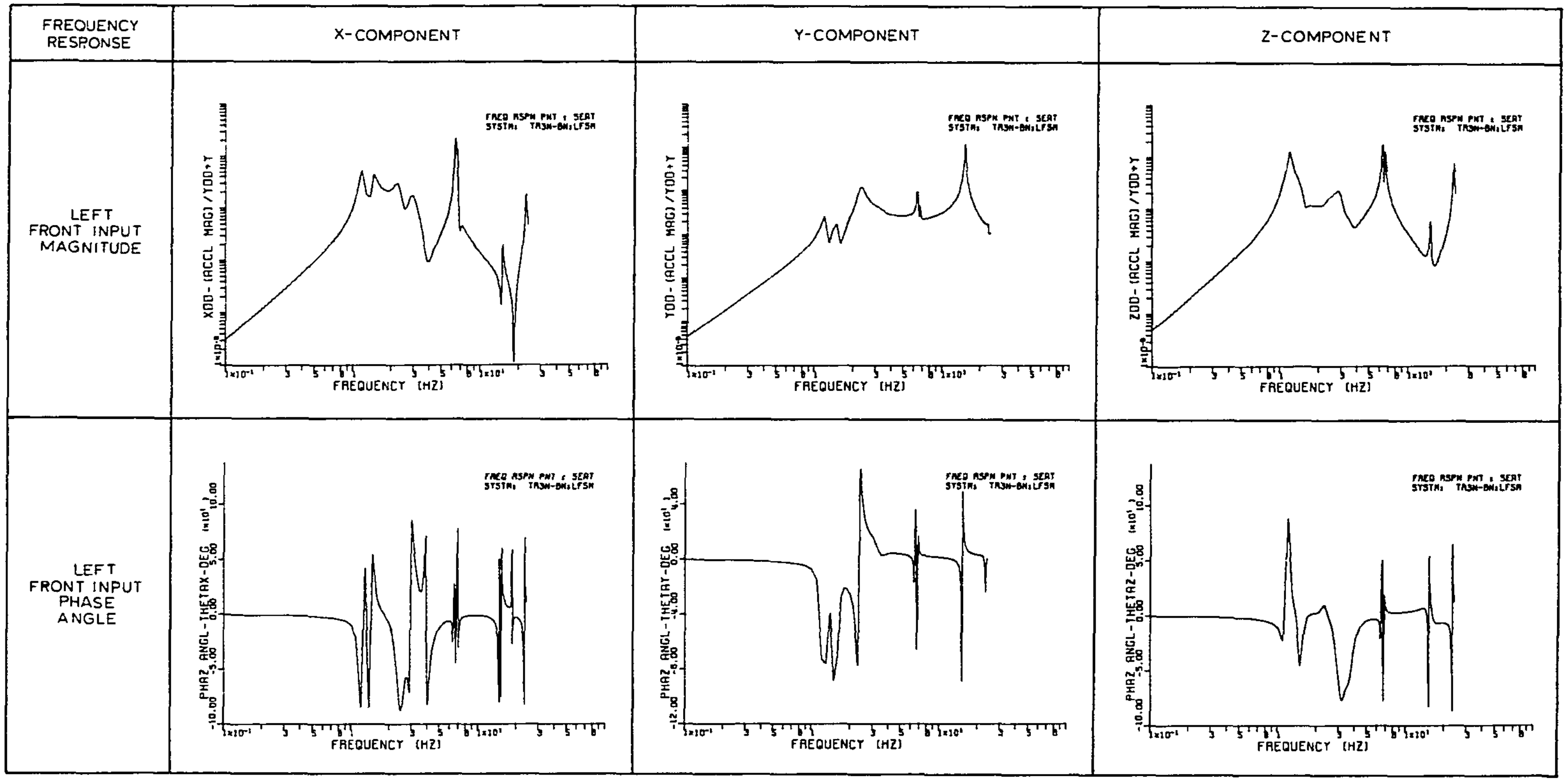


FIGURE 38. Frequency response for the tractor seat point with respect to the left front wheel input excitation

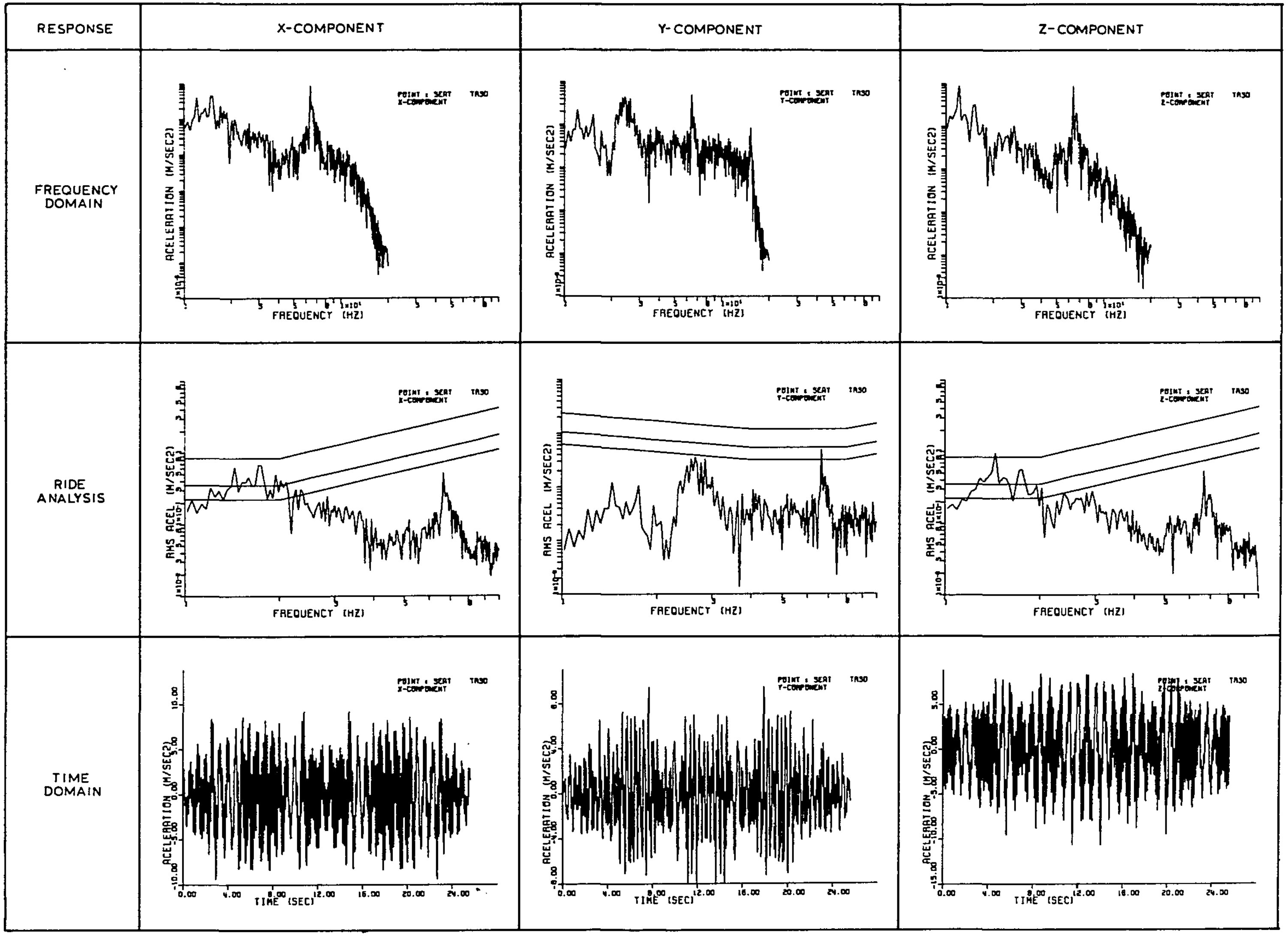


FIGURE 39. Acceleration response of the tractor seat point in the frequency- and time-domains

APPENDIX A: DERIVATION OF THE TRANSFER FUNCTION RELATIONSHIPS WITH FORCE AND TORQUE INPUT EXCITATIONS

With the IMP transfer function analysis procedure, the constrained mechanical system was excited by force and/or torque inputs through single degree-of-freedom revolute and prismatic joints, and/or forces and/or torques acting at points on the links.

For a constrained mechanical system, the vibratory motion of a revolute or prismatic joint, or a point of interest was expressed as the sum of the motion of the joint or point about its static equilibrium position plus the motion induced by the input excitations. In this case of the input forcing excitations causing the effective loading on the system, the vibratory motion was expressed in the form of Equation [17] to yield

$$[\ulcorner \alpha \lrcorner]\ \{q\} + [\ulcorner \beta \lrcorner]\ \{q\} = [\psi]^T\ \{P\} \qquad [A1]$$

where

$$[\psi]^T = \begin{bmatrix} (\psi_1)_1 & (\psi_2)_1 & \cdots & (\psi_{2N})_1 \\ (\psi_1)_2 & (\psi_2)_2 & \cdots & (\psi_{2N})_2 \\ & \vdots & & \\ (\psi_1)_{2N} & (\ _2)_{2N} & \cdots & (\ _{2N})_{2N} \end{bmatrix} \quad ;$$

$$[\psi]^T \{P\} = [\psi]^T \begin{Bmatrix} \{0\} \\ \{F\} \end{Bmatrix} =$$

$$\begin{Bmatrix} (\psi_{N+1})_1 F_1 + (\psi_{N+2})_1 F_2 + \cdots + (\psi_{N+N})_1 F_N \\ (\psi_{N+1})_2 F_1 + (\psi_{N+2})_2 F_2 + \cdots + (\psi_{N+N})_2 F_N \\ \vdots \\ (\psi_{N+1})_{2N} F_1 + (\psi_{N+2})_{2N} F_2 + \cdots + (\psi_{N+N})_{2N} F_N \end{Bmatrix}$$

Equation [A1] was rewritten in concise form as

$$[\alpha] \{\dot{q}\} + [\beta] \{q\} = \begin{Bmatrix} FF_1 \\ FF_2 \\ \vdots \\ FF_{2N} \end{Bmatrix} \qquad [A2]$$

Equations [A2] and [26] were combined together to yield the expression for the free generalized coordinates

$$\begin{Bmatrix} \eta_1 (s) \\ \eta_2 (s) \\ \vdots \\ \eta_{2N}(s) \end{Bmatrix} = \qquad [A3]$$

$$\begin{bmatrix} [(\psi_{N+1})_1 \dfrac{FF_1}{\alpha_1(s-\lambda_1)} + (\psi_{N+1})_2 \dfrac{FF_2}{\alpha_2(s-\lambda_2)} + \ldots + (\psi_{N+1})_{2N} \dfrac{FF_{2N}}{\alpha_{2N}(s-\lambda_{2N})}] \\ [(\psi_{N+2})_1 \dfrac{FF_1}{\alpha_1(s-\lambda_1)} + (\psi_{N+2})_2 \dfrac{FF_2}{\alpha_2(s-\lambda_2)} + \ldots + (\psi_{N+2})_{2N} \dfrac{FF_{2N}}{\alpha_{2N}(s-\lambda_{2N})}] \\ \vdots \\ [(\psi_{N+N})_1 \dfrac{FF_1}{\alpha_1(s-\lambda_1)} + (\psi_{N+N})_2 \dfrac{FF_2}{\alpha_2(s-\lambda_2)} + \ldots + (\psi_{N+N})_{2N} \dfrac{FF_{2N}}{\alpha_{2N}(s-\lambda_{2N})}] \end{bmatrix}$$

APPPENDIX B: DERIVATION OF THE TRANSFER FUNCTION RELATIONSHIPS WITH BASE INPUT EXCITATIONS

With the IMP transfer function analysis procedure, the constrained mechanical systems were excited by base motion inputs through single degree-of-freedom revolute and prismatic joints. The motions applied through the specified joints were accelerations, velocities, and/or displacements.

For a constrained mechanical system, the vibratory motion of a revolute or prismatic joint or a point of interest was expressed as the sum of the motion of the joint or point of interest about its static equilibrium position plus the base motion. This vibratory motion was expressed in matrix form to yield

$$\begin{bmatrix} [\ulcorner M \lrcorner]_{N*N} & [R]_{N*NS} \\ [0]_{NS*N} & [0]_{NS*NS} \end{bmatrix} \begin{Bmatrix} \{\ddot{\eta}\} \\ \{\ddot{\xi}\} \end{Bmatrix} + \begin{bmatrix} [\ulcorner C \lrcorner]_{N*N} & [S]_{N*NS} \\ [0]_{NS*N} & [0]_{NS*NS} \end{bmatrix} \begin{Bmatrix} \{\dot{\eta}\} \\ \{\dot{\xi}\} \end{Bmatrix}$$

$$+ \begin{bmatrix} [\ulcorner K \lrcorner]_{N*N} & [T]_{N*NS} \\ [0]_{NS*N} & [0]_{NS*NS} \end{bmatrix} \begin{Bmatrix} \{\eta\} \\ \{\xi\} \end{Bmatrix} = \begin{Bmatrix} \{0\} \\ \{0\} \end{Bmatrix} \qquad \text{[B1]}$$

Equation [B1] was rewritten to yield

$$[\ulcorner M \lrcorner]\{\ddot{\eta}\} + [\ulcorner C \lrcorner]\{\dot{\eta}\} + [\ulcorner K \lrcorner]\{\eta\} = -[R]\{\ddot{\xi}\} - [S]\{\dot{\xi}\} - [T]\{\xi\} \qquad \text{[B2]}$$

where $[\ulcorner M \lrcorner]$, $[\ulcorner C \lrcorner]$, and $[\ulcorner K \lrcorner]$ were the N by N real symmetric mass and inertia, viscous damping, and stiffness matrices, respectively; [R],

[S], and [T] were the N by NS real mass and inertia, viscous damping, and stiffness matrices of the system corresponding to the base motion inputs, respectively; $\eta_i(s)$ were the free generalized coordinates selected by the IMP algorithm; $\varepsilon_i(s)$ were the generalized coordinates specified for the base motion inputs.

The right-hand side of Equation [B2] represented the effective force or base motion input excitations in equation form

$$- [R] \{\ddot{\xi}\} - [S] \{\dot{\xi}\} - [T] \{\xi\} = \{F_i(t)\} \qquad [B3]$$

The value of the subscript variable, i, ranged from one to NS number of SGCs. The system responded to the base motion input accelerations, velocities, and displacements exactly as it would have to an external load F(t) which was equal to the products of masses and base accelerations, damping and base velocities, and stiffness and base displacements, respectively. The negative signs in Equation [B2] indicate the effective forces opposed the direction of the base motion input excitations.

In the case of the base motion inputs causing the effective loading on the system, Equation [17] was rewritten in concise form to yield

$$\lceil \alpha \rfloor \{\dot{q}\} + \lceil \beta \rfloor \{q\} = [\psi]^T \left\{ \begin{array}{c} \{0\} \\ \hline - [R] \{\ddot{\xi}\} - [S] \{\dot{\xi}\} - [T] \{\xi\} \end{array} \right\} \qquad [B4]$$

where

$$[\psi]^T = \begin{bmatrix} (\psi_1)_1 & (\psi_2)_1 & \cdots & (\psi_{2N})_1 \\ (\psi_1)_2 & (\psi_2)_2 & \cdots & (\psi_{2N})_2 \\ & \vdots & & \\ (\psi_1)_{2N} & (\psi_2)_{2N} & \cdots & (\psi_{2N})_{2N} \end{bmatrix} ;$$

$$[R]\{\ddot{\xi}\} = \begin{Bmatrix} R_{11}\ddot{\xi}_1 + R_{12}\ddot{\xi}_2 + \ldots + R_{1NS}\ddot{\xi}_{NS} \\ R_{21}\ddot{\xi}_1 + R_{22}\ddot{\xi}_2 + \ldots + R_{2NS}\ddot{\xi}_{NS} \\ \vdots \\ R_{N1}\ddot{\xi}_1 + R_{NS}\ddot{\xi}_2 + \ldots + R_{NNS}\ddot{\xi}_{NS} \end{Bmatrix} ;$$

$$[S]\{\dot{\xi}\} = \begin{Bmatrix} S_{11}\dot{\xi}_1 + S_{12}\dot{\xi}_2 + \ldots + S_{1NS}\dot{\xi}_{NS} \\ S_{21}\dot{\xi}_1 + S_{22}\dot{\xi}_2 + \ldots + S_{2NS}\dot{\xi}_{NS} \\ \vdots \\ S_{N1}\dot{\xi}_1 + S_{N2}\dot{\xi}_2 + \ldots + S_{NNS}\dot{\xi}_{NS} \end{Bmatrix} ;$$

$$[T]\{\xi\} = \begin{Bmatrix} T_{11}\xi_1 + T_{12}\xi_2 + \ldots + T_{1NS}\xi_{NS} \\ T_{21}\xi_1 + T_{22}\xi_2 + \ldots + T_{2NS}\xi_{NS} \\ \vdots \\ T_{N1}\xi_1 + T_{N2}\xi_2 + \ldots + T_{NNS}\xi_{NS} \end{Bmatrix} .$$

$$
[\psi]^T \left\{ \frac{\{0\}}{-[R]\ \{\ddot{\xi}\}} \right\} =
$$

$$
- \left\{ \begin{array}{l}
(\psi_{N+1})_1 \ R_{11} + (\psi_{N+2})_1 \ R_{21} + \ldots + (\psi_{2N})_1 \ R_{N1} \\
(\psi_{N+1})_2 \ R_{11} + (\psi_{N+2})_2 \ R_{21} + \ldots + (\psi_{2N})_2 \ R_{N1} \\
\vdots \\
(\psi_{N+1})_{2N} \ R_{11} + (\psi_{N+2})_{2N} \ R_{21} + \ldots + (\psi_{2N})_{2N} \ R_{N1}
\end{array} \right\} \ddot{\xi}_1
$$

$$
- \left\{ \begin{array}{l}
(\psi_{N+1})_1 \ R_{12} + (\psi_{N+2})_1 \ R_{22} + \ldots + (\psi_{2N})_1 \ R_{N2} \\
(\psi_{N+2})_2 \ R_{12} + (\psi_{N+2})_2 \ R_{22} + \ldots + (\psi_{2N})_2 \ R_{N2} \\
\vdots \\
(\psi_{N+1})_{2N} \ R_{12} + (\psi_{N+2})_{2N} \ R_{22} + \ldots + (\psi_{2N})_{2N} \ R_{N2}
\end{array} \right\} \ddot{\xi}_2
$$

$$
- \quad \cdots
$$

$$
- \left\{ \begin{array}{l}
(\psi_{N+1})_1 \ R_{1NS} + (\psi_{N+2})_1 \ R_{2NS} + \ldots + (\psi_{2N})_1 \ R_{NNS} \\
(\psi_{N+1})_2 \ R_{1NS} + (\psi_{N+2})_2 \ R_{2NS} + \ldots + (\psi_{2N})_2 \ R_{NNS} \\
\vdots \\
(\psi_{N+1})_{2N} \ R_{1NS} + (\psi_{N+2})_{2N} \ R_{2NS} + \ldots + (\psi_{2N})_{2N} \ R_{NNS}
\end{array} \right\} \ddot{\xi}_{NS};
$$

$$[\psi]^T \left\{ \begin{array}{c|c} & \{0\} \\ \hline -[S] & \{\dot{\xi}\} \end{array} \right\} =$$

$$\left\{ \begin{array}{l} (\psi_{N+1})_1 \; S_{11} + (\psi_{N+2})_1 \; S_{21} + \ldots + (\psi_{2N})_1 \; S_{N1} \\ (\psi_{N+1})_2 \; S_{11} + (\psi_{N+2})_2 \; S_{21} + \ldots + (\psi_{2N})_2 \; S_{N1} \\ \quad\vdots \\ (\psi_{N+1})_{2N} \; S_{11} + (\psi_{N+2})_{2N} \; S_{21} + \ldots + (\psi_{2N})_{2N} \; S_{N1} \end{array} \right\} \dot{\xi}_1$$

$$- \left\{ \begin{array}{l} (\psi_{N+1})_1 \; S_{12} + (\psi_{N+2})_1 \; S_{22} + \ldots + (\psi_{2N})_1 \; S_{N2} \\ (\psi_{N+1})_2 \; S_{12} + (\psi_{N+2})_2 \; S_{22} + \ldots + (\psi_{2N})_2 \; S_{N2} \\ \quad\vdots \\ (\psi_{N+1})_{2N} \; S_{12} + (\psi_{N+2})_{2N} \; S_{22} + \ldots + (\psi_{2N})_{2N} \; S_{N2} \end{array} \right\} \dot{\xi}_2$$

$$- \quad \cdots$$

$$- \left\{ \begin{array}{l} (\psi_{N+1})_1 \; S_{1NS} + (\psi_{N+2})_1 \; S_{2NS} + \ldots + (\psi_{2N})_1 \; S_{NNS} \\ (\psi_{N+1})_2 \; S_{1NS} + (\psi_{N+2})_2 \; S_{2NS} + \ldots + (\psi_{2N})_2 \; S_{NNS} \\ \quad\vdots \\ (\psi_{N+1})_{2N} \; S_{1NS} + (\psi_{N+2})_{2N} \; S_{2NS} + \ldots + (\psi_{2N})_{2N} \; S_{NNS} \end{array} \right\} \dot{\xi}_{NS};$$

$$[\psi]^T \left\{ \begin{array}{c} \{0\} \\ \hline -[T]\;\{\xi\} \end{array} \right\} =$$

$$- \left\{ \begin{array}{l} (\psi_{N+1})_1 \;\; T_{11} + (\psi_{N+2})_1 \;\; T_{21} + \ldots + (\psi_{2N})_1 \;\; T_{N1} \\ (\psi_{N+1})_2 \;\; T_{11} + (\psi_{N+2})_2 \;\; T_{21} + \ldots + (\psi_{2N})_2 \;\; T_{N1} \\ \quad \vdots \\ (\psi_{N+1})_{2N} \; T_{11} + (\psi_{N+2})_{2N} \; T_{21} + \ldots + (\psi_{2N})_{2N} \; T_{N1} \end{array} \right\} \xi_1$$

$$- \left\{ \begin{array}{l} (\psi_{N+1})_1 \;\; T_{12} + (\psi_{N+2})_1 \;\; T_{22} + \ldots + (\psi_{2N})_1 \;\; T_{N2} \\ (\psi_{N+1})_2 \;\; T_{12} + (\psi_{N+2})_2 \;\; T_{22} + \ldots + (\psi_{2N})_2 \;\; T_{N2} \\ \quad \vdots \\ (\psi_{N+1})_{2N} \; T_{12} + (\psi_{N+2})_2 \;\; T_{22} + \ldots + (\psi_{2N})_{2N} \; T_{N2} \end{array} \right\} \xi_2$$

$$- \quad \cdots$$

$$- \left\{ \begin{array}{l} (\psi_{N+1})_1 \;\; T_{1NS} + (\psi_{N+2})_1 \;\; T_{2NS} + \ldots + (\psi_{2N})_1 \;\; T_{NNS} \\ (\psi_{N+1})_2 \;\; T_{1NS} + (\psi_{N+2})_2 \;\; T_{2NS} + \ldots + (\psi_{2N})_2 \;\; T_{NNS} \\ \quad \vdots \\ (\psi_{N+1})_{2N} \; T_{1NS} + (\psi_{N+2})_{2N} \; T_{2NS} + \ldots + (\psi_{2N})_{2N} \; T_{NNS} \end{array} \right\} \xi_{NS}$$

Equation [B4] was rewritten in concise form to yield

$$\left[\alpha\right]\{\dot{q}\} + \left[\beta\right]\{q\} = \qquad [B5]$$

$$- \left[\begin{Bmatrix} g_{11} \\ g_{21} \\ \vdots \\ g_{2N1} \end{Bmatrix} \ddot{\xi}_1 + \begin{Bmatrix} g_{12} \\ g_{22} \\ \vdots \\ g_{2N2} \end{Bmatrix} \ddot{\xi}_2 + \ldots + \begin{Bmatrix} g_{1NS} \\ g_{2NS} \\ \vdots \\ g_{2NNS} \end{Bmatrix} \ddot{\xi}_{NS} \right]$$

$$- \left[\begin{Bmatrix} e_{11} \\ e_{21} \\ \vdots \\ e_{2N1} \end{Bmatrix} \dot{\xi}_1 + \begin{Bmatrix} e_{12} \\ e_{22} \\ \vdots \\ e_{2N2} \end{Bmatrix} \dot{\xi}_2 + \ldots + \begin{Bmatrix} e_{1NS} \\ e_{2NS} \\ \vdots \\ e_{2NNS} \end{Bmatrix} \dot{\xi}_{NS} \right]$$

$$- \left[\begin{Bmatrix} f_{11} \\ f_{21} \\ \vdots \\ f_{2N1} \end{Bmatrix} \xi_1 + \begin{Bmatrix} f_{12} \\ f_{22} \\ \vdots \\ f_{2N2} \end{Bmatrix} \xi_2 + \ldots + \begin{Bmatrix} f_{1NS} \\ f_{2NS} \\ \vdots \\ f_{2NNS} \end{Bmatrix} \xi_{NS} \right]$$

Equation [B5] was transformed to the frequency- or s-domain in the form of Equation [24] to yield

[B6]

$$
\begin{Bmatrix} q_1(s) \\ q_2(s) \\ \vdots \\ q_{2N}(s) \end{Bmatrix} = - \begin{bmatrix} \frac{g_{11}}{\alpha_1(s-\lambda_1)} & \frac{g_{12}}{\alpha_1(s-\lambda_1)} & \cdots & \frac{g_{1NS}}{\alpha_1(s-\lambda_1)} \\ \frac{g_{21}}{\alpha_2(s-\lambda_2)} & \frac{g_{22}}{\alpha_2(s-\lambda_2)} & \cdots & \frac{g_{2NS}}{\alpha_2(s-\lambda_2)} \\ \vdots & & & \\ \frac{g_{2N1}}{\alpha_{2N}(s-\lambda_{2N})} & \frac{g_{2N2}}{\alpha_{2N}(s-\lambda_{2N})} & \cdots & \frac{g_{2NNS}}{\alpha_{2N}(s-\lambda_{2N})} \end{bmatrix} \begin{Bmatrix} s^2\xi_1(s) \\ s^2\xi_2(s) \\ \vdots \\ s^2\xi_{NS}(s) \end{Bmatrix}
$$

$$
- \begin{bmatrix} \frac{e_{11}}{\alpha_1(s-\lambda_1)} & \frac{e_{12}}{\alpha_1(s-\lambda_1)} & \cdots & \frac{e_{1NS}}{\alpha_1(s-\lambda_1)} \\ \frac{e_{21}}{\alpha_2(s-\lambda_2)} & \frac{e_{22}}{\alpha_2(s-\lambda_2)} & \cdots & \frac{e_{2NS}}{\alpha_2(s-\lambda_2)} \\ \vdots & & & \\ \frac{e_{2N1}}{\alpha_{2N}(s-\lambda_{2N})} & \frac{e_{2N2}}{\alpha_{2N}(s-\lambda_{2N})} & \cdots & \frac{e_{2NNS}}{\alpha_{2N}(s-\lambda_{2N})} \end{bmatrix} \begin{Bmatrix} s\,\xi_1(s) \\ s\,\xi_2(s) \\ \vdots \\ s\,\xi_{NS}(s) \end{Bmatrix}
$$

$$
- \begin{bmatrix} \frac{f_{11}}{\alpha_1(s-\lambda_1)} & \frac{f_{12}}{\alpha_1(s-\lambda_1)} & \cdots & \frac{f_{1NS}}{\alpha_1(s-\lambda_1)} \\ \frac{f_{21}}{\alpha_2(s-\lambda_2)} & \frac{f_{22}}{\alpha_2(s-\lambda_2)} & \cdots & \frac{f_{2NS}}{\alpha_2(s-\lambda_2)} \\ \vdots & & & \\ \frac{f_{2N1}}{\alpha_{2N}(s-\lambda_{2N})} & \frac{f_{2N2}}{\alpha_{2N}(s-\lambda_{2N})} & \cdots & \frac{f_{2NNS}}{\alpha_{2N}(s-\lambda_{2N})} \end{bmatrix} \begin{Bmatrix} \xi_1(s) \\ \xi_2(s) \\ \vdots \\ \xi_{NS}(s) \end{Bmatrix}
$$

Equation [B6] was expressed in concise form to yield

$$\begin{Bmatrix} q_1(s) \\ q_2(s) \\ \vdots \\ q_{2N}(s) \end{Bmatrix} = - \begin{bmatrix} GG_{11} & GG_{12} & \cdots & GG_{1NS} \\ GG_{21} & GG_{22} & \cdots & GG_{2NS} \\ \vdots & & & \\ GG_{2N1} & GG_{2N2} & \cdots & GG_{2NNS} \end{bmatrix} \begin{Bmatrix} s^2\xi_1(s) \\ s^2\xi_2(s) \\ \vdots \\ s^2\xi_{NS}(s) \end{Bmatrix} - \begin{bmatrix} EE_{11} & EE_{12} & \cdots & EE_{1NS} \\ EE_{21} & EE_{22} & \cdots & EE_{2NS} \\ \vdots & & & \\ EE_{2N1} & EE_{2N2} & \cdots & EE_{2NNS} \end{bmatrix} \begin{Bmatrix} s\,\xi_1(s) \\ s\,\xi_2(s) \\ \vdots \\ s\,\xi_{NS}(s) \end{Bmatrix} - \begin{bmatrix} FF_{11} & FF_{12} & \cdots & FF_{1NS} \\ FF_{21} & FF_{22} & \cdots & FF_{2NS} \\ \vdots & & & \\ FF_{2N1} & FF_{2N2} & \cdots & FF_{2NNS} \end{bmatrix} \begin{Bmatrix} \xi_1(s) \\ \xi_2(s) \\ \vdots \\ \xi_{NS}(s) \end{Bmatrix} \qquad [B7]$$

Equations [26] and [B7] were combined together to yield the expressions for the free generalized coordinates

$$\begin{Bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{2N} \end{Bmatrix} = \begin{Bmatrix} s \begin{Bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_N \end{Bmatrix} \\ \hdashline \begin{Bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_N \end{Bmatrix} \end{Bmatrix} = [\psi] \begin{Bmatrix} q_1 \\ q_2 \\ \vdots \\ q_{2N} \end{Bmatrix} \qquad [B8]$$

From Equation [B8], the equations for the free generalized coordinates were written in concise form

$$\eta_i = \sum_{k=1}^{NS} \left[\sum_{j=1}^{2N} \psi_{N+i\ j}\ GG_{j\ k} \right] s^2 \xi_k \quad + \qquad \text{[B9]}$$
$$\sum_{k=1}^{NS} \left[\sum_{j=1}^{2N} \psi_{N+i\ j}\ EE_{j\ k} \right] s \xi_k \quad +$$
$$\sum_{k=1}^{NS} \left[\sum_{j=1}^{2N} \psi_{N+i\ j}\ FF_{j\ k} \right] \xi_k ,$$

The value of the subscript variable, i, ranged from one to N number of system FGCs.

APPENDIX C: PRINT/DYNAM ... STATEMENT TO REQUEST THE TRANSFER FUNCTION ANALYSIS ENTITY

This statement was used to request the transfer functions of the system motion resulting from the sinusoidally-varying force and torque inputs specified by the DATA/FORCE and/or DATA/TORQUE statements, and the sinusoidally-varying base motion inputs specified by the DATA/MOTION or DATA/POSITN statements.

FORMAT:

PRINT/DYNAM/FORCE/POSITN/ List of excitation forces / List of design variables /

PRINT/DYNAM/FORCE/VELCTY/ List of excitation forces / List of design variables /

PRINT/DYNAM/FORCE/ACCL/ List of excitation forces / List of design variables/

PRINT/DYNAM/MOTION/POSITN/ List of design variables /

PRINT/DYNAM/MOTION/VELCTY/ List of design variables /

PRINT/DYNAM/MOTION/ACCL/ List of design variables /

where

FORCE - requests the force transfer function analysis;

MOTION - requests the motion transfer function analysis;

POSITN - requests the position transfer functions of "List of design variables"

VELCTY - requests the velocity transfer functions of "List of design variables"

ACCL - requests the acceleration transfer functions of "List of design variables"

"List of excitation forces" - is a string of alphanumeric names of forces and torques that are selected to excite the mechanism system

"List of design variables" - is a string of alphanumeric names of the revolute and prismatic joints and points of interest on various links.

Example C1

PRINT/DYNAM/MOTION/POSITN/RLT1,PRM4,PNTA/ requested the position transfer function ratio sets resulting from the base motion inputs, for the displacements occurring within the revolute joint RLT1 and the prismatic joint PRM4, and the point PNTA along the global coordinate frame X, Y, and Z axes. If the pair variable for the revolute joint RLT1 was designated by θ, the transfer function corresponding to a position or base motion input BASE by a DATA/POSITN or DATA/MOTION statement had the general form

$$\frac{\theta(s)}{BASE(s)} = \sum_{i=1}^{m} \frac{N_i}{s-D_i} \qquad [C1]$$

where

$m = 2*FGC$, the number of free generalized coordinates;

N_i = complex numerators;

D_i = complex conjugate denominators, the eigenvalues of the mechanical system;

$s = j\omega$;

j = the symbol for complex notation.

The transfer function ratio set for PRM4 had the same general form as Equation [C1]. For the point PNTA, three transfer function ratio sets were computed to correspond to the x, y, and z coordinate motion for the point and were expressed in the general form

$$\frac{x(s)}{BASE(s)} = \sum_{i=1}^{m} \frac{N_{xi}}{s-D_i} + \sum_{n=1}^{k} BX_n \qquad [C2]$$

$$\frac{y(s)}{BASE(s)} = \sum_{i=1}^{m} \frac{N_{yi}(s)}{s-D_i} + \sum_{n=1}^{k} BY_n \qquad [C3]$$

$$\frac{z(s)}{BASE(s)} = \sum_{i=1}^{m} \frac{N_{zi}(s)}{s-D_i} + \sum_{n=1}^{k} BZ_n \qquad [C4]$$

where $N_{xi}(s)$, $N_{yi}(s)$, and $N_{zi}(s)$ are the complex modal numerators for the x, y, and z coordinate motions as measured in the global coordinate frame, respectively; BX, BY, and BZ were the base motion inputs for the input joints (SGCs) for the x, y, and z coordinate motions measured in the global coordinate frame, respectively; k was the total number base motion input excitations or the number of SGC variables.

<u>Example C2</u>

PRINT/DYNAM/FORCE/ACCL/FEXT/PRM2,PNTC/ requested the acceleration transfer function ratio sets for the system, resulting from a force excitation input, for the acceleration occurring within the prismatic joint PRM2 and for the point PNTC along the global coordinate frame x, y, and z axes. If the pair variable for the prismatic joint PRM2 was designated by V, the transfer function ratio set corresponding to the force input FEXT by a DATA/FORCE statement, had the general form

$$\frac{s^2V(s)}{FEXT(s)} = \omega^2 \sum_{i=1}^{m} \frac{N_i(s)}{s-D_i} \qquad [C5]$$

For the point PNTC, three transfer functions were computed to correspond to the x, y, and z global cooordinate motion for the point and were expressed in the general form

$$\frac{s^2x(s)}{FEXT(s)} = \omega^2 \sum_{i=1}^{m} \frac{N_{xi}(s)}{s-D_i} \qquad [C6]$$

$$\frac{s^2y(s)}{FEXT(s)} = \omega^2 \sum_{i=1}^{m} \frac{N_{xi}(s)}{s-D_i} \qquad [C7]$$

$$\frac{s^2z(s)}{FEXT(s)} = \omega^2 \sum_{i=1}^{m} \frac{N_{xi}(s)}{s-D_i} \qquad [C8]$$

Example C3

PRINT/DYNAM/FORCE/VELCTY/FEXT/PRM2/ requested the velocity transfer function ratio sets for the system, resulting from the force input excitation, for the velocity occurring within the prismatic joint PRM2. If the pair variable for the prismatic joint PRM2 was designated V, the velocity transfer function ratio sets, cooresponding to the force input excitation FEXT by a DATA/FORCE statement, had the general form

$$\frac{sV(s)}{FEXT(s)} = \omega \sum_{i=1}^{m} \frac{N_i(s)}{s-D_i} \qquad [C9]$$

Notes

1. At least one input force or torque must be specified by the DATA/FORCE or DATA/TORQUE statement for the force transfer function analysis, or at least one input position or motion must be specified by the DATA/POSITN or DATA/MOTION statement for the motion transfer fuction analysis. If no inputs are specified, only the common denominators D_i are printed.
2. The transfer functions for only the revolute or prismatic pair variables are permitted. The transfer functions for joints with more than one degree-of-freedom -e.g. a spherical joint, are not allowed. A message is printed out and the names of the multi-degree-of-freedom joints are ignored if the transfer functions for such joints are requested.
3. The names of the revolute and prismatic joints and the points

of interest must have been previously defined in the IMP statements. If an undefined name is encountered, it is ignored, and a message is printed.

4. For each requested revolute or prismatic joint name, there are as many transfer function ratio sets as the number of force or torque inputs defined by the DATA/FORCE and DATA/TORQUE statements, or the number of base motion inputs defined by the DATA/POSITN or DATA/MOTION statements.
5. For each requested point name, there are three transfer function ratio sets for each force, torque, or base motion input representing the x, y, and z coordinate motions as measured in the global coordinate frame.
6. For each requested revolute or prismatic joint or point of interest name, the transfer function ratio sets will always be computed after the last position of the mechanical system, i.e. after the EXECUTE/HOLD statement.
7. If the velocity transfer function analysis was requested, a message will be printed out to inform the program user that the transfer function ratio sets must be multiplied by the frequency of excitation ω. If the acceleration transfer function analysis was requested, a message will be printed out to inform the program user that the transfer function ratio sets must be multiplied by the square of the frequency of excitation ω^2.
8. With the force input excitations, the numerators for the

transfer function ratio sets had only one set of terms, the modal forces.

For a revolute or prismatic joint, the position transfer function ratio sets were written in the general form

$$\frac{\theta(s)}{F(s)} = \sum_{i=1}^{m} \frac{NF_i}{s-D_i} \qquad [C10]$$

where

$\theta(s)$ = motion of a revolute or prismatic joint;

$F(s)$ = force input excitation;

$NF_i(s)$ = the ith complex modal force numerator term;

$D_i(s)$ = the ith complex eigenvalue denominator term.

For a point, the position transfer function ratio sets were written in general form

$$\frac{x(s)}{F(s)} = \sum_{i=1}^{m} \frac{NF_{xi}}{s-D_i} \qquad [C11]$$

$$\frac{y(s)}{F(s)} = \sum_{i=1}^{m} \frac{NF_{yi}}{s-D_i} \qquad [C12]$$

$$\frac{z(s)}{F(s)} = \sum_{i=1}^{m} \frac{NF_{zi}}{s-D_i} \qquad [C13]$$

where

$NF_{xi}(s)$ = complex modal force numerator term in the global X-direction;

$NF_{yi}(s)$ = complex modal force numerator term in the global Y-direction;

$NF_{zi}(s)$ = complex modal force numerator term in the global Z-direction;

$x(s)$ = motion of the point in the global X-direction;

$y(s)$ = motion of the point in the global Y-direction;

$z(s)$ = motion of the point in the global Z-direction.

9. With the base motion input excitations, the base motion of the input joints must be included with the transfer function ratio sets to complete the total system response.

With the base motion input excitations, the numerators of the transfer function ratio sets consisted of three sets of terms - the modal mass, damping, and stiffness numerators, respectively. For a revolute or prismatic joint, the transfer function ratio sets were written in general form

$$\frac{\theta(s)}{B(s)} = \sum_{i=1}^{m} \left[\frac{\omega^2 NM_i(s) + \omega NC_i(s) + NK_i(s)}{s-D_i} \right] \qquad [C14]$$

where

$\Theta(s)$ = motion of a revolute or prismatic joint

$B(s)$ = base motion input excitation;

$NM_i(s)$ = the ith complex modal mass numerator term;

$NC_i(s)$ = the ith complex modal damping numerator term;

$NK_i(s)$ = the ith complex modal stiffness numerator term;

D_i = the ith complex eigenvalue denominator term.

For a point, the transfer function ratio sets were written in the general form

$$\frac{x(s)}{B_n(s)} = \sum_{i=1}^{m} \left[\frac{\omega^2 NM_{xi}(s) + \omega NC_{xi}(s) + NK_{xi}(s)}{s-D_i} \right] + \sum_{n=1}^{k} BX_n \quad [C15]$$

$$\frac{y(s)}{B_n(s)} = \sum_{i=1}^{m} \left[\frac{\omega^2 NM_{yi}(s) + \omega NC_{yi}(s) + NK_{yi}(s)}{s-D_i} \right] + \sum_{n=1}^{k} BY_n \quad [C16]$$

$$\frac{z(s)}{B_n(s)} = \sum_{i=1}^{m} \left[\frac{\omega^2 NM_{zi}(s) + NC_{zi}(s) + NK_{zi}(s)}{s-D_i} \right] + \sum_{n=1}^{k} BZ_n \quad [C17]$$

where the subscripts x, y, and z denoted the complex numerator term components in the global X, Y, and Z directions, respectively.

APPENDIX D: TIME- AND FREQUENCY-DOMAIN ANALYSIS OF A SINGLE DEGREE-OF-FREEDOM SYSTEM

```
      real m,k,kmc
      dimension t(512),hz(512),w(512)
      dimension f(512),x(512),xdd(512)
      dimension fm(512),ft(512),disp(512),accl(512)
      complex fc(512),tfx(512),tfxdd(512)
      complex d1,d2,n1,n2,sjw
      m=0.5
      c=1.0
      k=50.0
      time=8.0
      xst=200.0/k
      twopi=6.283185
      pi=3.14159265
      t1=2.0
      wn=sqrt(k/m)
      eta=c/(2.0*wn*m)
      seta=sqrt(1.0-eta*eta)
      wd=wn*sqrt(1.0-eta*eta)
      phi=asin(eta)
      kmc=sqrt((k/m)-(c*c)/(4.0*m*m))
      d1=-c/(2.0*m)+(0.0,1.0)*kmc
      d2=-c/(2.0*m)-(0.0,1.0)*kmc
      n1=(0.0,-1.0)/(2.0*kmc)
      n2=(0.0,1.0)/(2.0*kmc)
      npwr=8
      nfft=2**npwr
      sfnfft=sqrt(float(nfft))
      delt=time/float(nfft)
      delf=delt*nfft
      do 10 i=1,nfft
      t(i)=(i-1)*delt
      hz(i)=(i-1)/delf
      w(i)=hz(i)*twopi
10    continue
      do 20 i=1,nfft
      if(t(i).gt.2.0) goto 25
      f(i)=200.0
      f(1)=0.0
      if(t(i).eq.2.0) f(i)=0.0
      x(i)=xst*(1.0-exp(-eta*wn*t(i))*cos(wd*t(i)-phi)/seta)
      xdd(i)=xst*((wd*wd-eta*eta*wn*wn)*(exp(-eta*wn*t(i))*
     1        cos(wd*t(i)-phi)/seta)-2.0*wd*eta*wn*
     2        exp(-eta*wn*t(i))*sin(wd*t(i)-phi)/seta)
      goto 20
25    f(i)=0.0
      x(i)=xst*exp(-eta*wn*t(i))/seta*((exp(eta*wn*t1)*
     1      cos(wd*t1)-1.0)*cos(wd*t(i)-phi)+
     2      (exp(eta*wn*t1)*sin(wd*t1))*sin(wd*t(i)-phi))
      xdd(i)=(eta*eta*wn*wn-wd*wd)*exp(-eta*wn*t(i))*xst/seta
     1*((exp(eta*wn*t1)*cos(wd*t1)-1.0)*cos(wd*t(i)-phi)+
     2(exp(eta*wn*t1)*sin(wd*t1))*sin(wd*t(i)-phi))
     3-2.0*eta*wn*xst*exp(-eta*wn*t(i))*wd/seta*(
     4-1.0*(exp(eta*wn*t1)*cos(wd*t1)-1.0)*sin(wd*t(i)-phi)+
     5(exp(eta*wn*t1)*sin(wd*t1))*cos(wd*t(i)-phi))
20    continue
      do 30 i=1,nfft
      fc(i)=cmplx(f(i),0.0)
30    continue
      inv=-1
      call fft(fc,inv,npwr)
      do 40 i=1,nfft
      fc(i)=fc(i)/sfnfft
      fm(i)=cabs(fc(i))
40    continue
```

```
      do 50 i=1,nfft
      sjw=cmplx(0.0,w(i))
      tfx(i)=((fc(i)*n1)/(sjw-d1))+((fc(i)*n2)/(sjw-d2))
      tfxdd(i)=sjw*sjw*tfx(i)
50    continue
      inv=1
      call fft(fc,inv,npwr)
      do 70 i=1,nfft
      fc(i)=fc(i)*sfnfft
      ft(i)=real(fc(i))
70    continue
      nfft2=nfft/2
      nffti=nfft2
      do 75 i=2,nfft2
      tfx(nfft2+i)=conjg(tfx(nffti))
      tfxdd(nfft2+i)=conjg(tfxdd(nffti))
75    nffti=nffti-1
      inv=1
      call fft(tfx,inv,npwr)
      call fft(tfxdd,inv,npwr)
      do 80 i=1,nfft
      disp(i)=2.0*sfnfft*real(tfx(i))
      accl(i)=2.0*sfnfft*real(tfxdd(i))
80    continue
      print 1000
      do 90 i=1,nfft
      print 1010,hz(i),fm(i),t(i),f(i),ft(i)
90    continue
      print 1020
      do 100 i=1,nfft
      print 1010,t(i),x(i),disp(i),xdd(i),accl(i)
100   continue
1000  format(1h0,9x,2hhz,13x,2hfm,13x,2htm,13x,2hfh,13x,2hft)
1010  format(1h ,5f15.6)
1020  format(1h0,9x,2htm,14x,1hx,11x,4hdisp,12x,3hxdd,11x,
     1       4haccl)
1050  format(1h ,2i10)
1060  format(1h ,4e20.4)
      stop
      end
      subroutine fft(cx,inv,npwr)
      complex cx(1),carg,cexp,cw,ctemp
      lx=2**npwr
      is=inv
      j=1
      sc=sqrt(1.0/lx)
      do 30 i=1,lx
      if(i.gt.j) goto 10
      ctemp=cx(j)*sc
      cx(j)=cx(i)*sc
      cx(i)=ctemp
10    m=lx/2
20    if(j.le.m) goto 30
      j=j-m
      m=m/2
      if(m.ge.1) goto 20
30    j=j+m
      l=1
40    istep=2*l
      do 50 m=1,l
      carg=(0.0,1.0)*(3.14159265*is*(m-1))/l
      cw=cexp(carg)
      do 50 i=m,lx,istep
      ctemp=cw*cx(i+l)
      cx(i+l)=cx(i)-ctemp
```

```
50      cx(i)=cx(i)+ctemp
        l=istep
        if(l.lt.lx) goto 40
        return
        end
```

hz	fm	tm	fh	ft
0.000000	49.218750	0.000000	0.000000	-0.000004
0.125000	44.461132	0.031250	200.000000	200.000015
0.250000	31.824596	0.062500	200.000000	200.000031
0.375000	15.550921	0.093750	200.000000	200.000046
0.500000	0.781250	0.125000	200.000000	200.000031
0.625000	8.439435	0.156250	200.000000	200.000031
0.750000	10.591149	0.187500	200.000000	200.000031
0.875000	6.967431	0.218750	200.000000	200.000031
1.000000	0.781250	0.250000	200.000000	200.000031
1.125000	4.428977	0.281250	200.000000	200.000031
1.250000	6.334207	0.312500	200.000000	200.000015
1.375000	4.619886	0.343750	200.000000	200.000031
1.500000	0.781250	0.375000	200.000000	200.000046
1.625000	2.880901	0.406250	200.000000	200.000046
1.750000	4.502453	0.437500	200.000000	200.000031
1.875000	3.519508	0.468750	200.000000	200.000046
2.000000	0.781250	0.500000	200.000000	200.000031
2.125000	2.057034	0.531250	200.000000	200.000031
2.250000	3.479065	0.562500	200.000000	200.000031
2.375000	2.878589	0.593750	200.000000	200.000061
2.500000	0.781250	0.625000	200.000000	200.000031
2.625000	1.543517	0.656250	200.000000	200.000031
2.750000	2.823075	0.687500	200.000000	200.000031
2.875000	2.457384	0.718750	200.000000	200.000046
3.000000	0.781250	0.750000	200.000000	200.000046
3.125000	1.191354	0.781250	200.000000	200.000046
3.250000	2.364878	0.812500	200.000000	200.000046
3.375000	2.158215	0.843750	200.000000	200.000046
3.500000	0.781250	0.875000	200.000000	200.000046
3.625000	0.933749	0.906250	200.000000	200.000046
3.750000	2.025315	0.937500	200.000000	200.000046
3.875000	1.933812	0.968750	200.000000	200.000046
4.000000	0.781250	1.000000	200.000000	200.000031
4.125000	0.736288	1.031250	200.000000	200.000031
4.250000	1.762472	1.062500	200.000000	200.000015
4.375000	1.758511	1.093750	200.000000	200.000046
4.500000	0.781250	1.125000	200.000000	200.000031
4.625000	0.579433	1.156250	200.000000	200.000031
4.750000	1.552077	1.187500	200.000000	200.000031
4.875000	1.617172	1.218750	200.000000	200.000046
5.000000	0.781250	1.250000	200.000000	200.000031
5.125000	0.451267	1.281250	200.000000	200.000015
5.250000	1.379099	1.312500	200.000000	200.000031
5.375000	1.500284	1.343750	200.000000	200.000031
5.500000	0.781250	1.375000	200.000000	200.000046
5.625000	0.344105	1.406250	200.000000	200.000046
5.750000	1.233728	1.437500	200.000000	200.000046
5.875000	1.401569	1.468750	200.000000	200.000046
6.000000	0.781250	1.500000	200.000000	200.000031
6.125000	0.252770	1.531250	200.000000	200.000031
6.250000	1.109290	1.562500	200.000000	200.000031
6.375000	1.316716	1.593750	200.000000	200.000031
6.500000	0.781250	1.625000	200.000000	200.000046
6.625000	0.173641	1.656250	200.000000	200.000046
6.750000	1.001080	1.687500	200.000000	200.000046
6.875000	1.242663	1.718750	200.000000	200.000046
7.000000	0.781250	1.750000	200.000000	200.000031
7.125000	0.104110	1.781250	200.000000	200.000031
7.250000	0.905686	1.812500	200.000000	200.000046
7.375000	1.177176	1.843750	200.000000	200.000046
7.500000	0.781250	1.875000	200.000000	200.000046
7.625000	0.042250	1.906250	200.000000	200.000031

7.750000	0.820572	1.937500	200.000000	200.000031
7.875000	1.118580	1.968750	200.000000	200.000031
8.000000	0.781250	2.000000	0.000000	-0.000002
8.125000	0.013395	2.031250	0.000000	-0.000008
8.250000	0.743812	2.062500	0.000000	-0.000008
8.375000	1.065606	2.093750	0.000000	-0.000025
8.500000	0.781250	2.125000	0.000000	-0.000006
8.625000	0.063949	2.156250	0.000000	0.000000
8.750000	0.673911	2.187500	0.000000	-0.000027
8.875000	1.017254	2.218750	0.000000	0.000004
9.000000	0.781250	2.250000	0.000000	-0.000010
9.125000	0.110295	2.281250	0.000000	-0.000011
9.250000	0.609694	2.312500	0.000000	0.000000
9.375000	0.972740	2.343750	0.000000	0.000002
9.500000	0.781250	2.375000	0.000000	-0.000020
9.625000	0.153133	2.406250	0.000000	-0.000006
9.750000	0.550218	2.437500	0.000000	-0.000008
9.875000	0.931435	2.468750	0.000000	-0.000015
10.000000	0.781250	2.500000	0.000000	-0.000006
10.125000	0.193034	2.531250	0.000000	0.000003
10.250000	0.494721	2.562500	0.000000	-0.000002
10.375000	0.892823	2.593750	0.000000	-0.000017
10.500000	0.781250	2.625000	0.000000	0.000001
10.625000	0.230463	2.656250	0.000000	0.000003
10.750000	0.442573	2.687500	0.000000	-0.000006
10.875000	0.856480	2.718750	0.000000	-0.000012
11.000000	0.781250	2.750000	0.000000	-0.000018
11.125000	0.265809	2.781250	0.000000	0.000002
11.250000	0.393248	2.812500	0.000000	-0.000006
11.375000	0.822051	2.843750	0.000000	-0.000011
11.500000	0.781250	2.875000	0.000000	-0.000005
11.625000	0.299397	2.906250	0.000000	-0.000006
11.750000	0.346305	2.937500	0.000000	-0.000017
11.875000	0.789233	2.968750	0.000000	-0.000001
12.000000	0.781250	3.000000	0.000000	0.000002
12.125000	0.331507	3.031250	0.000000	0.000007
12.250000	0.301362	3.062500	0.000000	0.000009
12.375000	0.757770	3.093750	0.000000	0.000000
12.500000	0.781250	3.125000	0.000000	0.000004
12.625000	0.362380	3.156250	0.000000	0.000014
12.750000	0.258090	3.187500	0.000000	-0.000002
12.875000	0.727435	3.218750	0.000000	0.000006
13.000000	0.781250	3.250000	0.000000	-0.000001
13.125000	0.392226	3.281250	0.000000	0.000003
13.250000	0.216201	3.312500	0.000000	0.000011
13.375000	0.698030	3.343750	0.000000	0.000005
13.500000	0.781250	3.375000	0.000000	-0.000003
13.625000	0.421234	3.406250	0.000000	0.000008
13.750000	0.175436	3.437500	0.000000	-0.000007
13.875000	0.669377	3.468750	0.000000	0.000007
14.000000	0.781250	3.500000	0.000000	-0.000012
14.125000	0.449573	3.531250	0.000000	0.000015
14.250000	0.135560	3.562500	0.000000	-0.000016
14.375000	0.641314	3.593750	0.000000	0.000004
14.500000	0.781250	3.625000	0.000000	-0.000002
14.625000	0.477398	3.656250	0.000000	0.000003
14.750000	0.096358	3.687500	0.000000	-0.000006
14.875000	0.613690	3.718750	0.000000	-0.000007
15.000000	0.781250	3.750000	0.000000	-0.000005
15.125000	0.504855	3.781250	0.000000	0.000015
15.250000	0.057628	3.812500	0.000000	-0.000011
15.375000	0.586367	3.843750	0.000000	-0.000005
15.500000	0.781250	3.875000	0.000000	-0.000005
15.625000	0.532080	3.906250	0.000000	0.000013
15.750000	0.019178	3.937500	0.000000	0.000000

15.875000	0.559208	3.968750	0.000000	0.000006
16.000000	0.781250	4.000000	0.000000	0.000004
16.125000	0.559207	4.031250	0.000000	0.000000
16.250000	0.019178	4.062500	0.000000	-0.000015
16.375000	0.532080	4.093750	0.000000	-0.000015
16.500000	0.781250	4.125000	0.000000	-0.000008
16.625000	0.586366	4.156250	0.000000	-0.000008
16.750000	0.057628	4.187500	0.000000	-0.000008
16.875000	0.504855	4.218750	0.000000	-0.000023
17.000000	0.781250	4.250000	0.000000	0.000000
17.125000	0.613690	4.281250	0.000000	0.000008
17.250000	0.096358	4.312500	0.000000	-0.000015
17.375000	0.477399	4.343750	0.000000	-0.000015
17.500000	0.781250	4.375000	0.000000	-0.000008
17.625000	0.641313	4.406250	0.000000	-0.000008
17.750000	0.135559	4.437500	0.000000	-0.000008
17.875000	0.449574	4.468750	0.000000	-0.000031
18.000000	0.781250	4.500000	0.000000	-0.000015
18.125000	0.669377	4.531250	0.000000	-0.000008
18.250000	0.175436	4.562500	0.000000	-0.000023
18.375000	0.421234	4.593750	0.000000	-0.000023
18.500000	0.781250	4.625000	0.000000	-0.000015
18.625000	0.698030	4.656250	0.000000	-0.000015
18.750000	0.216201	4.687500	0.000000	-0.000015
18.875000	0.392226	4.718750	0.000000	-0.000031
19.000000	0.781250	4.750000	0.000000	-0.000008
19.125000	0.727435	4.781250	0.000000	-0.000023
19.250000	0.258090	4.812500	0.000000	-0.000023
19.375000	0.362380	4.843750	0.000000	-0.000015
19.500000	0.781250	4.875000	0.000000	-0.000008
19.625000	0.757770	4.906250	0.000000	-0.000031
19.750000	0.301361	4.937500	0.000000	-0.000008
19.875000	0.331507	4.968750	0.000000	-0.000031
20.000000	0.781250	5.000000	0.000000	-0.000015
20.125000	0.789233	5.031250	0.000000	-0.000008
20.250000	0.346304	5.062500	0.000000	-0.000015
20.375000	0.299397	5.093750	0.000000	-0.000038
20.500000	0.781250	5.125000	0.000000	-0.000015
20.625000	0.822050	5.156250	0.000000	-0.000031
20.750000	0.393248	5.187500	0.000000	-0.000008
20.875000	0.265809	5.218750	0.000000	-0.000023
21.000000	0.781250	5.250000	0.000000	-0.000015
21.125000	0.856480	5.281250	0.000000	0.000000
21.250000	0.442573	5.312500	0.000000	-0.000023
21.375000	0.230463	5.343750	0.000000	-0.000023
21.500000	0.781250	5.375000	0.000000	-0.000008
21.625000	0.892823	5.406250	0.000000	-0.000023
21.750000	0.494721	5.437500	0.000000	-0.000008
21.875000	0.193034	5.468750	0.000000	-0.000023
22.000000	0.781250	5.500000	0.000000	0.000000
22.125000	0.931435	5.531250	0.000000	-0.000015
22.250000	0.550218	5.562500	0.000000	0.000000
22.375000	0.153133	5.593750	0.000000	-0.000015
22.500000	0.781250	5.625000	0.000000	-0.000008
22.625000	0.972740	5.656250	0.000000	-0.000015
22.750000	0.609693	5.687500	0.000000	0.000000
22.875000	0.110295	5.718750	0.000000	-0.000023
23.000000	0.781250	5.750000	0.000000	0.000000
23.125000	1.017254	5.781250	0.000000	-0.000023
23.250000	0.673911	5.812500	0.000000	-0.000031
23.375000	0.063950	5.843750	0.000000	-0.000008
23.500000	0.781250	5.875000	0.000000	-0.000008
23.625000	1.065605	5.906250	0.000000	-0.000031
23.750000	0.743812	5.937500	0.000000	0.000008
23.875000	0.013397	5.968750	0.000000	-0.000015

24.000000	0.781250	6.000000	0.000000	0.000002
24.125000	1.118582	6.031250	0.000000	-0.000008
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24.375000	0.042250	6.093750	0.000000	-0.000005
24.500000	0.781250	6.125000	0.000000	-0.000009
24.625000	1.177176	6.156250	0.000000	-0.000016
24.750000	0.905686	6.187500	0.000000	0.000012
24.875000	0.104110	6.218750	0.000000	-0.000004
25.000000	0.781250	6.250000	0.000000	-0.000013
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25.250000	1.001079	6.312500	0.000000	0.000000
25.375000	0.173640	6.343750	0.000000	-0.000010
25.500000	0.781250	6.375000	0.000000	-0.000011
25.625000	1.316716	6.406250	0.000000	-0.000017
25.750000	1.109289	6.437500	0.000000	-0.000007
25.875000	0.252769	6.468750	0.000000	0.000000
26.000000	0.781250	6.500000	0.000000	-0.000002
26.125000	1.401569	6.531250	0.000000	-0.000018
26.250000	1.233729	6.562500	0.000000	0.000002
26.375000	0.344105	6.593750	0.000000	-0.000006
26.500000	0.781250	6.625000	0.000000	-0.000016
26.625000	1.500284	6.656250	0.000000	-0.000018
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26.875000	0.451267	6.718750	0.000000	-0.000003
27.000000	0.781250	6.750000	0.000000	-0.000012
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27.250000	1.552077	6.812500	0.000000	-0.000009
27.375000	0.579433	6.843750	0.000000	-0.000019
27.500000	0.781250	6.875000	0.000000	-0.000018
27.625000	1.758511	6.906250	0.000000	-0.000010
27.750000	1.762471	6.937500	0.000000	-0.000006
27.875000	0.736288	6.968750	0.000000	-0.000006
28.000000	0.781250	7.000000	0.000000	-0.000009
28.125000	1.933812	7.031250	0.000000	-0.000015
28.250000	2.025315	7.062500	0.000000	-0.000002
28.375000	0.933749	7.093750	0.000000	0.000000
28.500000	0.781250	7.125000	0.000000	-0.000019
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28.875000	1.191353	7.218750	0.000000	-0.000014
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29.250000	2.823075	7.312500	0.000000	-0.000011
29.375000	1.543517	7.343750	0.000000	-0.000005
29.500000	0.781250	7.375000	0.000000	-0.000027
29.625000	2.878589	7.406250	0.000000	-0.000023
29.750000	3.479063	7.437500	0.000000	-0.000024
29.875000	2.057034	7.468750	0.000000	-0.000014
30.000000	0.781250	7.500000	0.000000	-0.000011
30.125000	3.519509	7.531250	0.000000	-0.000022
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30.500000	0.781250	7.625000	0.000000	-0.000029
30.625000	4.619887	7.656250	0.000000	-0.000026
30.750000	6.334208	7.687500	0.000000	-0.000032
30.875000	4.428975	7.718750	0.000000	-0.000008
31.000000	0.781250	7.750000	0.000000	-0.000018
31.125000	6.967433	7.781250	0.000000	-0.000007
31.250000	10.591149	7.812500	0.000000	-0.000004
31.375000	8.439436	7.843750	0.000000	-0.000018
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31.625000	15.550923	7.906250	0.000000	-0.000013
31.750000	31.824602	7.937500	0.000000	-0.000023
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tm	x	disp	xdd	accl
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0.156250	3.601004	3.061166	-28.863785	24.878042
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0.218750	5.564449	5.136336	-209.556000	-172.865204
0.250000	6.281662	5.967553	-266.267029	-243.180237
0.281250	6.741176	6.563880	-294.539795	-285.852356
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0.343750	6.805415	6.912506	-264.857819	-283.916138
0.375000	6.438616	6.666482	-213.134918	-242.916763
0.406250	5.865057	6.185073	-144.532623	-181.263794
0.437500	5.151119	5.527578	-66.515121	-106.618904
0.468750	4.372348	4.766547	13.021210	-26.221880
0.500000	3.605798	3.979589	86.515991	51.529522
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0.593750	2.026272	2.166415	213.654816	206.326843
0.625000	1.877281	1.911749	215.046448	218.330124
0.656250	1.936639	1.868634	196.161026	209.244522
0.687500	2.186148	2.028231	160.000473	180.937164
0.718750	2.590860	2.363338	111.006836	137.465240
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0.906250	5.419273	5.309267	-154.807953	-148.606705
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0.968750	5.515974	5.558147	-145.111481	-153.942596
1.000000	5.347407	5.456358	-119.913033	-134.679260
1.031250	5.062533	5.224019	-84.981575	-103.940041
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1.437500	4.062604	3.913649	-25.095976	-10.833735
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1.500000	4.569238	4.466004	-69.580956	-61.320145
1.531250	4.730843	4.664152	-80.993622	-76.896614
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2.312500	-2.673601	-2.612528	276.032318	260.112610
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2.406250	-1.965136	-1.571745	161.272171	118.399117
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2.500000	0.085155	0.461144	-53.926949	-87.535423
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2.562500	1.318552	1.514188	-163.094620	-175.711014
2.593750	1.721512	1.803191	-192.187180	-192.841782
2.625000	1.938408	1.905349	-201.470840	-191.105743
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2.687500	1.796515	1.573381	-163.734589	-136.797516
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2.750000	1.032808	0.721237	-72.240784	-40.275471
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2.843750	-0.504860	-0.728598	80.221085	98.473267
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2.968750	-1.434302	-1.341230	140.881989	127.028885
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3.062500	-0.789582	-0.563518	56.676888	33.245533
3.093750	-0.420100	-0.193079	17.407463	-4.607660
3.125000	-0.033559	0.173047	-21.117075	-39.884762
3.156250	0.332716	0.500741	-55.324871	-69.134186
3.187500	0.645558	0.761562	-82.265144	-90.260460
3.218750	0.878828	0.935071	-99.848526	-101.495178
3.250000	1.015443	1.010265	-106.990852	-102.650841
3.281250	1.048432	0.986040	-103.654030	-93.969772
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3.343750	0.825492	0.680579	-70.173622	-53.920063
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3.406250	0.335367	0.169133	-15.687305	0.597819
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3.468750	-0.216358	-0.342328	37.972012	48.507389
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3.750000	-0.060246	0.052743	-7.109207	-17.533659
3.781250	0.138324	0.232617	-25.919340	-33.856117
3.812500	0.311876	0.379750	-41.156212	-46.085785
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4.312500	-0.262689	-0.200750	20.440777	13.843506
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4.468750	0.224273	0.247350	-26.522600	-27.729252
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4.531250	0.296202	0.285029	-30.054853	-27.902271
4.562500	0.288627	0.262658	-27.485262	-24.043015
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5.062500	0.068988	0.091497	-9.999174	-11.759251
5.093750	0.111915	0.126059	-13.548800	-14.392799
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6.250000	-0.018481	-0.008813	0.803384	-0.117599
6.281250	-0.002037	0.006804	-0.839339	-1.664276
6.312500	0.013602	0.020818	-2.303702	-2.879089
6.343750	0.027016	0.032046	-3.463044	-3.834480
6.375000	0.037081	0.039567	-4.226954	-4.284706
6.406250	0.043054	0.042934	-4.547677	-4.395355
6.437500	0.044622	0.042048	-4.421946	-3.993649
6.468750	0.041905	0.037286	-3.888609	-3.337140
6.500000	0.035418	0.029292	-3.022465	-2.308228
6.531250	0.025999	0.019052	-1.924973	-1.226814
6.562500	0.014708	0.007623	-0.713218	0.009384
6.593750	0.002721	-0.003809	0.492063	1.061447
6.625000	-0.008795	-0.014212	1.578143	2.061813
6.656250	-0.018788	-0.022628	2.450232	2.692726
6.687500	-0.026412	-0.028432	3.039392	3.151451
6.718750	-0.031093	-0.031191	3.307475	3.163879
6.750000	-0.032571	-0.030879	3.248740	3.004887
6.781250	-0.030901	-0.027660	2.888516	2.442888
6.812500	-0.026431	-0.022066	2.278881	1.820255
6.843750	-0.019750	-0.014712	1.492193	0.925388
6.875000	-0.011619	-0.006449	0.613072	0.141354
6.906250	-0.002889	0.001947	-0.270373	-0.756660
6.937500	0.005582	0.009622	-1.074904	-1.376038
6.968750	0.013017	0.015959	-1.729742	-1.987034
7.000000	0.018778	0.020381	-2.182498	-2.210686
7.031250	0.022428	0.022656	-2.402886	-2.382618
7.062500	0.023750	0.022630	-2.384290	-2.144986
7.093750	0.022762	0.020518	-2.142939	-1.902078
7.125000	0.019696	0.016567	-1.714986	-1.306898
7.156250	0.014967	0.011342	-1.151924	-0.820568
7.187500	0.009119	0.005325	-0.514825	-0.083534
7.218750	0.002769	-0.000782	0.132030	0.405514
7.250000	-0.003456	-0.006494	0.727296	1.046608
7.281250	-0.008980	-0.011198	1.218192	1.321587
7.312500	-0.013326	-0.014621	1.564970	1.694775
7.343750	-0.016157	-0.016405	1.743786	1.636522
7.375000	-0.017299	-0.016603	1.748026	1.689394
7.406250	-0.016748	-0.015164	1.587872	1.306496
7.437500	-0.014658	-0.012460	1.288319	1.111995
7.468750	-0.011318	-0.008671	0.885951	0.521694
7.500000	-0.007118	-0.004383	0.424763	0.238228
7.531250	-0.002503	0.000150	-0.048369	-0.389347
7.562500	0.002066	0.004291	-0.488283	-0.582161
7.593750	0.006165	0.007893	-0.855705	-1.092842
7.625000	0.009437	0.010407	-1.120515	-1.058290
7.656250	0.011624	0.011937	-1.264060	-1.369972
7.687500	0.012587	0.012086	-1.280218	-1.053333
7.718750	0.012310	0.011286	-1.175184	-1.182030
7.750000	0.010894	0.009244	-0.966162	-0.613272
7.781250	0.008541	0.006737	-0.679089	-0.666470
7.812500	0.005528	0.003402	-0.345637	0.075689
7.843750	0.002179	0.000396	0.000068	-0.074917
7.875000	-0.001172	-0.003067	0.324814	0.784458
7.906250	-0.004210	-0.005166	0.599392	0.299320
7.937500	-0.006669	-0.008024	0.801051	1.397106
7.968750	-0.008352	-0.007418	0.915270	0.033172

APPENDIX E: PARAMETER VALUES AND IMP ANALYSIS OF A TWO DEGREE-OF-FREEDOM SYSTEM WITH A SINGLE FORCE EXCITATION

$m = m_1 = m_2 = 193.044/386.088\ lb_f/sec^2\text{-}in$

$c = 2.0\ lb_f\text{-}sec/in$

$k = 50.0\ lb_f/in$

$g = 386.088\ in/sec^2$

```
**********************************************************************
*                              IMP-75                                *
*                  THE INTEGRATED MECHANISMS PROGRAM                 *
**********************************************************************

REMARK/  EXAMPLE#2 FOR IMP FORCE TRANSFER FUNCTION ANALYSIS

SYSTEM=  TWO DEGREE-OF-FREEDOM MASS-SPRING-DAMPER SYSTEM

GROUND=BASE

PRISM(BASE,LNK2)=PRM3

    DATA/LINK(BASE,PRM3)=0.0,2.0,0.0/0.0,3.0,0.0/0.0,2.0,1.0

    DATA/LINK(LNK2,PRM3)=0.0,2.0,0.0/0.0,3.0,0.0/0.0,2.0,1.0

PRISM(LNK2,LNK1)=PRM2

    DATA/LINK(LNK2,PRM2)=0.0,1.0,0.0/0.0,2.0,0.0/0.0,1.0,1.0

    DATA/LINK(LNK1,PRM2)=0.0,1.0,0.0/0.0,2.0,0.0/0.0,1.0,1.0

PRISM(LNK1,BASE)=PRM1

    DATA/LINK(LNK1,PRM1)=0.0,0.0,0.0/0.0,1.0,0.0/0.0,0.0,1.0

    DATA/LINK(BASE,PRM1)=0.0,0.0,0.0/0.0,1.0,0.0/0.0,0.0,1.0

    DATA/WEIGHT(LNK1,PRM1)=193.044,0.0,0.0,0.0

    DATA/WEIGHT(LNK2,PRM2)=193.044,0.0,0.0,0.0

    DATA/SPRING(PRM1)=50.0,-7.762

    DATA/SPRING(PRM2)=50.0,-3.881

    DATA/SPRING(PRM3)=50.0,-0.02

    DATA/DAMPER(PRM2)=2.0

POINT(LNK1)=PNTA,PNTB

    DATA/POINT(PNTA,PRM1)=0.0,0.0,0.0

    DATA/POINT(PNTB,PRM1)=0.0,0.0, 1.0

POINT(LNK2)=PNTC

    DATA/POINT(PNTC,PRM3)=0.0,0.0,0.0

FORCE(PNTA,PNTB,PNTA)=EXCT

    DATA/FORCE(EXCT)=1.0

REMARK/  GRAVITATIONAL CONSTANTS

    DATA/GRAV=0.0,-386.088,0.0
```

```
ZERO(EQULIB)=0.01
REMARK/  REQUEST STATEMENTS
FIND/EQULIB
PRINT/FREQ
PRINT/POSITN(ALL)
PRINT/FORCE(ALL)
PRINT/DYNAM/FORCE/POSITN/EXCT,PRM3,PNTA,PNTC,PRM1,PRM2,PRM3/
EXECUTE/HOLD
```

```
*********************************************************************
*                          ANALYSIS OF                              *
*         TWO DEGREE-OF-FREEDOM MASS-SPRING-DAMPER SYSTEMST         *
*                          STATIC MODE                              *
*                ON      i                AT    -               *
*********************************************************************

POSITION    0

THE GENERALIZED COORDINATES ARE
    JOINT             SET BY       POSITION       VELOCITY     ACCELERATION
                                    DEG,IN      RAD,IN/SEC    RAD,IN/SEC2
    PRM3    1          IMP           0.000           0.0             0.0
    PRM2    1          IMP           0.000           0.0             0.0

DEGREE OF FREEDOM =  2.  QUALITY INDEX =  0.100E 01

DYNAMIC CHARACTERISTICS
       FGC         NATURAL         DAMPING         DAMPED           DECAY
      JOINT       FREQUENCY         RATIO         FREQUENCY          RATE
       ---         RAD/SEC           ---           RAD/SEC           1/SEC
      PRM2         17.320           0.231          16.851          -4.000
      PRM3         10.000          -0.000          10.000           0.000

POSITION RESULTS (DEG,IN)              X               Y               Z

  JNT. PRM3            0.000

  JNT. PRM2            0.000

  JNT. PRM1           -0.000

  PNT. PNTA            0.000          0.0           0.000             0.0

  PNT. PNTB            1.000          0.0           1.000             0.0

  PNT. PNTC            2.000          0.0           2.000             0.0

FORCE RESULTS (IN-LB,LB)               X               Y               Z

  JNT. PRM3 FROM BASE ONTO LNK2
       FORC            1.006         ----            ----          -1.006
       TORQ            0.0           ----            ----            ----

  JNT. PRM2 FROM LNK2 ONTO LNK1
       FORC          194.050         ----            ----        -194.050
       TORQ            0.0           ----            ----            ----

  JNT. PRM1 FROM LNK1 ONTO BASE
       FORC          388.094         ----            ----        -388.094
       TORQ            0.0           ----            ----            ----

TRANSFER FUNCTION ANALYSIS : FORC INPUT(S)

  EIGENVECTORS(2*NDOF(FORCE) OR 2*NFGC(MOTION) :

  COLUMN :   1
  PSI( 1)=(  0.139141E 02,  0.650971E 01)
  PSI( 2)=( -0.695706E 01, -0.325469E 01)
```

```
PSI( 3)=(  0.180143E 00, -0.868394E 00)
PSI( 4)=( -0.900718E-01,  0.434201E 00)

COLUMN :  2
PSI( 1)=(  0.139141E 02, -0.650971E 01)
PSI( 2)=( -0.695706E 01,  0.325469E 01)
PSI( 3)=(  0.180143E 00,  0.868394E 00)
PSI( 4)=( -0.900718E-01, -0.434201E 00)

COLUMN :  3
PSI( 1)=(  0.158310E-03,  0.395857E-03)
PSI( 2)=( -0.945031E 01, -0.329941E 01)
PSI( 3)=(  0.305176E-04, -0.870228E-05)
PSI( 4)=( -0.329938E 00,  0.945021E 00)

COLUMN :  4
PSI( 1)=(  0.158310E-03, -0.395857E-03)
PSI( 2)=( -0.945031E 01,  0.329941E 01)
PSI( 3)=(  0.305176E-04,  0.870228E-05)
PSI( 4)=( -0.329938E 00, -0.945021E 00)

EIGENVALUES - COMMON DENOMINATORS
   D( 1) = S-( -0.399999E 01,  0.168515E 02)
   D( 2) = S-( -0.399999E 01, -0.168515E 02)
   D( 3) = S-(  0.991821E-04,  0.999972E 01)
   D( 4) = S-(  0.991821E-04, -0.999972E 01)

FORCE OR TORQUE INPUT EXCITATION : EXCT

POSITION TRANSFER FUNCTION : POIN - PNTA

FIRST PARTIAL DERIVATIVES FOR POINT : PNTA WRT -

FGC( 1) : PRM2
WMAT(11) =   0.0           WMAT(21) =  -0.100000E 01  WMAT(31) =   0.0
WMAT(12) =   0.0           WMAT(22) =   0.0           WMAT(32) =   0.0
WMAT(13) =   0.0           WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0           WMAT(24) =   0.0           WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : PNTA WRT -

FGC( 2) : PRM3
WMAT(11) =   0.0           WMAT(21) =  -0.100000E 01  WMAT(31) =   0.0
WMAT(12) =   0.0           WMAT(22) =   0.0           WMAT(32) =   0.0
WMAT(13) =   0.0           WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0           WMAT(24) =   0.0           WMAT(34) =   0.0

X-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.0         ,   0.0          )
NF( 2) = (   0.0         ,   0.0          )
NF( 3) = (   0.0         ,   0.0          )
NF( 4) = (   0.0         ,   0.0          )

Y-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (  -0.417233E-06,  -0.296684E-01)
NF( 2) = (  -0.417233E-06,   0.296684E-01)
NF( 3) = (  -0.109849E-05,  -0.499987E-01)
NF( 4) = (  -0.109849E-05,   0.499987E-01)

Z-COMPONENT NUMERATORS(2*NFGC) :
```

```
NF( 1) = (   0.0          ,  0.0          )
NF( 2) = (   0.0          ,  0.0          )
NF( 3) = (   0.0          ,  0.0          )
NF( 4) = (   0.0          ,  0.0          )

POSITION TRANSFER FUNCTION : POIN - PNTC

FIRST PARTIAL DERIVATIVES FOR POINT : PNTC WRT -

FGC( 1) : PRM2
WMAT(11) =   0.0            WMAT(21) =   0.0             WMAT(31) =   0.0
WMAT(12) =   0.0            WMAT(22) =   0.0             WMAT(32) =   0.0
WMAT(13) =   0.0            WMAT(23) =   0.0             WMAT(33) =   0.0
WMAT(14) =   0.0            WMAT(24) =   0.0             WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : PNTC WRT -

FGC( 2) : PRM3
WMAT(11) =   0.0            WMAT(21) =  -0.100000E 01    WMAT(31) =   0.0
WMAT(12) =   0.0            WMAT(22) =   0.0             WMAT(32) =   0.0
WMAT(13) =   0.0            WMAT(23) =   0.0             WMAT(33) =   0.0
WMAT(14) =   0.0            WMAT(24) =   0.0             WMAT(34) =   0.0

X-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.0          ,  0.0          )
NF( 2) = (   0.0          ,  0.0          )
NF( 3) = (   0.0          ,  0.0          )
NF( 4) = (   0.0          ,  0.0          )

Y-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.491738E-06,   0.296689E-01)
NF( 2) = (   0.491738E-06,  -0.296689E-01)
NF( 3) = (   0.197440E-06,  -0.499996E-01)
NF( 4) = (   0.197440E-06,   0.499996E-01)

Z-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.0          ,  0.0          )
NF( 2) = (   0.0          ,  0.0          )
NF( 3) = (   0.0          ,  0.0          )
NF( 4) = (   0.0          ,  0.0          )

POSITION TRANSFER FUNCTION : JOIN - PRM1
NF( 1) = (  -0.417233E-06,  -0.296683E-01)
NF( 2) = (  -0.417233E-06,   0.296683E-01)
NF( 3) = (  -0.109896E-05,  -0.499987E-01)
NF( 4) = (  -0.109896E-05,   0.499987E-01)

POSITION TRANSFER FUNCTION : JOIN - PRM2
NF( 1) = (   0.908971E-06,   0.593372E-01)
NF( 2) = (   0.908971E-06,  -0.593372E-01)
NF( 3) = (   0.129593E-05,  -0.912870E-06)
NF( 4) = (   0.129593E-05,   0.912870E-06)

FORCE OR TORQUE INPUT EXCITATION : PRM3

POSITION TRANSFER FUNCTION : JOIN - PRM3
NF( 1) = (  -0.569969E-06,  -0.296694E-01)
NF( 2) = (  -0.569969E-06,   0.296694E-01)
NF( 3) = (   0.150874E-05,  -0.500006E-01)
NF( 4) = (   0.150874E-05,   0.500006E-01)
```

```
POSITION TRANSFER FUNCTION : POIN - PNTA

FIRST PARTIAL DERIVATIVES FOR POINT : PNTA WRT -

FGC( 1) : PRM2
WMAT(11) =   0.0            WMAT(21) =  -0.100000E 01  WMAT(31) =   0.0
WMAT(12) =   0.0            WMAT(22) =   0.0           WMAT(32) =   0.0
WMAT(13) =   0.0            WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0            WMAT(24) =   0.0           WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : PNTA WRT -

FGC( 2) : PRM3
WMAT(11) =   0.0            WMAT(21) =  -0.100000E 01  WMAT(31) =   0.0
WMAT(12) =   0.0            WMAT(22) =   0.0           WMAT(32) =   0.0
WMAT(13) =   0.0            WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0            WMAT(24) =   0.0           WMAT(34) =   0.0

X-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.0          ,  0.0          )
NF( 2) = (   0.0          ,  0.0          )
NF( 3) = (   0.0          ,  0.0          )
NF( 4) = (   0.0          ,  0.0          )

Y-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (  -0.495464E-06,  -0.296689E-01)
NF( 2) = (  -0.495464E-06,   0.296689E-01)
NF( 3) = (  -0.212765E-06,   0.499997E-01)
NF( 4) = (  -0.212765E-06,  -0.499997E-01)

Z-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.0          ,  0.0          )
NF( 2) = (   0.0          ,  0.0          )
NF( 3) = (   0.0          ,  0.0          )
NF( 4) = (   0.0          ,  0.0          )

POSITION TRANSFER FUNCTION : POIN - PNTC

FIRST PARTIAL DERIVATIVES FOR POINT : PNTC WRT -

FGC( 1) : PRM2
WMAT(11) =   0.0            WMAT(21) =   0.0           WMAT(31) =   0.0
WMAT(12) =   0.0            WMAT(22) =   0.0           WMAT(32) =   0.0
WMAT(13) =   0.0            WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0            WMAT(24) =   0.0           WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : PNTC WRT -

FGC( 2) : PRM3
WMAT(11) =   0.0            WMAT(21) =  -0.100000E 01  WMAT(31) =   0.0
WMAT(12) =   0.0            WMAT(22) =   0.0           WMAT(32) =   0.0
WMAT(13) =   0.0            WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0            WMAT(24) =   0.0           WMAT(34) =   0.0

X-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.0          ,  0.0          )
NF( 2) = (   0.0          ,  0.0          )
NF( 3) = (   0.0          ,  0.0          )
NF( 4) = (   0.0          ,  0.0          )
```

```
Y-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (  0.569969E-06,  0.296694E-01)
NF( 2) = (  0.569969E-06, -0.296694E-01)
NF( 3) = ( -0.150874E-05,  0.500006E-01)
NF( 4) = ( -0.150874E-05, -0.500006E-01)

Z-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (  0.0         ,  0.0         )
NF( 2) = (  0.0         ,  0.0         )
NF( 3) = (  0.0         ,  0.0         )
NF( 4) = (  0.0         ,  0.0         )

POSITION TRANSFER FUNCTION : JOIN - PRM1
NF( 1) = ( -0.499189E-06, -0.296689E-01)
NF( 2) = ( -0.499189E-06,  0.296689E-01)
NF( 3) = ( -0.208616E-06,  0.499997E-01)
NF( 4) = ( -0.208616E-06, -0.499997E-01)

POSITION TRANSFER FUNCTION : JOIN - PRM2
NF( 1) = (  0.106543E-05,  0.593383E-01)
NF( 2) = (  0.106543E-05, -0.593383E-01)
NF( 3) = ( -0.129598E-05,  0.912854E-06)
NF( 4) = ( -0.129598E-05, -0.912854E-06)

********************
```

APPENDIX F: PARAMETER VALUES FOR A TWO DEGREE-OF-FREEDOM SYSTEM WITH TWO FORCE EXCITATIONS

$m = m_1 = m_2 = 193.044/386.088\ lb_f\text{-}sec^2/in$

$c = 2.0\ lb_f\text{-}sec/in$

$k = 50.0\ lb_f/in$

$g = 386.088\ in/sec^2$

APPENDIX G: PARAMETER VALUES FOR A PENDULUM SYSTEM

$m = 10.0/386.088\ lb_f\text{-}sec^2/in$

$l = 10.0\ in$

$I = 83.3\ lb_f\text{-}sec^2\text{-}in$

$g = 386.088\ in/sec^2$

APPENDIX H: PARAMETER VALUES AND IMP ANALYSIS OF A DOUBLE PENDULUM SYSTEM WITH A FLEXIBLE COUPLER

$m = m_1 = m_2 = 10.0/386.088\ lb_f\text{-}sec^2/in$

$I = 83.3\ lb_f\text{-}sec^2\text{-}in$

$l = 10.0\ in$

$k = 50.0\ lb_f/in$

$c = 0.1\ lb_f\text{-}sec/in$

$g = 386.088\ in/sec^2$

```
************************************************************************
*                              IMP-75                                  *
*                  THE INTEGRATED MECHANISMS PROGRAM                   *
************************************************************************

SYSTEM=  DOUBLE-PENDULUM WITH FLEXIBLE COUPLING

REVLUT(BASE,LNK1)=RLT1

    DATA/LINK(BASE,RLT1)=0.0,0.0,0.0/0.0,0.0,1.0/1.0,0.0,0.0

    DATA/LINK(LNK1,RLT1)=0.0,0.0,0.0/0.0,0.0,1.0/0.0,-1.0,0.0

REVLUT(LNK1,LNK2)=RLT2

    DATA/LINK(LNK1,RLT2)=0.0,-5.0,0.0/0.0,-5.0,1.0/0.0,-6.0,0.0

    DATA/LINK(LNK2,RLT2)=0.0,-5.0,0.0/0.0,-5.0,1.0/1.0,-5.0,0.0

PRISM(LNK2,LNK3)=PRM1

    DATA/LINK(LNK2,PRM1)=0.0,-5.0,0.0/1.0,-5.0,0.0/0.0,-4.0,0.0

    DATA/LINK(LNK3,PRM1)=10.0,-5.0,0.0/11.0,-5.0,0.0/10.0,-4.0,0.0

REVLUT(LNK3,LNK4)=RLT3

    DATA/LINK(LNK3,RLT3)=10.0,-5.0,0.0/10.0,-5.0,1.0/11.0,-5.0,0.0

    DATA/LINK(LNK4,RLT3)=10.0,-5.0,0.0/10.0,-5.0,1.0/10.0,-4.0,0.0

REVLUT(LNK4,BASE)=RLT4

    DATA/LINK(LNK4,RLT4)=10.0,0.0,0.0/10.0,0.0,1.0/10.0,-1.0,0.0

    DATA/LINK(BASE,RLT4)=10.0,0.0,0.0/10.0,0.0,1.0/11.0,0.0,0.0

    DATA/WEIGHT(LNK1,RLT2)=10.0,0.0,0.0,0.0

    DATA/INERTIA(LNK1,RLT2)=0.0,83.4,83.4,0.0,0.0,0.0

    DATA/WEIGHT(LNK4,RLT3)=10.0,0.0,0.0,0.0

    DATA/INERTIA(LNK4,RLT3)=0.0,83.4,83.4,0.0,0.0,0.0

    DATA/SPRING(PRM1)=50.0,10.0

    DATA/DAMPER(PRM1)=0.1

    DATA/TORQUE(RLT1)=5.0

    DATA/SPRING(RLT1)=50.0,-90.1

POINT(LNK1)=PNT2

    DATA/POINT(PNT2,RLT2)=0.0,0.0,0.0

    DATA/GRAVITY=0.0,-386.088,0.0
```

```
REMARK/  IMP STATEMENTS
FIND/EQULIB
PRINT/FREQ
PRINT/POSITN(ALL)
PRINT/FORCE(ALL)
PRINT/DYNAM/FORCE/POSITN/RLT1/PNT2,RLT2,RLT3,RLT4,PRM1/
EXECUTE/HOLD
```

```
*********************************************************************
*                         ANALYSIS OF                               *
*              DOUBLE-PENDULUM WITH FLEXIBLE COUPLINGLI             *
*                         STATIC MODE                               *
*                 ON      i              AT   -                   *
*********************************************************************

POSITION    0

THE GENERALIZED COORDINATES ARE
    JOINT              SET BY         POSITION         VELOCITY      ACCELERATION
                                       DEG,IN        RAD,IN/SEC      RAD,IN/SEC2
    PRM1    1          IMP             10.000            0.0              0.0
    RLT3    1          IMP             90.000            0.0              0.0

DEGREE OF FREEDOM =  2.  QUALITY INDEX =  0.500E 02

DYNAMIC CHARACTERISTICS
       FGC          NATURAL          DAMPING          DAMPED             DECAY
      JOINT        FREQUENCY          RATIO          FREQUENCY           RATE
       ---          RAD/SEC            ---            RAD/SEC            1/SEC
      RLT3          73.138            0.033           73.098            -2.405
      PRM1          31.048            0.016           31.044            -0.488

POSITION RESULTS (DEG,IN)                X                Y                Z

  JNT. RLT1         -90.000

  JNT. RLT2          90.000

  JNT. PRM1          10.000

  JNT. RLT3          90.000

  JNT. RLT4          90.000

  PNT. PNT2           5.000             0.000           -5.000            0.0

FORCE RESULTS (IN-LB,LB)                 X                Y                Z

  JNT. RLT1 FROM BASE ONTO LNK1
       FORC          10.000          -10.000            0.000             ----
       TORQ           0.000             ----             ----            0.000

  JNT. RLT2 FROM LNK1 ONTO LNK2
       FORC           0.000             0.000            0.000             ----
       TORQ           0.000             ----             ----            0.000

  JNT. PRM1 FROM LNK2 ONTO LNK3
       FORC           0.000             0.000             ----            0.000
       TORQ           0.000             ----           -0.000             ----

  JNT. RLT3 FROM LNK3 ONTO LNK4
       FORC           0.000             0.000           -0.000             ----
       TORQ           0.000             ----             ----           -0.000

  JNT. RLT4 FROM LNK4 ONTO BASE
       FORC          10.000             0.000          -10.000             ----
       TORQ           0.000             ----             ----            0.000
```

```
TRANSFER FUNCTION ANALYSIS : FORC INPUT(S)

  EIGENVECTORS(2*NDOF(FORCE) OR 2*NFGC(MOTION) :

  COLUMN :  1
  PSI( 1)=(  0.182893E 01, -0.176431E 01)
  PSI( 2)=(  0.305961E 02, -0.345740E 02)
  PSI( 3)=( -0.252325E-01, -0.244156E-01)
  PSI( 4)=( -0.486378E 00, -0.402662E 00)

  COLUMN :  2
  PSI( 1)=(  0.182893E 01,  0.176431E 01)
  PSI( 2)=(  0.305961E 02,  0.345740E 02)
  PSI( 3)=( -0.252325E-01,  0.244156E-01)
  PSI( 4)=( -0.486378E 00,  0.402662E 00)

  COLUMN :  3
  PSI( 1)=(  0.613163E 01,  0.145571E 02)
  PSI( 2)=(  0.204543E 02,  0.449521E 02)
  PSI( 3)=(  0.465766E 00, -0.205059E 00)
  PSI( 4)=(  0.143738E 01, -0.681695E 00)

  COLUMN :  4
  PSI( 1)=(  0.613163E 01, -0.145571E 02)
  PSI( 2)=(  0.204543E 02, -0.449521E 02)
  PSI( 3)=(  0.465766E 00,  0.205059E 00)
  PSI( 4)=(  0.143738E 01,  0.681695E 00)

  EIGENVALUES - COMMON DENOMINATORS
     D( 1) = S-( -0.240483E 01,  0.731084E 02)
     D( 2) = S-( -0.240483E 01, -0.731084E 02)
     D( 3) = S-( -0.491287E 00,  0.310439E 02)
     D( 4) = S-( -0.491287E 00, -0.310439E 02)

  FORCE OR TORQUE INPUT EXCITATION : RLT1

  POSITION TRANSFER FUNCTION : JOIN - RLT1
  NF( 1) = (   -0.187111E-03,   -0.689619E-02)
  NF( 2) = (   -0.187111E-03,    0.689619E-02)
  NF( 3) = (    0.188961E-03,   -0.230107E-02)
  NF( 4) = (    0.188961E-03,    0.230107E-02)

  POSITION TRANSFER FUNCTION : POIN - PNT2

  FIRST PARTIAL DERIVATIVES FOR POINT : PNT2 WRT -

  FGC( 1) : RLT3
  WMAT(11) =    0.476837E-05  WMAT(21) =  -0.762939E-05  WMAT(31) =    0.0
  WMAT(12) =    0.0           WMAT(22) =  -0.999999E 00  WMAT(32) =    0.0
  WMAT(13) =    0.999999E 00  WMAT(23) =    0.0          WMAT(33) =    0.0
  WMAT(14) =    0.0           WMAT(24) =    0.0          WMAT(34) =    0.0

  FIRST PARTIAL DERIVATIVES FOR POINT : PNT2 WRT -

  FGC( 2) : PRM1
  WMAT(11) =    0.0           WMAT(21) =    0.560466E-0   WMAT(31) =    0.0
  WMAT(12) =    0.0           WMAT(22) =    0.200000E 00  WMAT(32) =    0.0
  WMAT(13) =  -0.200000E 00   WMAT(23) =    0.0           WMAT(33) =    0.0
  WMAT(14) =    0.0           WMAT(24) =    0.0           WMAT(34) =    0.0
```

```
X-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.935551E-03,   0.344810E-01)
NF( 2) = (   0.935551E-03,  -0.344810E-01)
NF( 3) = (  -0.944812E-03,   0.115054E-01)
NF( 4) = (  -0.944812E-03,  -0.115054E-01)

Y-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.190515E-08,   0.197162E-07)
NF( 2) = (   0.190515E-08,  -0.197162E-07)
NF( 3) = (  -0.193166E-08,   0.405023E-07)
NF( 4) = (  -0.193166E-08,  -0.405023E-07)

Z-COMPONENT NUMERATORS(2*NFGC) :
NF( 1) = (   0.0         ,   0.0          )
NF( 2) = (   0.0         ,   0.0          )
NF( 3) = (   0.0         ,   0.0          )
NF( 4) = (   0.0         ,   0.0          )

POSITION TRANSFER FUNCTION : JOIN - RLT2
NF( 1) = (   0.187112E-03,   0.689619E-02)
NF( 2) = (   0.187112E-03,  -0.689619E-02)
NF( 3) = (  -0.188961E-03,   0.230108E-02)
NF( 4) = (  -0.188961E-03,  -0.230108E-02)

POSITION TRANSFER FUNCTION : JOIN - RLT3
NF( 1) = (  -0.212120E-03,   0.264398E-02)
NF( 2) = (  -0.212120E-03,  -0.264398E-02)
NF( 3) = (   0.215805E-03,  -0.614554E-02)
NF( 4) = (   0.215805E-03,   0.614554E-02)

POSITION TRANSFER FUNCTION : JOIN - RLT4
NF( 1) = (   0.212120E-03,  -0.264398E-02)
NF( 2) = (   0.212120E-03,   0.264398E-02)
NF( 3) = (  -0.215806E-03,   0.614553E-02)
NF( 4) = (  -0.215806E-03,  -0.614553E-02)

POSITION TRANSFER FUNCTION : JOIN - PRM1
NF( 1) = (  -0.125047E-03,   0.477009E-01)
NF( 2) = (  -0.125047E-03,  -0.477009E-01)
NF( 3) = (   0.134222E-03,  -0.192223E-01)
NF( 4) = (   0.134222E-03,   0.192223E-01)

********************
```

APPENDIX I: PARAMETER VALUES FOR A SINGLE DEGREE-OF-FREEDOM SYSTEM WITH BASE MOTION INPUT EXCITATIONS

$m = 386.088/386.088\ lb_f\text{-}sec^2/in$

$c = 1.0\ lb_f\text{-}sec/in$

$k = 100.0\ lb_f/in$

$g = 386.088\ in/sec^2$

$x = 0.0\ in$

$y = 5.0\ in$

$z = 0.0\ in$

APPENDIX J: PARAMETER VALUES FOR A SINGLE DEGREE-OF-FREEDOM SYSTEM WITH BASE MOTION INPUT EXCITATIONS

$m = 386.088/386.088\ lb_f\text{-}sec^2/in$

$c = 1.0\ lb_f\text{-}sec/in$

$k = 100.0\ lb_f/in$

$g = 386.088\ in/sec^2$

$x = 10.0\ in$

$y = 5.0\ in$

$z = 0.0\ in$

APPENDIX K: PARAMETER VALUES FOR A PENDULUM SYSTEM WITH BASE MOTION INPUT EXCITATIONS

$m = 386.088/386.088\ lb_f\text{-}sec^2/in$

$c = 1.0\ lb{=}f^{-sec/in}$

$k = 100.0\ lb_f/in$

$l = 10.0\ in$

$l_1 = 4.0\ in$

$l_2 = 8.0\ in$

$g = 386.088\ in/sec^2$

APPENDIX L: IMP ANALYSIS OF AN AGRICULTURAL WHEEL TRACTOR WITH BASE MOTION INPUT EXCITATIONS

```
*********************************************************************
*                             IMP-75                                *
*                THE INTEGRATED MECHANISMS PROGRAM                  *
*********************************************************************

REMARK/  JOHN DEERE 4020 AGRI. ROW-CROP WHEEL TRACTOR W/NON/OSCL FT AXLE
SYSTEM=  THREE-DIM TRACTOR W/CAB & NONOSCILLATING FT. AXLE
GROUND=TRACK
REMARK/  TRACTOR CHASSIS
REVLUT(CHASIS,RRWHEL)=RRAXL
    DATA/LINK(CHAS,RRAXL)=0.0,0.0,40.0/0.0,0.0,41.0/1.0,0.0,40.0
    DATA/LINK(RRWH,RRAXL)=0.0,0.0,40.0/0.0,0.0,41.0/1.0,0.0,40.0
REVLUT(CHASIS,LRWHEL)=LRAXL
    DATA/LINK(CHAS,LRAXL)=0.0,0.0,-40.0/0.0,0.0,-39.0/1.0,0.0,-40.0
    DATA/LINK(LRWH,LRAXL)=0.0,0.0,-40.0/0.0,0.0,-39.0/1.0,0.0,-40.0
REMARK/  TRACTOR CHASSIS CENTER OF GRAVITY
POINT(CHASIS)=CGCH
    DATA/POINT(CGCH,RRAXL)=33.572,33.71,-40.0
REMARK/  CHASSIS WEIGHT AND INERTIAL PROPERTIES
    DATA/WEIGHT(CHASIS,RRAXL)=8850.0,33.572,33.71,-40.0
    DATA/INERTIA(CHASIS,RRAXL)=54945600.0,67317900.0,56598760.0,$
                               20031290.0,-23768970.0,-23866670.0
REMARK/  V-VERTICAL(BOUNCE)     A-AFT(LONGITUDINAL)    R-ROLL
REMARK/  RIGHT REAR TRACTOR TIRE
PRISM(RRWHEL,RRT1)=VRRTR
    DATA/LINK(RRWH,VRRTR)=0.0,0.0,40.0/0.0,1.0,40.0/0.0,0.0,41.0
    DATA/LINK(RRT1,VRRTR)=0.0,0.0,40.0/0.0,1.0,40.0/0.0,0.0,41.0
PRISM(RRT1,RRT2)=ARRTR
    DATA/LINK(RRT1,ARRTR)=0.0,0.0,40.0/1.0,0.0,40.0/0.0,1.0,40.0
    DATA/LINK(RRT2,ARRTR)=0.0,0.0,40.0/1.0,0.0,40.0/0.0,1.0,40.0
REVLUT(RRT2,RRT3)=RRRTR
    DATA/LINK(RRT2,RRRTR)=0.0,0.0,40.0/1.0,0.0,40.0/0.0,1.0,40.0
    DATA/LINK(RRT3,RRRTR)=0.0,0.0,40.0/1.0,0.0,40.0/0.0,1.0,40.0
```

```
PRISM(RRT3,RRT4)=LRRTR
    DATA/LINK(RRT3,LRRTR)=0.0,0.0,40.0/0.0,0.0,41.0/1.0,0.0,40.0
    DATA/LINK(RRT4,LRRTR)=0.0,0.0,40.0/0.0,0.0,41.0/1.0,0.0,40.0
REVLUT(RRT4,RRT5)=YRRTR
    DATA/LINK(RRT4,YRRTR)=0.0,0.0,40.0/0.0,1.0,40.0/0.0,0.0,41.0
    DATA/LINK(RRT5,YRRTR)=0.0,0.0,40.0/0.0,1.0,40.0/0.0,0.0,41.0
PRISM(RRT5,TRACK)=RRSMP
    DATA/LINK(RRT5,RRSMP)=0.0,0.0,40.0/0.0,1.0,40.0/0.0,0.0,41.0
    DATA/LINK(TRAC,RRSMP)=0.0,0.0,40.0/0.0,1.0,40.0/0.0,0.0,41.0
REMARK/  TIRE SPRING & DAMPING RATES
    DATA/SPRING(VRRTR)=1780,-2.0
    DATA/SPRING(ARRTR)=2000.0, 0.09
    DATA/SPRING(LRRTR)=2000.0,0.0
    DATA/DAMPER(VRRTR)=17.7
    DATA/DAMPER(ARRTR)=17.7
    DATA/DAMPER(LRRTR)=17.7
REMARK/  LEFT REAR TRACTOR TIRE
PRISM(LRWHEL,LRT1)=VLRTR
    DATA/LINK(LRWH,VLRTR)=0.0,0.0,-40/0.0,1.0,-40/0.0,0.0,-39
    DATA/LINK(LRT1,VLRTR)=0.0,0.0,-40/0.0,1.0,-40/0.0,0.0,-39
PRISM(LRT1,LRT2)=ALRTR
    DATA/LINK(LRT1,ALRTR)=0.0,0.0,-40/1.0,0.0,-40/0.0,1.0,-40
    DATA/LINK(LRT2,ALRTR)=0.0,0.0,-40/1.0,0.0,-40/0.0,1.0,-40
REVLUT(LRT2,LRT3)=RLRTR
    DATA/LINK(LRT2,RLRTR)=0.0,0.0,-40/1.0,0.0,-40/0.0,1.0,-40
    DATA/LINK(LRT3,RLRTR)=0.0,0.0,-40/1.0,0.0,-40/0.0,1.0,-40
PRISM(LRT3,LRT4)=LLRTR
    DATA/LINK(LRT3,LLRTR)=0.0,0.0,-40/0.0,0.0,-39/1.0,0.0,-40
    DATA/LINK(LRT4,LLRTR)=0.0,0.0,-40/0.0,0.0,-39/1.0,0.0,-40
PRISM(LRT4,TRACK)=LRSMP
```

```
    DATA/LINK(LRT4,LRSMP)=0.0,0.0,-40/0.0,1.0,-40/0.0,0.0,-39
    DATA/LINK(TRAC,LRSMP)=0.0,0.0,-40/0.0,1.0,-40/0.0,0.0,-39
REMARK/  TIRE SPRING & DAMPING RATES
    DATA/SPRING(VLRTR)=1780,-2.0
    DATA/SPRING(ALRTR)=2000.0, 0.09
    DATA/SPRING(LLRTR)=2000.0,0.0
    DATA/DAMPER(VLRTR)=17.7
    DATA/DAMPER(ALRTR)=17.7
    DATA/DAMPER(LLRTR)=17.7
REMARK/  NON-OSCILLATING FRONT AXLE
REVLUT(CHASIS,RFWHEL)=RFSPNDL
    DATA/LINK(CHAS,RFSPN)=100.25,0.0,40.0/100.25,0.0,41./101.25,0.0,40.0
    DATA/LINK(RFWH,RFSPN)=100.25,0.0,40.0/100.25,0.0,41./101.25,0.0,40.0
REVLUT(CHASIS,LFWHEL)=LFSPNDL
    DATA/LINK(CHAS,LFSPN)=100.25,0.0,-41/100.25,0.0,-39/101.25,0.0,-40
    DATA/LINK(LFWH,LFSPN)=100.25,0.0,-41/100.25,0.0,-39/101.25,0.0,-40
REMARK/  RIGHT FRONT TRACTOR TIRE
PRISM(RFWHEL,RFT1)=VRFTR
    DATA/LINK(RFWH,VRFTR)=100.25,0.0,40/100.25,1.0,40/100.25,0.0,41
    DATA/LINK(RFT1,VRFTR)=100.25,0.0,40/100.25,1.0,40/100.25,0.0,41
PRISM(RFT1,RFT2)=ARFTR
    DATA/LINK(RFT1,ARFTR)=100.25,0.0,40/101.25,0.0,40/100.25,1.0,40
    DATA/LINK(RFT2,ARFTR)=100.25,0.0,40/101.25,0.0,40/100.25,1.0,40
REVLUT(RFT2,RFT3)=RRFTR
    DATA/LINK(RFT2,RRFTR)=100.25,0.0,40/101.25,0.0,40/100.25,1.0,40
    DATA/LINK(RFT3,RRFTR)=100.25,0.0,40/101.25,0.0,40/100.25,1.0,40
PRISM(RFT3,RFT4)=LRFTR
    DATA/LINK(RFT3,LRFTR)=100.25,0.0,40/100.25,0.0,41/101.25,0.0,40
    DATA/LINK(RFT4,LRFTR)=100.25,0.0,40/100.25,0.0,41/101.25,0.0,40
REVLUT(RFT4,RFT5)=YRFTR
```

```
    DATA/LINK(RFT4,YRFTR)=100.25,0.0,40/100.25,1.0,40/100.25,0.0,41
    DATA/LINK(RFT5,YRFTR)=100.25,0.0,40/100.25,1.0,40/100.25,0.0,41
PRISM(RFT5,TRACK)=RFSMP
    DATA/LINK(RFT5,RFSMP)=100.25,0.0,40/100.25,1.0,40/100.25,0.0,41
    DATA/LINK(TRAC,RFSMP)=100.25,0.0,40/100.25,1.0,40/100.25,0.0,41
REMARK/  TIRE SPRING & DAMPING RATES
    DATA/SPRING(VRFTR)=1123,-1.44
    DATA/SPRING(ARFTR)=1235.0, 0.065
    DATA/SPRING(LRFTR)=1235.0,0.0
    DATA/DAMPER(VRFTR)=8.25
    DATA/DAMPER(ARFTR)=8.25
    DATA/DAMPER(LRFTR)=8.25
REMARK/  LEFT FRONT TRACTOR TIRE
PRISM(LFWHEL,LFT1)=VLFTR
    DATA/LINK(LFWH,VLFTR)=100.25,0.0,-40/100.25,1.0,-40/100.25,0.0,-39
    DATA/LINK(LFT1,VLFTR)=100.25,0.0,-40/100.25,1.0,-40/100.25,0.0,-39
PRISM(LFT1,LFT2)=ALFTR
    DATA/LINK(LFT1,ALFTR)=100.25,0.0,-40/101.25,0.0,-40/100.25,1.0,-40
    DATA/LINK(LFT2,ALFTR)=100.25,0.0,-40/101.25,0.0,-40/100.25,1.0,-40
REVLUT(LFT2,LFT3)=RLFTR
    DATA/LINK(LFT2,RLFTR)=100.25,0.0,-40/101.25,0.0,-40/100.25,1.0,-40
    DATA/LINK(LFT3,RLFTR)=100.25,0.0,-40/101.25,0.0,-40/100.25,1.0,-40
PRISM(LFT3,LFT4)=LLFTR
    DATA/LINK(LFT3,LLFTR)=100.25,0.0,-40/100.25,0.0,-39/101.25,0.0,-40
    DATA/LINK(LFT4,LLFTR)=100.25,0.0,-40/100.25,0.0,-39/101.25,0.0,-40
REVLUT(LFT4,LFT5)=YLFTR
    DATA/LINK(LFT4,YLFTR)=100.25,0.0,-40/100.25,1.0,-40/100.25,0.0,-39
    DATA/LINK(LFT5,YLFTR)=100.25,0.0,-40/100.25,1.0,-40/100.25,0.0,-39
PRISM(LFT5,TRACK)=LFSMP
    DATA/LINK(LFT5,LFSMP)=100.25,0.0,-40/100.25,1.0,-40/100.25,0.0,-39
```

```
    DATA/LINK(TRAC,LFSMP)=100.25,0.0,-40/100.25,1.0,-40/100.25,0.0,-39
REMARK/  TIRE SPRING & DAMPING RATES
    DATA/SPRING(VLFTR)=1123,-1.44
    DATA/SPRING(ALFTR)=1235.0, 0.065
    DATA/SPRING(LLFTR)=1235.0,0.0
    DATA/DAMPER(VLFTR)=8.25
    DATA/DAMPER(ALFTR)=8.25
    DATA/DAMPER(LLFTR)=8.25
REMARK/  TRACTOR OPERATOR ENCLOSURE CAB
REMARK/  P-PITCH     B-BOUNCE     A-AFT(LONGITUDINAL)     R-ROLL
REMARK/  RIGHT REAR ISOMOUNT PAD
REVLUT(CAB ,RRP1)=PRRPD
    DATA/LINK(CAB ,PRRPD)=0.0,43.16,18/0.0,43.16,19/1.0,43.16,18
    DATA/LINK(RRP1,PRRPD)=0.0,43.16,18/0.0,43.16,19/1.0,43.16,18
PRISM(RRP1,RRP2)=BRRPD
    DATA/LINK(RRP1,BRRPD)=0.0,43.16,18/0.0,44.16,18/0.0,43.16,19
    DATA/LINK(RRP2,BRRPD)=0.0,43.16,18/0.0,44.16,18/0.0,43.16,19
PRISM(RRP2,RRP3)=ARRPD
    DATA/LINK(RRP2,ARRPD)=0.0,43.16,18/1.0,43.16,18/0.0,44.16,18
    DATA/LINK(RRP3,ARRPD)=0.0,43.16,18/1.0,43.16,18/0.0,44.16,18
REVLUT(RRP3,RRP4)=RRRPD
    DATA/LINK(RRP3,RRRPD)=0.0,43.16,18/1.0,43.16,18/0.0,44.16,18
    DATA/LINK(RRP4,RRRPD)=0.0,43.16,18/1.0,43.16,18/0.0,44.16,18
PRISM(RRP4,RRP5)=LRRPD
    DATA/LINK(RRP4,LRRPD)=0.0,43.16,18/0.0,43.16,19/1.0,43.16,18
    DATA/LINK(RRP5,LRRPD)=0.0,43.16,18/0.0,43.16,19/1.0,43.16,18
REVLUT(RRP5,CHASIS)=YRRPD
    DATA/LINK(RRP5,YRRPD)=0.0,43.16,18/0.0,44.16,18/0.0,43.16,19
    DATA/LINK(CHAS,YRRPD)=0.0,43.16,18/0.0,44.16,18/0.0,43.16,19
REMARK/  ISOMOUNT SPRING & DAMPING RATES
```

```
    DATA/SPRING(BRRPD)=7525.0,-0.05
    DATA/SPRING(ARRPD)=7525.,0.0
    DATA/SPRING(LRRPD)=7525.0,0.0
    DATA/DAMPER(BRRPD)=0.85
    DATA/DAMPER(ARRPD)=0.85
    DATA/DAMPER(LRRPD)=0.85
REMARK/  CAB CENTER OF GRAVITY AND SEAT  ATTACH POINT
POINT(CAB)=CGCB,SEAT
    DATA/POINT(CGCB,PRRPD)=18,30,-18
    DATA/POINT(SEAT,PRRPD)=6.75,20.3,-18
REMARK/  CAB WEIGHT & INERTIAL PROPERTIES
    DATA/WEIGHT(CAB,PRRPD)=1505,18,30,-18
    DATA/INERTIA(CAB,PRRPD)=2890750,2139905,2890750,812700,$
                            -487620,-812700
REMARK/  LEFT REAR ISOMOUNT PAD
REVLUT(CAB ,LRP1)=PLRPD
    DATA/LINK(CAB ,PLRPD)=0.0,43.16,-18/0.0,43.16,-17/1.0,43.16,-18
    DATA/LINK(LRP1,PLRPD)=0.0,43.16,-18/0.0,43.16,-17/1.0,43.16,-18
PRISM(LRP1,LRP2)=BLRPD
    DATA/LINK(LRP1,BLRPD)=0.0,43.16,-18/0.0,44.16,-18/0.0,43.16,-17
    DATA/LINK(LRP2,BLRPD)=0.0,43.16,-18/0.0,44.16,-18/0.0,43.16,-17
PRISM(LRP2,LRP3)=ALRPD
    DATA/LINK(LRP2,ALRPD)=0.0,43.16,-18/1.0,43.16,-18/0.0,44.16,-18
    DATA/LINK(LRP3,ALRPD)=0.0,43.16,-18/1.0,43.16,-18/0.0,44.16,-18
REVLUT(LRP3,LRP4)=RLRPD
    DATA/LINK(LRP3,RLRPD)=0.0,43.16,-18/1.0,43.16,-18/0.0,44.16,-18
    DATA/LINK(LRP4,RLRPD)=0.0,43.16,-18/1.0,43.16,-18/0.0,44.16,-18
PRISM(LRP4,CHASIS)=LLRPD
    DATA/LINK(LRP4,LLRPD)=0.0,43.16,-18/0.0,43.16,-17/1.0,43.16,-18
    DATA/LINK(CHAS,LLRPD)=0.0,43.16,-18/0.0,43.16,-17/1.0,43.16,-18
```

```
REMARK/  ISOMOUNT SPRING & DAMPING RATES
    DATA/SPRING(BLRPD)=7525.0,-0.05
    DATA/SPRING(ALRPD)=7525.,0.0
    DATA/SPRING(LLRPD)=7525.0,0.0
    DATA/DAMPER(BLRPD)=0.85
    DATA/DAMPER(ALRPD)=0.85
    DATA/DAMPER(LLRPD)=0.85
REMARK/  RIGHT FRONT ISOMOUNT PAD
REVLUT(CAB ,RFP1)=PRFPD
    DATA/LINK(CAB ,PRFPD)=36.0,43.16,18/36.0,43.16,19/37.0,43.16,18
    DATA/LINK(RFP1,PRFPD)=36.0,43.16,18/36.0,43.16,19/37.0,43.16,18
PRISM(RFP1,RFP2)=BRFPD
    DATA/LINK(RFP1,BRFPD)=36.0,43.16,18/36.0,44.16,18/36.0,43.16,19
    DATA/LINK(RFP2,BRFPD)=36.0,43.16,18/36.0,44.16,18/36.0,43.16,19
PRISM(RFP2,RFP3)=ARFPD
    DATA/LINK(RFP2,ARFPD)=36.0,43.16,18/37.0,43.16,18/36.0,44.16,18
    DATA/LINK(RFP3,ARFPD)=36.0,43.16,18/37.0,43.16,18/36.0,44.16,18
REVLUT(RFP3,RFP4)=RRFPD
    DATA/LINK(RFP3,RRFPD)=36.0,43.16,18/37.0,43.16,18/36.0,44.16,18
    DATA/LINK(RFP4,RRFPD)=36.0,43.16,18/37.0,43.16,18/36.0,44.16,18
PRISM(RFP4,RFP5)=LRFPD
    DATA/LINK(RFP4,LRFPD)=36.0,43.16,18/36.0,43.16,19/37.0,43.16,18
    DATA/LINK(RFP5,LRFPD)=36.0,43.16,18/36.0,43.16,19/37.0,43.16,18
REVLUT(RFP5,CHASIS)=YRFPD
    DATA/LINK(RFP5,YRFPD)=36.0,43.16,18/36.0,44.16,18/36.0,43.16,19
    DATA/LINK(CHAS,YRFPD)=36.0,43.16,18/36.0,44.16,18/36.0,43.16,19
REMARK/  ISOMOUNT SPRING & DAMPING RATES
    DATA/SPRING(BRFPD)=7525.0,-0.05
    DATA/SPRING(ARFPD)=7525.0,0.0
    DATA/SPRING(LRFPD)=7525.0,0.0
```

```
    DATA/DAMPER(BRFPD)=0.85
    DATA/DAMPER(ARFPD)=0.85
    DATA/DAMPER(LRFPD)=0.85
REMARK/  LEFT FRONT ISOMOUNT PAD
REVLUT(CAB ,LFP1)=PLFPD
    DATA/LINK(CAB ,PLFPD)=36.0,43.16,-18/36.0,43.16,-17/37.0,43.16,-18
    DATA/LINK(LFP1,PLFPD)=36.0,43.16,-18/36.0,43.16,-17/37.0,43.16,-18
PRISM(LFP1,LFP2)=BLFPD
    DATA/LINK(LFP1,BLFPD)=36.0,43.16,-18/36.0,44.16,-18/36.0,43.16,-17
    DATA/LINK(LFP2,BLFPD)=36.0,43.16,-18/36.0,44.16,-18/36.0,43.16,-17
PRISM(LFP2,LFP3)=ALFPD
    DATA/LINK(LFP2,ALFPD)=36.0,43.16,-18/37.0,43.16,-18/36.0,44.16,-18
    DATA/LINK(LFP3,ALFPD)=36.0,43.16,-18/37.0,43.16,-18/36.0,44.16,-18
REVLUT(LFP3,LFP4)=RLFPD
    DATA/LINK(LFP3,RLFPD)=36.0,43.16,-18/37.0,43.16,-18/36.0,44.16,-18
    DATA/LINK(LFP4,RLFPD)=36.0,43.16,-18/37.0,43.16,-18/36.0,44.16,-18
PRISM(LFP4,LFP5)=LLFPD
    DATA/LINK(LFP4,LLFPD)=36.0,43.16,-18/36.0,43.16,-17/37.0,43.16,-18
    DATA/LINK(LFP5,LLFPD)=36.0,43.16,-18/36.0,43.16,-17/37.0,43.16,-18
REVLUT(LFP5,CHASIS)=YLFPD
    DATA/LINK(LFP5,YLFPD)=36.0,43.16,-18/36.0,44.16,-18/36.0,43.16,-17
    DATA/LINK(CHAS,YLFPD)=36.0,43.16,-18/36.0,44.16,-18/36.0,43.16,-17
REMARK/  ISOMOUNT SPRING & DAMPING RATES
    DATA/SPRING(BLFPD)=7525.0,-0.05
    DATA/SPRING(ALFPD)=7525.0,0.0
    DATA/SPRING(LLFPD)=7525.0,0.0
    DATA/DAMPER(BLFPD)=0.85
    DATA/DAMPER(ALFPD)=0.85
    DATA/DAMPER(LLFPD)=0.85
REMARK/  TRACTIVE ROLLING RESISTANCE
```

```
    DATA/FORCE(ARRTR)=-180.0
    DATA/FORCE(ALRTR)=-180.0
    DATA/FORCE(ARFTR)=-80.0
    DATA/FORCE(ALFTR)=-80.0
REMARK/  SIMULATOR PADS - INPUT TRACK PROFILE
    DATA/POSITN(RRSMP)=-3.938
    DATA/POSITN(RFSMP)=-3.938
    DATA/POSITN(LRSMP)=-3.742
    DATA/POSITN(LFSMP)=-3.742
REMARK/  GRAVITATIONAL CONSTANTS
    DATA/GRAV=0,-386.088,0
REMARK/  IMP TOLERANCES
ZERO(EQULIB)=0.01
REMARK/  REQUEST STATEMENTS
FIND/EQULIB
PRINT/FREQ
PRINT/POSITN(ALL)
PRINT/FORCE(ALL)
PRINT/DYNAM/MOTION/ACCEL/SEAT,CGCH/
EXECUTE/HOLD
```

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**********************************************************************
*                          ANALYSIS OF                               *
*          THREE-DIM TRACTOR W/CAB & NONOSCILLATING FT. AXLEA        *
*                          STATIC MODE                               *
*              ON      i               AT   -                    *
**********************************************************************
```

POSITION 1

THE GENERALIZED COORDINATES ARE

JOINT		SET BY	POSITION DEG,IN	VELOCITY RAD,IN/SEC	ACCELERATION RAD,IN/SEC2
RRSM		USER	-3.938	0.0	0.0
RFSM		USER	-3.938	0.0	0.0
LRSM		USER	-3.742	0.0	0.0
LFSM		USER	-3.742	0.0	0.0
LLFP	1	IMP	0.000	0.0	0.0
BRRP	1	IMP	-0.000	0.0	0.0
BRFP	1	IMP	-0.000	0.0	0.0
ALRT	1	IMP	0.000	0.0	0.0
VLRT	1	IMP	0.005	0.0	0.0
VRFT	1	IMP	-0.005	0.0	0.0
VLFT	1	IMP	0.004	0.0	0.0
LLFT	1	IMP	-0.000	0.0	0.0
ALFP	1	IMP	0.000	0.0	0.0
BLRP	1	IMP	0.000	0.0	0.0

DEGREE OF FREEDOM = 14.   QUALITY INDEX = 0.104E 08

DYNAMIC CHARACTERISTICS

FGC JOINT	NATURAL FREQUENCY RAD/SEC	DAMPING RATIO	DAMPED FREQUENCY RAD/SEC	DECAY RATE 1/SEC
---		---		
BLRP	145.815	0.009	145.810	-1.245
ALFP	145.160	0.009	145.154	-1.257
LLFT	95.504	0.007	95.502	-0.697
VLFT	42.229	0.005	42.229	-0.228
VRFT	40.623	0.004	40.622	-0.160
VLRT	22.036	0.089	21.948	-1.967
ALRT	18.656	0.075	18.604	-1.396
BRFP	14.685	0.065	14.654	-0.954
BRRP	9.313	0.040	9.306	-0.374
LLFP	7.542	0.034	7.537	-0.257

POSITION RESULTS (DEG,IN)		X	Y	Z
JNT. RRAX	-0.000			
JNT. LRAX	-0.000			
JNT. VRRT	-0.004			
JNT. ARRT	0.000			
JNT. RRRT	0.147			
JNT. LRRT	0.000			
JNT. YRRT	-0.000			

JNT. RRSM	-3.938
JNT. VLRT	0.005
JNT. ALRT	0.000
JNT. RLRT	0.147
JNT. LLRT	-0.000
JNT. LRSM	-3.742
JNT. RFSP	-0.000
JNT. LFSP	-0.000
JNT. VRFT	-0.005
JNT. ARFT	0.000
JNT. RRFT	0.147
JNT. LRFT	0.000
JNT. YRFT	-0.000
JNT. RFSM	-3.938
JNT. VLFT	0.004
JNT. ALFT	0.000
JNT. RLFT	0.147
JNT. LLFT	-0.000
JNT. YLFT	-0.000
JNT. LFSM	-3.742
JNT. PRRP	-0.000
JNT. BRRP	-0.000
JNT. ARRP	-0.000
JNT. RRRP	0.001
JNT. LRRP	0.000
JNT. YRRP	-0.000
JNT. PLRP	0.000
JNT. BLRP	0.000
JNT. ALRP	-0.000
JNT. RLRP	0.001

JNT. LLRP	0.000			
JNT. PRFP	-0.000			
JNT. BRFP	-0.000			
JNT. ARFP	-0.000			
JNT. RRFP	0.001			
JNT. LRFP	0.000			
JNT. YRFP	-0.000			
JNT. PLFP	-0.000			
JNT. BLFP	0.000			
JNT. ALFP	0.000			
JNT. RLFP	0.001			
JNT. LLFP	0.000			
JNT. YLFP	-0.000			
PNT. CGCH	50.369	33.572	37.550	-0.086
PNT. CGCB	79.076	18.000	76.999	-0.188
PNT. SEAT	67.637	6.750	67.299	-0.163

FORCE RESULTS (IN-LB,LB)		X	Y	Z
JNT. RRAX FROM CHAS ONTO RRWH				
FORC	3552.502	-0.170	-3552.490	-9.441
TORQ	0.051	0.051	-0.000	-0.000
JNT. LRAX FROM CHAS ONTO LRWH				
FORC	3568.559	-0.190	-3568.548	-8.911
TORQ	8.947	-0.157	8.945	-0.016
JNT. VRRT FROM RRWH ONTO RRT1				
FORC	3552.502	-9.441	-0.170	-3552.490
TORQ	0.011	0.001	0.011	-0.000
JNT. ARRT FROM RRT1 ONTO RRT2				
FORC	3552.502	-3552.490	-9.441	-0.170
TORQ	0.320	-0.001	0.303	0.104
JNT. RRRT FROM RRT2 ONTO RRT3				
FORC	3552.502	-3552.502	-0.336	-0.170
TORQ	0.320	-0.000	0.303	0.104
JNT. LRRT FROM RRT3 ONTO RRT4				
FORC	3552.503	-0.170	-3552.503	-0.336
TORQ	0.674	-0.602	-0.000	0.303
JNT. YRRT FROM RRT4 ONTO RRT5				
FORC	3552.503	-0.336	-0.170	-3552.503

	TORQ	0.673	0.303	-0.601	-0.000
JNT. RRSM FROM RRT5 ONTO TRAC					
	FORC	3552.503	-0.336	-0.170	-3552.503
	TORQ	2.157	0.972	-1.925	-0.000
JNT. VLRT FROM LRWH ONTO LRT1					
	FORC	3568.561	-8.911	-0.189	-3568.550
	TORQ	8.946	-0.017	-0.113	8.945
JNT. ALRT FROM LRT1 ONTO LRT2					
	FORC	3568.561	-3568.550	-8.911	-0.189
	TORQ	8.942	8.937	0.286	-0.132
JNT. RLRT FROM LRT2 ONTO LRT3					
	FORC	3568.561	-3568.561	0.234	-0.189
	TORQ	8.943	8.938	0.263	-0.132
JNT. LLRT FROM LRT3 ONTO LRT4					
	FORC	3568.550	-0.189	-3568.550	0.234
	TORQ	8.958	0.407	8.945	0.263
JNT. LRSM FROM LRT4 ONTO TRAC					
	FORC	3568.550	0.234	-0.189	-3568.550
	TORQ	9.082	0.972	1.235	8.945
JNT. RFSP FROM CHAS ONTO RFWH					
	FORC	1611.896	0.170	-1611.890	-4.339
	TORQ	0.483	-0.001	-0.001	0.483
JNT. LFSP FROM CHAS ONTO LFWH					
	FORC	1622.025	0.190	-1622.021	-3.847
	TORQ	1622.991	1622.990	1.507	0.712
JNT. VRFT FROM RFWH ONTO RFT1					
	FORC	1611.896	-4.339	0.170	-1611.891
	TORQ	0.429	0.428	-0.023	-0.001
JNT. ARFT FROM RFT1 ONTO RFT2					
	FORC	1611.896	-1611.891	-4.339	0.170
	TORQ	0.566	-0.001	0.565	0.016
JNT. RRFT FROM RFT2 ONTO RFT3					
	FORC	1611.896	-1611.896	-0.208	0.170
	TORQ	0.552	0.000	0.551	0.016
JNT. LRFT FROM RFT3 ONTO RFT4					
	FORC	1611.894	0.170	-1611.895	-0.208
	TORQ	0.334	-0.294	0.000	0.157
JNT. YRFT FROM RFT4 ONTO RFT5					
	FORC	1611.894	-0.208	0.170	-1611.895
	TORQ	0.351	0.157	-0.313	0.000
JNT. RFSM FROM RFT5 ONTO TRAC					
	FORC	1611.894	-0.208	0.170	-1611.895
	TORQ	1.242	-0.512	-1.131	0.000
JNT. VLFT FROM LFWH ONTO LFT1					
	FORC	1622.025	-3.847	0.190	-1622.021

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      TORQ          1.527          0.734          0.244          1.317

JNT.  ALFT  FROM  LFT1  ONTO  LFT2
      FORC       1622.025      -1622.021         -3.847          0.190
      TORQ          1.595          1.317          0.865          0.244

JNT.  RLFT  FROM  LFT2  ONTO  LFT3
      FORC       1622.022      -1622.022          0.310          0.190
      TORQ          2.665          1.319          2.294          0.315

JNT.  LLFT  FROM  LFT3  ONTO  LFT4
      FORC       1622.030          0.190      -1622.030          0.310
      TORQ          2.834          0.814          1.319          2.373

JNT.  YLFT  FROM  LFT4  ONTO  LFT5
      FORC       1622.030          0.310          0.190      -1622.030
      TORQ          2.834          2.373          0.814          1.319

JNT.  LFSM  FROM  LFT5  ONTO  TRAC
      FORC       1622.030          0.310          0.190      -1622.030
      TORQ          2.886          1.710          1.914          1.319

JNT.  PRRP  FROM  CAB   ONTO  RRP1
      FORC        374.670          0.256       -374.668         -1.113
      TORQ          0.006          0.006          0.002          0.002

JNT.  BRRP  FROM  RRP1  ONTO  RRP2
      FORC        374.670         -1.113          0.256       -374.668
      TORQ          0.006          0.002          0.006          0.002

JNT.  ARRP  FROM  RRP2  ONTO  RRP3
      FORC        374.670       -374.668         -1.113          0.256
      TORQ          0.013          0.002         -0.011          0.006

JNT.  RRRP  FROM  RRP3  ONTO  RRP4
      FORC        374.670       -374.668         -1.108          0.256
      TORQ          0.013          0.002         -0.011          0.006

JNT.  LRRP  FROM  RRP4  ONTO  RRP5
      FORC        374.670          0.256       -374.668         -1.108
      TORQ          0.051         -0.049          0.002         -0.011

JNT.  YRRP  FROM  RRP5  ONTO  CHAS
      FORC        374.670         -1.108          0.256       -374.668
      TORQ          0.047         -0.011         -0.046          0.002

JNT.  PLRP  FROM  CAB   ONTO  LRP1
      FORC        377.835          0.351       -377.832         -1.560
      TORQ          7.892          0.459          7.878         -0.006

JNT.  BLRP  FROM  LRP1  ONTO  LRP2
      FORC        377.835         -1.560          0.351       -377.832
      TORQ          7.892         -0.006          0.459          7.878

JNT.  ALRP  FROM  LRP2  ONTO  LRP3
      FORC        377.835       -377.832         -1.560          0.351
      TORQ          7.892          7.878         -0.024          0.459

JNT.  RLRP  FROM  LRP3  ONTO  LRP4
      FORC        377.836       -377.833         -1.555          0.351
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     TORQ           7.891           7.878          -0.025           0.447

 JNT. LLRP FROM LRP4 ONTO CHAS
     FORC         377.836           0.351        -377.833          -1.555
     TORQ           7.888           0.382           7.878          -0.025

 JNT. PRFP FROM CAB  ONTO RFP1
     FORC         374.600           0.256        -374.598          -1.370
     TORQ           0.005           0.005          -0.001           0.001

 JNT. BRFP FROM RFP1 ONTO RFP2
     FORC         374.600          -1.370           0.256        -374.598
     TORQ           0.005           0.001           0.005          -0.001

 JNT. ARFP FROM RFP2 ONTO RFP3
     FORC         374.600        -374.598          -1.370           0.256
     TORQ           0.014          -0.001          -0.012           0.007

 JNT. RRFP FROM RFP3 ONTO RFP4
     FORC         374.600        -374.598          -1.366           0.256
     TORQ           0.019          -0.001          -0.017           0.007

 JNT. LRFP FROM RFP4 ONTO RFP5
     FORC         374.600           0.256        -374.598          -1.366
     TORQ           0.065          -0.063          -0.001          -0.017

 JNT. YRFP FROM RFP5 ONTO CHAS
     FORC         374.600          -1.366           0.256        -374.598
     TORQ           0.062          -0.017          -0.059          -0.001

 JNT. PLFP FROM CAB  ONTO LFP1
     FORC         377.896          -0.868        -377.895           0.167
     TORQ           0.437          -0.434           0.049          -0.012

 JNT. BLFP FROM LFP1 ONTO LFP2
     FORC         377.896           0.167          -0.868        -377.895
     TORQ           0.437          -0.012          -0.434           0.049

 JNT. ALFP FROM LFP2 ONTO LFP3
     FORC         377.896        -377.895           0.167          -0.868
     TORQ           0.437           0.049          -0.012          -0.434

 JNT. RLFP FROM LFP3 ONTO LFP4
     FORC         377.896        -377.895           0.171          -0.868
     TORQ           0.437           0.049          -0.012          -0.434

 JNT. LLFP FROM LFP4 ONTO LFP5
     FORC         377.895          -0.868        -377.894           0.171
     TORQ           0.506          -0.503           0.049          -0.012

 JNT. YLFP FROM LFP5 ONTO CHAS
     FORC         377.895           0.171          -0.868        -377.894
     TORQ           0.506          -0.012          -0.504           0.049

TRANSFER FUNCTION ANALYSIS : MOTI INPUT(S)

 EIGENVECTORS(2*NDOF(FORCE) OR 2*NFGC(MOTION) :

 COLUMN :  1
 PSI( 1)=( -0.695884E 00, -0.435948E 01)
```

```
PSI( 2)=(  0.103150E-01,  0.183060E 00)
PSI( 3)=( -0.108245E 00, -0.791422E 00)
PSI( 4)=( -0.452313E-02, -0.115825E-01)
PSI( 5)=( -0.163901E-01, -0.517584E-01)
PSI( 6)=(  0.125408E-01,  0.345720E-01)
PSI( 7)=( -0.314048E-02, -0.192652E-01)
PSI( 8)=( -0.680005E 00, -0.427665E 01)
PSI( 9)=( -0.690940E 00, -0.441325E 01)
PSI(10)=( -0.203432E 01, -0.128978E 02)
PSI(11)=( -0.299104E-01,  0.503838E-02)
PSI(12)=(  0.126048E-02, -0.823215E-04)
PSI(13)=( -0.546032E-02,  0.794958E-03)
PSI(14)=( -0.310312E-04,  0.229837E-04)
PSI(15)=( -0.402241E-03,  0.123977E-03)
PSI(16)=(  0.119291E-03, -0.693211E-04)
PSI(17)=( -0.740430E-04,  0.119361E-04)
PSI(18)=( -0.284577E-01,  0.476038E-02)
PSI(19)=( -0.294712E-01,  0.485665E-02)
PSI(20)=( -0.893849E-01,  0.148847E-01)

COLUMN :  2
PSI( 1)=( -0.695884E 00,  0.435948E 01)
PSI( 2)=(  0.103150E-01, -0.183060E 00)
PSI( 3)=( -0.108245E 00,  0.791422E 00)
PSI( 4)=( -0.452313E-02,  0.115825E-01)
PSI( 5)=( -0.163901E-01,  0.517584E-01)
PSI( 6)=(  0.125408E-01, -0.345720E-01)
PSI( 7)=( -0.314048E-02,  0.192652E-01)
PSI( 8)=( -0.680005E 00,  0.427665E 01)
PSI( 9)=( -0.690940E 00,  0.441325E 01)
PSI(10)=( -0.203432E 01,  0.128978E 02)
PSI(11)=( -0.299104E-01, -0.503838E-02)
PSI(12)=(  0.126048E-02,  0.823215E-04)
PSI(13)=( -0.546032E-02, -0.794958E-03)
PSI(14)=( -0.310312E-04, -0.229837E-04)
PSI(15)=( -0.402241E-03, -0.123977E-03)
PSI(16)=(  0.119291E-03,  0.693211E-04)
PSI(17)=( -0.740430E-04, -0.119361E-04)
PSI(18)=( -0.284577E-01, -0.476038E-02)
PSI(19)=( -0.294712E-01, -0.485665E-02)
PSI(20)=( -0.893849E-01, -0.148847E-01)

COLUMN :  3
PSI( 1)=( -0.160973E 00,  0.290937E 01)
PSI( 2)=( -0.315648E 00,  0.628413E 01)
PSI( 3)=( -0.127208E-01,  0.135441E 00)
PSI( 4)=( -0.160631E-02, -0.131434E-01)
PSI( 5)=(  0.379786E-02,  0.349297E-01)
PSI( 6)=( -0.256446E-02, -0.243528E-01)
PSI( 7)=(  0.270057E-01, -0.383926E 00)
PSI( 8)=( -0.158319E 00,  0.292901E 01)
PSI( 9)=(  0.551219E-01, -0.135412E 01)
PSI(10)=( -0.155016E 00,  0.229219E 01)
PSI(11)=(  0.200250E-01,  0.936508E-03)
PSI(12)=(  0.433697E-01,  0.180209E-02)
PSI(13)=(  0.930109E-03,  0.799112E-04)
PSI(14)=( -0.110251E-03,  0.116222E-04)
PSI(15)=(  0.222353E-03, -0.290093E-04)
PSI(16)=( -0.129738E-03,  0.156038E-04)
PSI(17)=( -0.265793E-02, -0.162814E-03)
```

```
PSI(18)=(  0.201512E-01,  0.901222E-03)
PSI(19)=( -0.937820E-02, -0.309169E-03)
PSI(20)=(  0.158330E-01,  0.927269E-03)

COLUMN :  4
PSI( 1)=( -0.160973E 00, -0.290937E 01)
PSI( 2)=( -0.315648E 00, -0.628413E 01)
PSI( 3)=( -0.127208E-01, -0.135441E 00)
PSI( 4)=( -0.160631E-02,  0.131434E-01)
PSI( 5)=(  0.379786E-02, -0.349297E-01)
PSI( 6)=( -0.256446E-02,  0.243528E-01)
PSI( 7)=(  0.270057E-01,  0.383926E 00)
PSI( 8)=( -0.158319E 00, -0.292901E 01)
PSI( 9)=(  0.551219E-01,  0.135412E 01)
PSI(10)=( -0.155016E 00, -0.229219E 01)
PSI(11)=(  0.200250E-01, -0.936508E-03)
PSI(12)=(  0.433697E-01, -0.180209E-02)
PSI(13)=(  0.930109E-03, -0.799112E-04)
PSI(14)=( -0.110251E-03, -0.116222E-04)
PSI(15)=(  0.222353E-03,  0.290093E-04)
PSI(16)=( -0.129738E-03, -0.156038E-04)
PSI(17)=( -0.265793E-02,  0.162814E-03)
PSI(18)=(  0.201512E-01, -0.901222E-03)
PSI(19)=( -0.937820E-02,  0.309169E-03)
PSI(20)=(  0.158330E-01, -0.927269E-03)

COLUMN :  5
PSI( 1)=( -0.392533E 02, -0.395165E 02)
PSI( 2)=(  0.223312E-01,  0.950869E-01)
PSI( 3)=( -0.346385E 00, -0.393924E 00)
PSI( 4)=(  0.428643E 01,  0.449631E 01)
PSI( 5)=( -0.350814E 01, -0.364386E 01)
PSI( 6)=( -0.670473E 01, -0.704923E 01)
PSI( 7)=(  0.803951E 00,  0.883534E 00)
PSI( 8)=(  0.370872E 02,  0.373238E 02)
PSI( 9)=(  0.383065E 02,  0.385268E 02)
PSI(10)=( -0.523777E 00, -0.533038E 00)
PSI(11)=( -0.409808E 00,  0.413038E 00)
PSI(12)=(  0.146639E-02, -0.941098E-03)
PSI(13)=( -0.391632E-02,  0.348691E-02)
PSI(14)=(  0.465435E-01, -0.450222E-01)
PSI(15)=( -0.376944E-01,  0.368227E-01)
PSI(16)=( -0.728267E-01,  0.702820E-01)
PSI(17)=(  0.894937E-02, -0.823038E-02)
PSI(18)=(  0.385225E 00, -0.388461E 00)
PSI(19)=(  0.397661E 00, -0.401089E 00)
PSI(20)=( -0.111866E-02,  0.130856E-02)

COLUMN :  6
PSI( 1)=( -0.392533E 02,  0.395165E 02)
PSI( 2)=(  0.223312E-01, -0.950869E-01)
PSI( 3)=( -0.346385E 00,  0.393924E 00)
PSI( 4)=(  0.428643E 01, -0.449631E 01)
PSI( 5)=( -0.350814E 01,  0.364386E 01)
PSI( 6)=( -0.670473E 01,  0.704923E 01)
PSI( 7)=(  0.803951E 00, -0.883534E 00)
PSI( 8)=(  0.370872E 02, -0.373238E 02)
PSI( 9)=(  0.383065E 02, -0.385268E 02)
PSI(10)=( -0.523777E 00,  0.533038E 00)
PSI(11)=( -0.409808E 00, -0.413038E 00)
```

```
PSI(12)=(  0.146639E-02,  0.941098E-03)
PSI(13)=( -0.391632E-02, -0.348691E-02)
PSI(14)=(  0.465435E-01,  0.450222E-01)
PSI(15)=( -0.376944E-01, -0.368227E-01)
PSI(16)=( -0.728267E-01, -0.702820E-01)
PSI(17)=(  0.894937E-02,  0.823038E-02)
PSI(18)=(  0.385225E 00,  0.388461E 00)
PSI(19)=(  0.397661E 00,  0.401089E 00)
PSI(20)=( -0.111866E-02, -0.130856E-02)

COLUMN :  7
PSI( 1)=( -0.277687E 02, -0.165554E 02)
PSI( 2)=(  0.428821E 01,  0.258896E 01)
PSI( 3)=(  0.288868E 00, -0.973788E-01)
PSI( 4)=( -0.176626E 01, -0.125051E 01)
PSI( 5)=(  0.104908E 02,  0.689941E 01)
PSI( 6)=( -0.695920E 01, -0.453060E 01)
PSI( 7)=( -0.225450E 00,  0.856974E-01)
PSI( 8)=( -0.283034E 02, -0.169184E 02)
PSI( 9)=( -0.352714E 01, -0.193992E 01)
PSI(10)=(  0.520160E 01,  0.307811E 01)
PSI(11)=( -0.388828E 00,  0.660369E 00)
PSI(12)=(  0.598307E-01, -0.100116E 00)
PSI(13)=( -0.228356E-02, -0.690654E-02)
PSI(14)=( -0.295073E-01,  0.422746E-01)
PSI(15)=(  0.162263E 00, -0.249643E 00)
PSI(16)=( -0.106345E 00,  0.165188E 00)
PSI(17)=(  0.206417E-02,  0.533019E-02)
PSI(18)=( -0.398061E 00,  0.674586E 00)
PSI(19)=( -0.452976E-01,  0.837096E-01)
PSI(20)=(  0.735159E-01, -0.125571E 00)

COLUMN :  8
PSI( 1)=( -0.277687E 02,  0.165554E 02)
PSI( 2)=(  0.428821E 01, -0.258896E 01)
PSI( 3)=(  0.288868E 00,  0.973788E-01)
PSI( 4)=( -0.176626E 01,  0.125051E 01)
PSI( 5)=(  0.104908E 02, -0.689941E 01)
PSI( 6)=( -0.695920E 01,  0.453060E 01)
PSI( 7)=( -0.225450E 00, -0.856974E-01)
PSI( 8)=( -0.283034E 02,  0.169184E 02)
PSI( 9)=( -0.352714E 01,  0.193992E 01)
PSI(10)=(  0.520160E 01, -0.307811E 01)
PSI(11)=( -0.388828E 00, -0.660369E 00)
PSI(12)=(  0.598307E-01,  0.100116E 00)
PSI(13)=( -0.228356E-02,  0.690654E-02)
PSI(14)=( -0.295073E-01, -0.422746E-01)
PSI(15)=(  0.162263E 00,  0.249643E 00)
PSI(16)=( -0.106345E 00, -0.165188E 00)
PSI(17)=(  0.206417E-02, -0.533019E-02)
PSI(18)=( -0.398061E 00, -0.674586E 00)
PSI(19)=( -0.452976E-01, -0.837096E-01)
PSI(20)=(  0.735159E-01,  0.125571E 00)

COLUMN :  9
PSI( 1)=( -0.141153E 00,  0.432772E 00)
PSI( 2)=(  0.178584E 01, -0.280977E 01)
PSI( 3)=( -0.962844E 00,  0.131036E 01)
PSI( 4)=( -0.176652E 01,  0.280133E 01)
PSI( 5)=(  0.298133E 00, -0.569885E 00)
```

```
PSI( 6)=(  0.294372E 00, -0.383821E 00)
PSI( 7)=( -0.118133E 01,  0.159769E 01)
PSI( 8)=( -0.763254E-01,  0.331344E 00)
PSI( 9)=(  0.102646E 02, -0.158532E 02)
PSI(10)=( -0.178119E 01,  0.272190E 01)
PSI(11)=(  0.101557E-01,  0.317782E-02)
PSI(12)=( -0.679643E-01, -0.428181E-01)
PSI(13)=(  0.323165E-01,  0.235451E-01)
PSI(14)=(  0.696627E-01,  0.435398E-01)
PSI(15)=( -0.143684E-01, -0.753589E-02)
PSI(16)=( -0.101851E-01, -0.761485E-02)
PSI(17)=(  0.396732E-01,  0.290768E-01)
PSI(18)=(  0.127285E-01,  0.479119E-02)
PSI(19)=( -0.389003E 00, -0.249738E 00)
PSI(20)=(  0.642126E-01,  0.415858E-01)

COLUMN : 10
PSI( 1)=( -0.141153E 00, -0.432772E 00)
PSI( 2)=(  0.178584E 01,  0.280977E 01)
PSI( 3)=( -0.962844E 00, -0.131036E 01)
PSI( 4)=( -0.176652E 01, -0.280133E 01)
PSI( 5)=(  0.298133E 00,  0.569885E 00)
PSI( 6)=(  0.294372E 00,  0.383821E 00)
PSI( 7)=( -0.118133E 01, -0.159769E 01)
PSI( 8)=( -0.763254E-01, -0.331344E 00)
PSI( 9)=(  0.102646E 02,  0.158532E 02)
PSI(10)=( -0.178119E 01, -0.272190E 01)
PSI(11)=(  0.101557E-01, -0.317782E-02)
PSI(12)=( -0.679643E-01,  0.428181E-01)
PSI(13)=(  0.323165E-01, -0.235451E-01)
PSI(14)=(  0.696627E-01, -0.435398E-01)
PSI(15)=( -0.143684E-01,  0.753589E-02)
PSI(16)=( -0.101851E-01,  0.761485E-02)
PSI(17)=(  0.396732E-01, -0.290768E-01)
PSI(18)=(  0.127285E-01, -0.479119E-02)
PSI(19)=( -0.389003E 00,  0.249738E 00)
PSI(20)=(  0.642126E-01, -0.415858E-01)

COLUMN : 11
PSI( 1)=(  0.139303E 00, -0.246316E 00)
PSI( 2)=( -0.447154E-02,  0.613773E-02)
PSI( 3)=( -0.112642E 01,  0.358454E 01)
PSI( 4)=( -0.344290E 00,  0.133415E 01)
PSI( 5)=(  0.147164E 01, -0.494827E 01)
PSI( 6)=( -0.910167E 00,  0.283375E 01)
PSI( 7)=(  0.100473E 01, -0.331312E 01)
PSI( 8)=(  0.103886E 00, -0.171036E 00)
PSI( 9)=( -0.119540E-01,  0.253622E-01)
PSI(10)=(  0.919056E-02, -0.311450E-01)
PSI(11)=( -0.107051E-01, -0.513136E-02)
PSI(12)=( -0.197351E-03,  0.393391E-04)
PSI(13)=(  0.166494E 00,  0.363807E-01)
PSI(14)=(  0.610474E-01,  0.100510E-01)
PSI(15)=( -0.230072E 00, -0.465407E-01)
PSI(16)=(  0.133130E 00,  0.299112E-01)
PSI(17)=( -0.154059E 00, -0.320337E-01)
PSI(18)=( -0.892222E-02, -0.417590E-02)
PSI(19)=(  0.521719E-03,  0.332236E-03)
PSI(20)=( -0.243306E-03, -0.426769E-04)
```

```
COLUMN : 12
PSI( 1)=(  0.139303E 00,  0.246316E 00)
PSI( 2)=( -0.447154E-02, -0.613773E-02)
PSI( 3)=( -0.112642E 01, -0.358454E 01)
PSI( 4)=( -0.344290E 00, -0.133415E 01)
PSI( 5)=(  0.147164E 01,  0.494827E 01)
PSI( 6)=( -0.910167E 00, -0.283375E 01)
PSI( 7)=(  0.100473E 01,  0.331312E 01)
PSI( 8)=(  0.103886E 00,  0.171036E 00)
PSI( 9)=( -0.119540E-01, -0.253622E-01)
PSI(10)=(  0.919056E-02,  0.311450E-01)
PSI(11)=( -0.107051E-01,  0.513136E-02)
PSI(12)=( -0.197351E-03, -0.393391E-04)
PSI(13)=(  0.166494E 00, -0.363807E-01)
PSI(14)=(  0.610474E-01, -0.100510E-01)
PSI(15)=( -0.230072E 00,  0.465407E-01)
PSI(16)=(  0.133130E 00, -0.299112E-01)
PSI(17)=( -0.154059E 00,  0.320337E-01)
PSI(18)=( -0.892222E-02,  0.417590E-02)
PSI(19)=(  0.521719E-03, -0.332236E-03)
PSI(20)=( -0.243306E-03,  0.426769E-04)

COLUMN : 13
PSI( 1)=( -0.819890E-02, -0.300606E-01)
PSI( 2)=( -0.444531E-02, -0.398213E-01)
PSI( 3)=( -0.579245E 00, -0.194805E 01)
PSI( 4)=(  0.550678E 00,  0.188116E 01)
PSI( 5)=( -0.184984E 00, -0.627882E 00)
PSI( 6)=( -0.116982E 00, -0.308679E 00)
PSI( 7)=( -0.552398E 00, -0.196330E 01)
PSI( 8)=(  0.781059E-03,  0.839004E-02)
PSI( 9)=( -0.651447E-02, -0.829455E-01)
PSI(10)=(  0.266583E-02,  0.342486E-01)
PSI(11)=( -0.178635E-03,  0.737607E-05)
PSI(12)=( -0.224417E-02,  0.431109E-03)
PSI(13)=( -0.101773E 00,  0.387472E-01)
PSI(14)=(  0.980868E-01, -0.368916E-01)
PSI(15)=( -0.330206E-01,  0.124948E-01)
PSI(16)=( -0.154554E-01,  0.725723E-02)
PSI(17)=( -0.102816E 00,  0.374523E-01)
PSI(18)=(  0.587702E-04,  0.693239E-04)
PSI(19)=( -0.493670E-02,  0.900567E-03)
PSI(20)=(  0.224791E-02, -0.447869E-03)

COLUMN : 14
PSI( 1)=( -0.819890E-02,  0.300606E-01)
PSI( 2)=( -0.444531E-02,  0.398213E-01)
PSI( 3)=( -0.579245E 00,  0.194805E 01)
PSI( 4)=(  0.550678E 00, -0.188116E 01)
PSI( 5)=( -0.184984E 00,  0.627882E 00)
PSI( 6)=( -0.116982E 00,  0.308679E 00)
PSI( 7)=( -0.552398E 00,  0.196330E 01)
PSI( 8)=(  0.781059E-03, -0.839004E-02)
PSI( 9)=( -0.651447E-02,  0.829455E-01)
PSI(10)=(  0.266583E-02, -0.342486E-01)
PSI(11)=( -0.178635E-03, -0.737607E-05)
PSI(12)=( -0.224417E-02, -0.431109E-03)
PSI(13)=( -0.101773E 00, -0.387472E-01)
PSI(14)=(  0.980868E-01,  0.368916E-01)
PSI(15)=( -0.330206E-01, -0.124948E-01)
```

```
PSI(16)=( -0.154554E-01, -0.725723E-02)
PSI(17)=( -0.102816E 00, -0.374523E-01)
PSI(18)=(  0.587702E-04, -0.693239E-04)
PSI(19)=( -0.493670E-02, -0.900567E-03)
PSI(20)=(  0.224791E-02,  0.447869E-03)

COLUMN : 15
PSI( 1)=( -0.251893E-01, -0.518876E-01)
PSI( 2)=( -0.154209E-02, -0.638593E-03)
PSI( 3)=(  0.798678E-01,  0.447006E-01)
PSI( 4)=( -0.558757E 01, -0.427790E 01)
PSI( 5)=(  0.558624E 01,  0.426671E 01)
PSI( 6)=(  0.537312E 01,  0.419946E 01)
PSI( 7)=( -0.936677E 00, -0.589157E 00)
PSI( 8)=(  0.233562E 00,  0.218474E 00)
PSI( 9)=(  0.903549E-01,  0.100364E 00)
PSI(10)=(  0.391417E-01,  0.242795E-01)
PSI(11)=( -0.566959E-02,  0.543535E-02)
PSI(12)=( -0.156689E-02,  0.221676E-02)
PSI(13)=(  0.293022E-02, -0.592327E-02)
PSI(14)=( -0.264968E 00,  0.397112E 00)
PSI(15)=(  0.266758E 00, -0.400576E 00)
PSI(16)=(  0.259013E 00, -0.380047E 00)
PSI(17)=( -0.351677E-01,  0.652285E-01)
PSI(18)=(  0.121406E-01, -0.145432E-01)
PSI(19)=(  0.622463E-02, -0.626630E-02)
PSI(20)=( -0.161171E-03,  0.131428E-03)

COLUMN : 16
PSI( 1)=( -0.251893E-01,  0.518876E-01)
PSI( 2)=( -0.154209E-02,  0.638593E-03)
PSI( 3)=(  0.798678E-01, -0.447006E-01)
PSI( 4)=( -0.558757E 01,  0.427790E 01)
PSI( 5)=(  0.558624E 01, -0.426671E 01)
PSI( 6)=(  0.537312E 01, -0.419946E 01)
PSI( 7)=( -0.936677E 00,  0.589157E 00)
PSI( 8)=(  0.233562E 00, -0.218474E 00)
PSI( 9)=(  0.903549E-01, -0.100364E 00)
PSI(10)=(  0.391417E-01, -0.242795E-01)
PSI(11)=( -0.566959E-02, -0.543535E-02)
PSI(12)=( -0.156689E-02, -0.221676E-02)
PSI(13)=(  0.293022E-02,  0.592327E-02)
PSI(14)=( -0.264968E 00, -0.397112E 00)
PSI(15)=(  0.266758E 00,  0.400576E 00)
PSI(16)=(  0.259013E 00,  0.380047E 00)
PSI(17)=( -0.351677E-01, -0.652285E-01)
PSI(18)=(  0.121406E-01,  0.145432E-01)
PSI(19)=(  0.622463E-02,  0.626630E-02)
PSI(20)=( -0.161171E-03, -0.131428E-03)

COLUMN : 17
PSI( 1)=( -0.385205E 00, -0.791083E-02)
PSI( 2)=(  0.102724E 00,  0.454023E-02)
PSI( 3)=( -0.115092E 01,  0.416769E-01)
PSI( 4)=(  0.281954E 01, -0.719610E-01)
PSI( 5)=( -0.692335E 01,  0.262621E 00)
PSI( 6)=(  0.576581E 01, -0.368226E 00)
PSI( 7)=(  0.193839E 01, -0.806383E-01)
PSI( 8)=( -0.250298E 00, -0.161300E-01)
PSI( 9)=(  0.165185E 00, -0.458986E-02)
```

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PSI(10)=( -0.301857E-01,  0.478438E-02)
PSI(11)=(  0.424683E-03,  0.389020E-01)
PSI(12)=( -0.119448E-03, -0.109070E-01)
PSI(13)=(  0.939274E-02,  0.123095E 00)
PSI(14)=( -0.196009E-01, -0.299986E 00)
PSI(15)=(  0.581121E-01,  0.741035E 00)
PSI(16)=( -0.648346E-01, -0.620176E 00)
PSI(17)=( -0.169134E-01, -0.207329E 00)
PSI(18)=(  0.134230E-03,  0.334951E-01)
PSI(19)=( -0.283360E-03, -0.121464E-01)
PSI(20)=( -0.759959E-04, -0.637758E-02)

COLUMN : 18
PSI( 1)=( -0.385205E 00,  0.791083E-02)
PSI( 2)=(  0.102724E 00, -0.454023E-02)
PSI( 3)=( -0.115092E 01, -0.416769E-01)
PSI( 4)=(  0.281954E 01,  0.719610E-01)
PSI( 5)=( -0.692335E 01, -0.262621E 00)
PSI( 6)=(  0.576581E 01,  0.368226E 00)
PSI( 7)=(  0.193839E 01,  0.806383E-01)
PSI( 8)=( -0.250298E 00,  0.161300E-01)
PSI( 9)=(  0.165185E 00,  0.458986E-02)
PSI(10)=( -0.301857E-01, -0.478438E-02)
PSI(11)=(  0.424683E-03, -0.389020E-01)
PSI(12)=( -0.119448E-03,  0.109070E-01)
PSI(13)=(  0.939274E-02, -0.123095E 00)
PSI(14)=( -0.196009E-01,  0.299986E 00)
PSI(15)=(  0.581121E-01, -0.741035E 00)
PSI(16)=( -0.648346E-01,  0.620176E 00)
PSI(17)=( -0.169134E-01,  0.207329E 00)
PSI(18)=(  0.134230E-03, -0.334951E-01)
PSI(19)=( -0.283360E-03,  0.121464E-01)
PSI(20)=( -0.759959E-04,  0.637758E-02)

COLUMN : 19
PSI( 1)=( -0.333514E-01,  0.589250E-01)
PSI( 2)=( -0.865978E-02,  0.152073E-01)
PSI( 3)=( -0.559035E 00,  0.120569E 01)
PSI( 4)=( -0.289031E 01,  0.628952E 01)
PSI( 5)=( -0.648808E 00,  0.157208E 01)
PSI( 6)=(  0.102269E 01, -0.224493E 01)
PSI( 7)=( -0.234214E 00,  0.491885E 00)
PSI( 8)=( -0.411887E-01,  0.764437E-01)
PSI( 9)=( -0.111519E 00,  0.205245E 00)
PSI(10)=(  0.378304E-01, -0.741993E-01)
PSI(11)=(  0.102919E-01,  0.493661E-02)
PSI(12)=(  0.369090E-02,  0.187002E-02)
PSI(13)=(  0.162216E 00,  0.685248E-01)
PSI(14)=(  0.846458E 00,  0.354481E 00)
PSI(15)=(  0.211280E 00,  0.788504E-01)
PSI(16)=( -0.302252E 00, -0.125264E 00)
PSI(17)=(  0.664844E-01,  0.288832E-01)
PSI(18)=(  0.121624E-01,  0.583068E-02)
PSI(19)=(  0.278113E-01,  0.137537E-01)
PSI(20)=( -0.902647E-02, -0.438887E-02)

COLUMN : 20
PSI( 1)=( -0.333514E-01, -0.589250E-01)
PSI( 2)=( -0.865978E-02, -0.152073E-01)
PSI( 3)=( -0.559035E 00, -0.120569E 01)
```

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PSI( 4)=( -0.289031E 01, -0.628952E 01)
PSI( 5)=( -0.648808E 00, -0.157208E 01)
PSI( 6)=(  0.102269E 01,  0.224493E 01)
PSI( 7)=( -0.234214E 00, -0.491885E 00)
PSI( 8)=( -0.411887E-01, -0.764437E-01)
PSI( 9)=( -0.111519E 00, -0.205245E 00)
PSI(10)=(  0.378304E-01,  0.741993E-01)
PSI(11)=(  0.102919E-01, -0.493661E-02)
PSI(12)=(  0.369090E-02, -0.187002E-02)
PSI(13)=(  0.162216E 00, -0.685248E-01)
PSI(14)=(  0.846458E 00, -0.354481E 00)
PSI(15)=(  0.211280E 00, -0.788504E-01)
PSI(16)=( -0.302252E 00,  0.125264E 00)
PSI(17)=(  0.664844E-01, -0.288832E-01)
PSI(18)=(  0.121624E-01, -0.583068E-02)
PSI(19)=(  0.278113E-01, -0.137537E-01)
PSI(20)=( -0.902647E-02,  0.438887E-02)

EIGENVALUES - COMMON DENOMINATORS
   D( 1) = S-( -0.123413E 01,  0.145839E 03)
   D( 2) = S-( -0.123413E 01, -0.145839E 03)
   D( 3) = S-( -0.124365E 01,  0.145310E 03)
   D( 4) = S-( -0.124365E 01, -0.145310E 03)
   D( 5) = S-( -0.701248E 00,  0.954895E 02)
   D( 6) = S-( -0.701248E 00, -0.954895E 02)
   D( 7) = S-( -0.226822E 00,  0.422107E 02)
   D( 8) = S-( -0.226822E 00, -0.422107E 02)
   D( 9) = S-( -0.158783E 00,  0.405879E 02)
   D(10) = S-( -0.158783E 00, -0.405879E 02)
   D(11) = S-( -0.196765E 01,  0.219543E 02)
   D(12) = S-( -0.196765E 01, -0.219543E 02)
   D(13) = S-( -0.139571E 01,  0.186094E 02)
   D(14) = S-( -0.139571E 01, -0.186094E 02)
   D(15) = S-( -0.953362E 00,  0.146475E 02)
   D(16) = S-( -0.953362E 00, -0.146475E 02)
   D(17) = S-( -0.374243E 00,  0.930475E 01)
   D(18) = S-( -0.374243E 00, -0.930475E 01)
   D(19) = S-( -0.257315E 00,  0.753748E 01)
   D(20) = S-( -0.257315E 00, -0.753748E 01)

ACCELERATION TRANSFER FUNCTION : POIN - SEAT
   MULTIPLY BY OMEGA**2 FOR FREQUENCY RESPONSE

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 1) : BLRP
WMAT(11) =  -0.153051E-05  WMAT(21) =   0.496920E 00  WMAT(31) =  -0.130684E 01
WMAT(12) =   0.0           WMAT(22) =  -0.675224E-08  WMAT(32) =  -0.121245E-06
WMAT(13) =   0.675224E-08  WMAT(23) =   0.0           WMAT(33) =   0.277778E-01
WMAT(14) =   0.121245E-06  WMAT(24) =  -0.277778E-01  WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 2) : ALFP
WMAT(11) =  -0.100000E 01  WMAT(21) =  -0.388507E-05  WMAT(31) =   0.125751E-05
WMAT(12) =   0.0           WMAT(22) =  -0.372232E-08  WMAT(32) =   0.970257E-11
WMAT(13) =   0.372232E-08  WMAT(23) =   0.0           WMAT(33) =  -0.265425E-07
WMAT(14) =  -0.970257E-11  WMAT(24) =   0.265425E-07  WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -
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FGC( 3) : LLFT
WMAT(11) =   0.492299E-05  WMAT(21) =  -0.164465E-05  WMAT(31) =   0.100000E 01
WMAT(12) =   0.0           WMAT(22) =   0.121217E-07  WMAT(32) =   0.369430E-08
WMAT(13) =  -0.121217E-07  WMAT(23) =   0.0           WMAT(33) =  -0.163618E-08
WMAT(14) =  -0.369430E-08  WMAT(24) =   0.163618E-08  WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 4) : VLFT
WMAT(11) =   0.383011E-01  WMAT(21) =  -0.499995E 00  WMAT(31) =  -0.467750E-01
WMAT(12) =   0.0           WMAT(22) =   0.997501E-02  WMAT(32) =  -0.255640E-04
WMAT(13) =  -0.997501E-02  WMAT(23) =   0.0           WMAT(33) =   0.125000E-01
WMAT(14) =   0.255640E-04  WMAT(24) =  -0.125000E-01  WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 5) : VRFT
WMAT(11) =  -0.473934E-07  WMAT(21) =  -0.499998E 00  WMAT(31) =  -0.467750E-01
WMAT(12) =   0.0           WMAT(22) =  -0.746209E-08  WMAT(32) =   0.274239E-09
WMAT(13) =   0.746209E-08  WMAT(23) =   0.0           WMAT(33) =   0.125000E-01
WMAT(14) =  -0.274239E-09  WMAT(24) =  -0.125000E-01  WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 6) : VLRT
WMAT(11) =   0.383010E-01  WMAT(21) =  -0.999994E 00  WMAT(31) =   0.256288E-02
WMAT(12) =   0.0           WMAT(22) =   0.997502E-02  WMAT(32) =  -0.255645E-04
WMAT(13) =  -0.997502E-02  WMAT(23) =   0.0           WMAT(33) =  -0.745056E-08
WMAT(14) =   0.255645E-04  WMAT(24) =   0.745056E-08  WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 7) : ALRT
WMAT(11) =  -0.999994E 00  WMAT(21) =   0.159444E-07  WMAT(31) =   0.222685E-07
WMAT(12) =   0.0           WMAT(22) =  -0.688777E-09  WMAT(32) =  -0.245671E-09
WMAT(13) =   0.688777E-09  WMAT(23) =   0.0           WMAT(33) =  -0.409500E-09
WMAT(14) =   0.245671E-09  WMAT(24) =   0.409500E-09  WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 8) : BRFP
WMAT(11) =  -0.130555E 01  WMAT(21) =  -0.591222E-05  WMAT(31) =   0.168207E-07
WMAT(12) =   0.0           WMAT(22) =  -0.277777E-01  WMAT(32) =   0.715467E-04
WMAT(13) =   0.277777E-01  WMAT(23) =   0.0           WMAT(33) =   0.150289E-11
WMAT(14) =  -0.715467E-04  WMAT(24) =  -0.150289E-11  WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 9) : BRRP
WMAT(11) =   0.130555E 01  WMAT(21) =  -0.503071E 00  WMAT(31) =  -0.130426E 01
WMAT(12) =   0.0           WMAT(22) =   0.277777E-01  WMAT(32) =  -0.716679E-04
WMAT(13) =  -0.277777E-01  WMAT(23) =   0.0           WMAT(33) =   0.277778E-01
WMAT(14) =   0.716679E-04  WMAT(24) =  -0.277778E-01  WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC(10) : LLFP
WMAT(11) =   0.956763E-07  WMAT(21) =  -0.256382E-02  WMAT(31) =  -0.999997E 00
WMAT(12) =   0.0           WMAT(22) =   0.948528E-08  WMAT(32) =   0.208816E-09
WMAT(13) =  -0.948528E-08  WMAT(23) =   0.0           WMAT(33) =   0.279630E-07
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WMAT(14) = -0.208816E-09 WMAT(24) = -0.279630E-07 WMAT(34) = 0.0

X-COMPONENT NUMERATORS(2*NFGC) :

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BASE INPUT EXCITATION( 1) : RRSM
NC( 1) = ( -0.152681E-07,  0.978093E-06)
NK( 1) = ( -0.146950E-05,  0.977887E-04)
NM( 1) = (  0.172444E-13,  0.907534E-13)
NC( 2) = ( -0.152681E-07, -0.978093E-06)
NK( 2) = ( -0.146950E-05, -0.977887E-04)
NM( 2) = (  0.172444E-13, -0.907534E-13)
NC( 3) = ( -0.453925E-06,  0.861283E-05)
NK( 3) = ( -0.461174E-04,  0.873825E-03)
NM( 3) = ( -0.193147E-11, -0.474817E-09)
NC( 4) = ( -0.453925E-06, -0.861283E-05)
NK( 4) = ( -0.461174E-04, -0.873825E-03)
NM( 4) = ( -0.193147E-11,  0.474817E-09)
NC( 5) = ( -0.283329E-05,  0.596791E-05)
NK( 5) = ( -0.284906E-03,  0.600158E-03)
NM( 5) = ( -0.133986E-12,  0.379779E-12)
NC( 6) = ( -0.283329E-05, -0.596791E-05)
NK( 6) = ( -0.284906E-03, -0.600158E-03)
NM( 6) = ( -0.133986E-12, -0.379779E-12)
NC( 7) = (  0.163406E-04, -0.619188E-03)
NK( 7) = (  0.164453E-02, -0.622468E-01)
NM( 7) = ( -0.163529E-10, -0.699362E-09)
NC( 8) = (  0.163406E-04,  0.619188E-03)
NK( 8) = (  0.164453E-02,  0.622468E-01)
NM( 8) = ( -0.163529E-10,  0.699362E-09)
NC( 9) = (  0.336923E-04,  0.134269E-02)
NK( 9) = (  0.338962E-02,  0.134985E 00)
NM( 9) = ( -0.197993E-10, -0.115466E-08)
NC(10) = (  0.336923E-04, -0.134269E-02)
NK(10) = (  0.338962E-02, -0.134985E 00)
NM(10) = ( -0.197993E-10,  0.115466E-08)
NC(11) = ( -0.340099E-04, -0.424166E-03)
NK(11) = ( -0.342239E-02, -0.427054E-01)
NM(11) = (  0.644211E-11,  0.112889E-09)
NC(12) = ( -0.340099E-04,  0.424166E-03)
NK(12) = ( -0.342239E-02,  0.427054E-01)
NM(12) = (  0.644211E-11, -0.112889E-09)
NC(13) = ( -0.117354E-03,  0.296707E-02)
NK(13) = ( -0.118033E-01,  0.298554E 00)
NM(13) = (  0.330135E-10, -0.237899E-09)
NC(14) = ( -0.117354E-03, -0.296707E-02)
NK(14) = ( -0.118033E-01, -0.298554E 00)
NM(14) = (  0.330135E-10,  0.237899E-09)
NC(15) = (  0.429041E-03,  0.427959E-02)
NK(15) = (  0.431462E-01,  0.430377E 00)
NM(15) = ( -0.133755E-09, -0.124875E-08)
NC(16) = (  0.429041E-03, -0.427959E-02)
NK(16) = (  0.431462E-01, -0.430377E 00)
NM(16) = ( -0.133755E-09,  0.124875E-08)
NC(17) = ( -0.366006E-03, -0.678199E-02)
NK(17) = ( -0.368048E-01, -0.682288E 00)
NM(17) = (  0.161788E-09, -0.560746E-08)
NC(18) = ( -0.366006E-03,  0.678199E-02)
NK(18) = ( -0.368048E-01,  0.682288E 00)
NM(18) = (  0.161788E-09,  0.560746E-08)
NC(19) = (  0.536771E-04, -0.990029E-02)
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NK(19) = (  0.539552E-02, -0.995461E 00)
NM(19) = ( -0.376525E-10,  0.575197E-08)
NC(20) = (  0.536771E-04,  0.990029E-02)
NK(20) = (  0.539552E-02,  0.995461E 00)
NM(20) = ( -0.376525E-10, -0.575197E-08)

BASE INPUT EXCITATION( 2) : RFSM
NC( 1) = (  0.226358E-07, -0.104031E-05)
NK( 1) = (  0.159653E-04, -0.235600E-03)
NM( 1) = ( -0.626818E-07,  0.939927E-06)
NC( 2) = (  0.226358E-07,  0.104031E-05)
NK( 2) = (  0.159653E-04,  0.235600E-03)
NM( 2) = ( -0.626818E-07, -0.939927E-06)
NC( 3) = (  0.397307E-06, -0.717586E-05)
NK( 3) = ( -0.330805E-04,  0.230744E-02)
NM( 3) = (  0.117758E-05, -0.233359E-04)
NC( 4) = (  0.397307E-06,  0.717586E-05)
NK( 4) = ( -0.330805E-04, -0.230744E-02)
NM( 4) = (  0.117758E-05,  0.233359E-04)
NC( 5) = (  0.283364E-05, -0.596930E-05)
NK( 5) = (  0.286009E-03, -0.602244E-03)
NM( 5) = (  0.166779E-06, -0.153290E-06)
NC( 6) = (  0.283364E-05,  0.596930E-05)
NK( 6) = (  0.286009E-03,  0.602244E-03)
NM( 6) = (  0.166779E-06,  0.153290E-06)
NC( 7) = ( -0.160958E-04,  0.619400E-03)
NK( 7) = ( -0.115870E-02,  0.626512E-01)
NM( 7) = ( -0.614553E-04,  0.160840E-03)
NC( 8) = ( -0.160958E-04, -0.619400E-03)
NK( 8) = ( -0.115870E-02, -0.626512E-01)
NM( 8) = ( -0.614553E-04, -0.160840E-03)
NC( 9) = ( -0.335373E-04, -0.134632E-02)
NK( 9) = ( -0.343598E-02, -0.141591E 00)
NM( 9) = (  0.255591E-04, -0.101049E-03)
NC(10) = ( -0.335373E-04,  0.134632E-02)
NK(10) = ( -0.343598E-02,  0.141591E 00)
NM(10) = (  0.255591E-04,  0.101049E-03)
NC(11) = (  0.336815E-04,  0.417118E-03)
NK(11) = (  0.304948E-02,  0.345389E-01)
NM(11) = (  0.293780E-04,  0.435948E-03)
NC(12) = (  0.336815E-04, -0.417118E-03)
NK(12) = (  0.304948E-02, -0.345389E-01)
NM(12) = (  0.293780E-04, -0.435948E-03)
NC(13) = (  0.117166E-03, -0.294506E-02)
NK(13) = (  0.116614E-01, -0.282111E 00)
NM(13) = (  0.705915E-04, -0.439185E-03)
NC(14) = (  0.117166E-03,  0.294506E-02)
NK(14) = (  0.116614E-01,  0.282111E 00)
NM(14) = (  0.705915E-04,  0.439185E-03)
NC(15) = ( -0.429092E-03, -0.427981E-02)
NK(15) = ( -0.431152E-01, -0.430183E 00)
NM(15) = ( -0.342807E-03, -0.333317E-02)
NC(16) = ( -0.429092E-03,  0.427981E-02)
NK(16) = ( -0.431152E-01,  0.430183E 00)
NM(16) = ( -0.342807E-03,  0.333317E-02)
NC(17) = (  0.366267E-03,  0.676540E-02)
NK(17) = (  0.364485E-01,  0.741054E 00)
NM(17) = (  0.414999E-03, -0.119044E-01)
NC(18) = (  0.366267E-03, -0.676540E-02)
NK(18) = (  0.364485E-01, -0.741054E 00)
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NM(18) = (  0.414999E-03,  0.119044E-01)
NC(19) = ( -0.538129E-04,  0.990796E-02)
NK(19) = ( -0.501677E-02,  0.945781E 00)
NM(19) = ( -0.151985E-03,  0.196758E-01)
NC(20) = ( -0.538129E-04, -0.990796E-02)
NK(20) = ( -0.501677E-02, -0.945781E 00)
NM(20) = ( -0.151985E-03, -0.196758E-01)

BASE INPUT EXCITATION( 3) : LRSM
NC( 1) = (  0.257826E-07, -0.106688E-05)
NK( 1) = (  0.278305E-05, -0.109605E-03)
NM( 1) = (  0.416583E-07, -0.171491E-06)
NC( 2) = (  0.257826E-07,  0.106688E-05)
NK( 2) = (  0.278305E-05,  0.109605E-03)
NM( 2) = (  0.416583E-07,  0.171491E-06)
NC( 3) = (  0.373217E-06, -0.656621E-05)
NK( 3) = (  0.467756E-04, -0.724488E-02)
NM( 3) = (  0.346280E-05,  0.796029E-05)
NC( 4) = (  0.373217E-06,  0.656621E-05)
NK( 4) = (  0.467756E-04,  0.724488E-02)
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NC( 5) = (  0.283406E-05, -0.596907E-05)
NK( 5) = (  0.282791E-03, -0.595688E-03)
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NK( 6) = (  0.282791E-03,  0.595688E-03)
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NK( 8) = ( -0.189802E-02, -0.620759E-01)
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NK( 9) = ( -0.341824E-02, -0.141749E 00)
NM( 9) = (  0.208216E-04, -0.600614E-04)
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NK(10) = ( -0.341824E-02,  0.141749E 00)
NM(10) = (  0.208216E-04,  0.600614E-04)
NC(11) = (  0.335417E-04,  0.414119E-03)
NK(11) = (  0.366083E-02,  0.468890E-01)
NM(11) = (  0.524762E-05, -0.235816E-03)
NC(12) = (  0.335417E-04, -0.414119E-03)
NK(12) = (  0.366083E-02, -0.468890E-01)
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NK(13) = (  0.115166E-01, -0.283966E 00)
NM(13) = (  0.757867E-04, -0.553183E-03)
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NK(14) = (  0.115166E-01,  0.283966E 00)
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NK(15) = ( -0.432974E-01, -0.430857E 00)
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NK(19) = ( -0.512311E-02,  0.980046E 00)
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BASE INPUT EXCITATION( 4) : LFSM
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NM( 1) = (  0.595201E-07, -0.924808E-06)
NC( 2) = ( -0.331503E-07, -0.112909E-05)
NK( 2) = ( -0.172784E-04, -0.247412E-03)
NM( 2) = (  0.595201E-07,  0.924808E-06)
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NK( 3) = (  0.324009E-04,  0.405920E-02)
NM( 3) = ( -0.166468E-05,  0.236328E-04)
NC( 4) = ( -0.316605E-06, -0.512871E-05)
NK( 4) = (  0.324009E-04, -0.405920E-02)
NM( 4) = ( -0.166468E-05, -0.236328E-04)
NC( 5) = ( -0.283486E-05,  0.597093E-05)
NK( 5) = ( -0.283889E-03,  0.597834E-03)
NM( 5) = ( -0.101965E-06,  0.113723E-06)
NC( 6) = ( -0.283486E-05, -0.597093E-05)
NK( 6) = ( -0.283889E-03, -0.597834E-03)
NM( 6) = ( -0.101965E-06, -0.113723E-06)
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NK( 7) = (  0.141212E-02, -0.624807E-01)
NM( 7) = (  0.809131E-05, -0.757802E-05)
NC( 8) = (  0.157456E-04,  0.619704E-03)
NK( 8) = (  0.141212E-02,  0.624807E-01)
NM( 8) = (  0.809131E-05,  0.757802E-05)
NC( 9) = (  0.333143E-04,  0.135149E-02)
NK( 9) = (  0.346454E-02,  0.148355E 00)
NM( 9) = ( -0.495866E-04,  0.202481E-03)
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NK(10) = (  0.346454E-02, -0.148355E 00)
NM(10) = ( -0.495866E-04, -0.202481E-03)
NC(11) = ( -0.332136E-04, -0.407070E-03)
NK(11) = ( -0.328789E-02, -0.387225E-01)
NM(11) = (  0.637022E-05, -0.104706E-07)
NC(12) = ( -0.332136E-04,  0.407070E-03)
NK(12) = ( -0.328789E-02,  0.387225E-01)
NM(12) = (  0.637022E-05,  0.104706E-07)
NC(13) = ( -0.116897E-03,  0.291363E-02)
NK(13) = ( -0.113749E-01,  0.267523E 00)
NM(13) = ( -0.736873E-04,  0.182556E-02)
NC(14) = ( -0.116897E-03, -0.291363E-02)
NK(14) = ( -0.113749E-01, -0.267523E 00)
NM(14) = ( -0.736873E-04, -0.182556E-02)
NC(15) = (  0.429168E-03,  0.428011E-02)
NK(15) = (  0.432661E-01,  0.430665E 00)
NM(15) = (  0.236016E-05,  0.746141E-03)
NC(16) = (  0.429168E-03, -0.428011E-02)
NK(16) = (  0.432661E-01, -0.430665E 00)
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NC(17) = ( -0.366639E-03, -0.674173E-02)
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NK(17) = ( -0.366475E-01, -0.657372E 00)
NM(17) = ( -0.187038E-03, -0.156842E-01)
NC(18) = ( -0.366639E-03,  0.674173E-02)
NK(18) = ( -0.366475E-01,  0.657372E 00)
NM(18) = ( -0.187038E-03,  0.156842E-01)
NC(19) = (  0.540104E-04, -0.991891E-02)
NK(19) = (  0.474455E-02, -0.930366E 00)
NM(19) = (  0.306579E-03, -0.287918E-01)
NC(20) = (  0.540104E-04,  0.991891E-02)
NK(20) = (  0.474455E-02,  0.930366E 00)
NM(20) = (  0.306579E-03,  0.287918E-01)
BASE( 1) - INPUT DISPL WVECT( 1) :       0.000
BASE( 2) - INPUT DISPL WVECT( 4) :       0.000
BASE( 3) - INPUT DISPL WVECT( 7) :      -0.633
BASE( 4) - INPUT DISPL WVECT(10) :       0.633

Y-COMPONENT NUMERATORS(2*NFGC) :

BASE INPUT EXCITATION( 1) : RRSM
NC( 1) = (  0.934288E-07,  0.247661E-06)
NK( 1) = (  0.935551E-05,  0.247593E-04)
NM( 1) = (  0.133659E-13,  0.211231E-13)
NC( 2) = (  0.934288E-07, -0.247661E-06)
NK( 2) = (  0.935551E-05, -0.247593E-04)
NM( 2) = (  0.133659E-13, -0.211231E-13)
NC( 3) = (  0.179890E-06, -0.331318E-05)
NK( 3) = (  0.182756E-04, -0.336141E-03)
NM( 3) = (  0.453036E-12,  0.182669E-09)
NC( 4) = (  0.179890E-06,  0.331318E-05)
NK( 4) = (  0.182756E-04,  0.336141E-03)
NM( 4) = (  0.453036E-12, -0.182669E-09)
NC( 5) = ( -0.554696E-04,  0.373141E-02)
NK( 5) = ( -0.557808E-02,  0.375245E 00)
NM( 5) = (  0.202992E-10,  0.226645E-09)
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NC( 7) = (  0.432062E-05, -0.271802E-03)
NK( 7) = (  0.435176E-03, -0.273241E-01)
NM( 7) = ( -0.103956E-10, -0.306835E-09)
NC( 8) = (  0.432062E-05,  0.271802E-03)
NK( 8) = (  0.435176E-03,  0.273241E-01)
NM( 8) = ( -0.103956E-10,  0.306835E-09)
NC( 9) = (  0.779545E-05,  0.687388E-03)
NK( 9) = (  0.784942E-03,  0.691050E-01)
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NC(10) = (  0.779545E-05, -0.687388E-03)
NK(10) = (  0.784942E-03, -0.691050E-01)
NM(10) = ( -0.200841E-11,  0.591062E-09)
NC(11) = ( -0.201551E-04,  0.665593E-03)
NK(11) = ( -0.203193E-02,  0.670119E-01)
NM(11) = (  0.942265E-11, -0.176694E-09)
NC(12) = ( -0.201551E-04, -0.665593E-03)
NK(12) = ( -0.203193E-02, -0.670119E-01)
NM(12) = (  0.942265E-11,  0.176694E-09)
NC(13) = (  0.632144E-04, -0.514590E-03)
NK(13) = (  0.635988E-02, -0.517795E-01)
NM(13) = ( -0.917571E-11,  0.409193E-10)
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NK(15) = (  0.311582E-01, -0.218160E 01)
NM(15) = ( -0.474814E-10,  0.633491E-08)
NC(16) = (  0.309814E-03,  0.216936E-01)
NK(16) = (  0.311582E-01,  0.218160E 01)
NM(16) = ( -0.474814E-10, -0.633491E-08)
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NK(17) = ( -0.239214E-01, -0.293839E 00)
NM(17) = (  0.364125E-11, -0.242042E-08)
NC(18) = ( -0.237851E-03,  0.292077E-02)
NK(18) = ( -0.239214E-01,  0.293839E 00)
NM(18) = (  0.364125E-11,  0.242042E-08)
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NK(19) = ( -0.640799E-02, -0.451274E 00)
NM(19) = (  0.340912E-10,  0.260761E-08)
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NM(20) = (  0.340912E-10, -0.260761E-08)

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NM( 1) = (  0.777007E-07,  0.242766E-06)
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NK( 2) = ( -0.194124E-04,  0.608716E-04)
NM( 2) = (  0.777007E-07, -0.242766E-06)
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NK( 3) = (  0.141359E-04, -0.887685E-03)
NM( 3) = ( -0.467282E-06,  0.897691E-05)
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NK( 4) = (  0.141359E-04,  0.887685E-03)
NM( 4) = ( -0.467282E-06, -0.897691E-05)
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NK( 5) = (  0.564032E-02, -0.376608E 00)
NM( 5) = (  0.497362E-04, -0.117906E-03)
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NK( 6) = (  0.564032E-02,  0.376608E 00)
NM( 6) = (  0.497362E-04,  0.117906E-03)
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NK( 7) = ( -0.220110E-03,  0.274994E-01)
NM( 7) = ( -0.262289E-04,  0.708665E-04)
NC( 8) = ( -0.421219E-05, -0.271894E-03)
NK( 8) = ( -0.220110E-03, -0.274994E-01)
NM( 8) = ( -0.262289E-04, -0.708665E-04)
NC( 9) = ( -0.769061E-05, -0.689240E-03)
NK( 9) = ( -0.762190E-03, -0.724862E-01)
NM( 9) = (  0.137914E-04, -0.515340E-04)
NC(10) = ( -0.769061E-05,  0.689240E-03)
NK(10) = ( -0.762190E-03,  0.724862E-01)
NM(10) = (  0.137914E-04,  0.515340E-04)
NC(11) = (  0.194520E-04, -0.654574E-03)
NK(11) = (  0.120545E-02, -0.542459E-01)
NM(11) = (  0.293882E-04, -0.683120E-03)
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NK(12) = (  0.120545E-02,  0.542459E-01)
NM(12) = (  0.293882E-04,  0.683120E-03)
NC(13) = ( -0.628643E-04,  0.510762E-03)
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NM(13) = ( -0.186176E-04,  0.754015E-04)
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NK(15) = ( -0.312032E-01,  0.218062E 01)
NM(15) = ( -0.197958E-03,  0.169010E-01)
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NK(16) = ( -0.312032E-01, -0.218062E 01)
NM(16) = ( -0.197958E-03, -0.169010E-01)
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NK(17) = (  0.244609E-01,  0.319189E 00)
NM(17) = (  0.385810E-04, -0.513930E-02)
NC(18) = (  0.237768E-03, -0.291362E-02)
NK(18) = (  0.244609E-01, -0.319189E 00)
NM(18) = (  0.385810E-04,  0.513930E-02)
NC(19) = (  0.637295E-04,  0.449158E-02)
NK(19) = (  0.613783E-02,  0.428751E 00)
NM(19) = (  0.106105E-03,  0.892005E-02)
NC(20) = (  0.637295E-04, -0.449158E-02)
NK(20) = (  0.613783E-02, -0.428751E 00)
NM(20) = (  0.106105E-03, -0.892005E-02)

BASE INPUT EXCITATION( 3) : LRSM
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NK( 1) = ( -0.101999E-04, -0.278627E-04)
NM( 1) = ( -0.657008E-08, -0.473034E-07)
NC( 2) = ( -0.996101E-07,  0.271038E-06)
NK( 2) = ( -0.101999E-04,  0.278627E-04)
NM( 2) = ( -0.657008E-08,  0.473034E-07)
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NK( 3) = ( -0.224202E-04,  0.278716E-02)
NM( 3) = ( -0.132731E-05, -0.306452E-05)
NC( 4) = ( -0.147591E-06, -0.252586E-05)
NK( 4) = ( -0.224202E-04, -0.278716E-02)
NM( 4) = ( -0.132731E-05,  0.306452E-05)
NC( 5) = (  0.555346E-04, -0.373253E-02)
NK( 5) = (  0.552544E-02, -0.372454E 00)
NM( 5) = (  0.823106E-04, -0.758059E-04)
NC( 6) = (  0.555346E-04,  0.373253E-02)
NK( 6) = (  0.552544E-02,  0.372454E 00)
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NK( 7) = ( -0.547196E-03,  0.272502E-01)
NM( 7) = (  0.197977E-04, -0.437641E-04)
NC( 8) = ( -0.416569E-05, -0.271934E-03)
NK( 8) = ( -0.547196E-03, -0.272502E-01)
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NC( 9) = ( -0.764562E-05, -0.690035E-03)
NK( 9) = ( -0.751998E-03, -0.725669E-01)
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NC(10) = ( -0.764562E-05,  0.690035E-03)
NK(10) = ( -0.751998E-03,  0.725669E-01)
NM(10) = (  0.110785E-04,  0.305912E-04)
NC(11) = (  0.191523E-04, -0.649885E-03)
NK(11) = (  0.238161E-02, -0.735602E-01)
NM(11) = ( -0.487756E-04,  0.365877E-03)
NC(12) = (  0.191523E-04,  0.649885E-03)
NK(12) = (  0.238161E-02,  0.735602E-01)
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NC(13) = ( -0.627144E-04,  0.509125E-03)
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NK(13) = ( -0.609957E-02,  0.492452E-01)
NM(13) = ( -0.211658E-04,  0.951625E-04)
NC(14) = ( -0.627144E-04, -0.509125E-03)
NK(14) = ( -0.609957E-02, -0.492452E-01)
NM(14) = ( -0.211658E-04, -0.951625E-04)
NC(15) = ( -0.309620E-03,  0.216951E-01)
NK(15) = ( -0.306772E-01,  0.218410E 01)
NM(15) = ( -0.625110E-03,  0.200988E-01)
NC(16) = ( -0.309620E-03, -0.216951E-01)
NK(16) = ( -0.306772E-01, -0.218410E 01)
NM(16) = ( -0.625110E-03, -0.200988E-01)
NC(17) = (  0.237733E-03,  0.291056E-02)
NK(17) = (  0.230202E-01,  0.257744E 00)
NM(17) = (  0.519319E-03,  0.146989E-01)
NC(18) = (  0.237733E-03, -0.291056E-02)
NK(18) = (  0.230202E-01, -0.257744E 00)
NM(18) = (  0.519319E-03, -0.146989E-01)
NC(19) = (  0.637310E-04,  0.449308E-02)
NK(19) = (  0.639437E-02,  0.444284E 00)
NM(19) = (  0.432880E-04,  0.462219E-02)
NC(20) = (  0.637310E-04, -0.449308E-02)
NK(20) = (  0.639437E-02, -0.444284E 00)
NM(20) = (  0.432880E-04, -0.462219E-02)

BASE INPUT EXCITATION( 4) : LFSM
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NK( 1) = (  0.202571E-04,  0.639831E-04)
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NC( 2) = (  0.103944E-06, -0.287429E-06)
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NK( 3) = ( -0.998561E-05, -0.156164E-02)
NM( 3) = (  0.654858E-06, -0.909078E-05)
NC( 4) = (  0.124934E-06,  0.197287E-05)
NK( 4) = ( -0.998561E-05,  0.156164E-02)
NM( 4) = (  0.654858E-06,  0.909078E-05)
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NK( 5) = ( -0.558777E-02,  0.373815E 00)
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NK( 6) = ( -0.558777E-02, -0.373815E 00)
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NK( 7) = (  0.332102E-03, -0.274257E-01)
NM( 7) = (  0.351594E-05, -0.336281E-05)
NC( 8) = (  0.405717E-05,  0.272025E-03)
NK( 8) = (  0.332102E-03,  0.274257E-01)
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NC( 9) = (  0.754012E-05,  0.691886E-03)
NK( 9) = (  0.729210E-03,  0.759482E-01)
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NC(11) = ( -0.184491E-04,  0.638866E-03)
NK(11) = ( -0.155516E-02,  0.607938E-01)
NM(11) = ( -0.990986E-05, -0.108084E-05)
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NM(13) = (  0.391518E-04, -0.316591E-03)
NC(14) = (  0.623640E-04,  0.505295E-03)
NK(14) = (  0.583778E-02,  0.463860E-01)
NM(14) = (  0.391518E-04,  0.316591E-03)
NC(15) = (  0.309484E-03, -0.216962E-01)
NK(15) = (  0.307210E-01, -0.218312E 01)
NM(15) = (  0.417057E-03, -0.374060E-02)
NC(16) = (  0.309484E-03,  0.216962E-01)
NK(16) = (  0.307210E-01,  0.218312E 01)
NM(16) = (  0.417057E-03,  0.374060E-02)
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NK(17) = ( -0.235597E-01, -0.283094E 00)
NM(17) = ( -0.265660E-03, -0.676244E-02)
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NK(18) = ( -0.235597E-01,  0.283094E 00)
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NK(19) = ( -0.612415E-02, -0.421762E 00)
NM(19) = ( -0.117110E-03, -0.130536E-01)
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BASE( 2) - INPUT DISPL WVECT( 5) :      -0.498
BASE( 3) - INPUT DISPL WVECT( 8) :      -0.933
BASE( 4) - INPUT DISPL WVECT(11) :       0.431

Z-COMPONENT NUMERATORS(2*NFGC) :

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NM( 1) = ( -0.196990E-11, -0.669518E-11)
NC( 2) = ( -0.611006E-05,  0.736492E-04)
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NK( 3) = ( -0.224712E-04,  0.322810E-03)
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NM( 5) = (  0.129182E-13,  0.111555E-11)
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NM( 7) = ( -0.291420E-10, -0.875043E-09)
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NK( 8) = (  0.128590E-02,  0.779217E-01)
NM( 8) = ( -0.291420E-10,  0.875043E-09)
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NK( 9) = ( -0.155982E-02, -0.126565E 00)
NM( 9) = (  0.472362E-11,  0.108253E-08)
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NK(11) = ( -0.279045E-02, -0.445573E-01)
NM(11) = (  0.466256E-11,  0.117737E-09)
NC(12) = ( -0.277344E-04,  0.442562E-03)
NK(12) = ( -0.279045E-02,  0.445573E-01)
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NM(13) = ( -0.297150E-10,  0.238231E-09)
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NK(14) = (  0.767910E-02,  0.298562E 00)
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NK(15) = (  0.504229E-02,  0.535163E-01)
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NM(16) = ( -0.156948E-10,  0.155282E-09)
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NK(17) = ( -0.201371E-01, -0.407346E 00)
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NK(18) = ( -0.201371E-01,  0.407346E 00)
NM(18) = (  0.111617E-09,  0.334657E-08)
NC(19) = (  0.109978E-03,  0.240300E-01)
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NM( 1) = ( -0.226207E-05, -0.711317E-04)
NC( 2) = (  0.601770E-05, -0.783816E-04)
NK( 2) = (  0.547944E-03, -0.178315E-01)
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NC( 3) = (  0.191407E-06, -0.265080E-05)
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NM( 3) = (  0.580134E-06, -0.862115E-05)
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NK( 4) = ( -0.265421E-04, -0.852975E-03)
NM( 4) = (  0.580134E-06,  0.862115E-05)
NC( 5) = (  0.169262E-05, -0.182203E-04)
NK( 5) = (  0.170971E-03, -0.183858E-02)
NM( 5) = (  0.287966E-06, -0.557350E-06)
NC( 6) = (  0.169262E-05,  0.182203E-04)
NK( 6) = (  0.170971E-03,  0.183858E-02)
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NK( 7) = ( -0.672873E-03,  0.784218E-01)
NM( 7) = ( -0.749154E-04,  0.202052E-03)
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NK( 8) = ( -0.672873E-03, -0.784218E-01)
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NK(11) = (  0.255025E-02,  0.360418E-01)
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NK(12) = (  0.255025E-02, -0.360418E-01)
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NK(13) = ( -0.776415E-02,  0.282125E 00)
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NK(14) = ( -0.776415E-02, -0.282125E 00)
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NK(17) = (  0.197668E-01,  0.442421E 00)
NM(17) = (  0.279654E-03, -0.710445E-02)
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NK(18) = (  0.197668E-01, -0.442421E 00)
NM(18) = (  0.279654E-03,  0.710445E-02)
NC(19) = ( -0.109837E-03, -0.240486E-01)
NK(19) = ( -0.107760E-01, -0.229560E 01)
NM(19) = ( -0.108609E-03, -0.477581E-01)
NC(20) = ( -0.109837E-03,  0.240486E-01)
NK(20) = ( -0.107760E-01,  0.229560E 01)
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BASE INPUT EXCITATION( 3) : LRSM
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NK( 2) = (  0.604097E-03, -0.826112E-02)
NM( 2) = ( -0.185948E-05, -0.132022E-04)
NC( 3) = (  0.178719E-06, -0.242553E-05)
NK( 3) = (  0.622271E-04, -0.267849E-02)
NM( 3) = (  0.123100E-05,  0.296478E-05)
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NK( 4) = (  0.622271E-04,  0.267849E-02)
NM( 4) = (  0.123100E-05, -0.296478E-05)
NC( 5) = (  0.169316E-05, -0.182217E-04)
NK( 5) = (  0.168846E-03, -0.181839E-02)
NM( 5) = (  0.431184E-06, -0.339153E-06)
NC( 6) = (  0.169316E-05,  0.182217E-04)
NK( 6) = (  0.168846E-03,  0.181839E-02)
NM( 6) = (  0.431184E-06,  0.339153E-06)
NC( 7) = ( -0.123269E-04,  0.775489E-03)
NK( 7) = ( -0.160524E-02,  0.777109E-01)
NM( 7) = (  0.565307E-04, -0.124773E-03)
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NM( 9) = ( -0.202361E-04,  0.560473E-04)
NC(10) = (  0.152232E-04, -0.126378E-02)
NK(10) = (  0.150561E-02, -0.132905E 00)
NM(10) = ( -0.202361E-04, -0.560473E-04)
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NK(11) = (  0.296298E-02,  0.489208E-01)
NM(11) = (  0.974875E-05, -0.245604E-03)
NC(12) = (  0.274291E-04, -0.432085E-03)
NK(12) = (  0.296298E-02, -0.489208E-01)
NM(12) = (  0.974875E-05,  0.245604E-03)
NC(13) = ( -0.765329E-04,  0.293574E-02)
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NM(13) = ( -0.681172E-04,  0.553939E-03)
NC(14) = ( -0.765329E-04, -0.293574E-02)
NK(14) = ( -0.759370E-02, -0.283978E 00)
NM(14) = ( -0.681172E-04, -0.553939E-03)
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NM(15) = ( -0.381762E-04, -0.493939E-03)
NC(16) = ( -0.501485E-04,  0.532197E-03)
NK(16) = ( -0.506089E-02,  0.535778E-01)
NM(16) = ( -0.381762E-04,  0.493939E-03)
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NK(17) = (  0.204805E-01,  0.357398E 00)
NM(17) = (  0.708038E-04,  0.203471E-01)
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NK(18) = (  0.204805E-01, -0.357398E 00)
NM(18) = (  0.708038E-04, -0.203471E-01)
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NK(19) = ( -0.113493E-01, -0.237877E 01)
NM(19) = (  0.632724E-05, -0.247467E-01)
NC(20) = ( -0.109770E-03,  0.240566E-01)
NK(20) = ( -0.113493E-01,  0.237877E 01)
NM(20) = (  0.632724E-05,  0.247467E-01)

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NC( 2) = ( -0.588595E-05,  0.851345E-04)
NK( 2) = ( -0.536886E-03,  0.187296E-01)
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NC( 4) = ( -0.148872E-06, -0.189437E-05)
NK( 4) = ( -0.131945E-04, -0.150109E-02)
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NM( 5) = ( -0.156991E-06,  0.392713E-06)
NC( 6) = ( -0.169338E-05, -0.182284E-04)
NK( 6) = ( -0.169680E-03, -0.182498E-02)
NM( 6) = ( -0.156991E-06, -0.392713E-06)
NC( 7) = (  0.120173E-04, -0.775749E-03)
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NM( 7) = (  0.100322E-04, -0.958422E-05)
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NK(10) = ( -0.146985E-02,  0.139098E 00)
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NK(11) = ( -0.272265E-02, -0.404049E-01)
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NC(12) = ( -0.272151E-04,  0.424735E-03)
NK(12) = ( -0.272265E-02,  0.404049E-01)
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NK(13) = (  0.767893E-02, -0.267541E 00)
NM(13) = (  0.484686E-04, -0.182563E-02)
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NC(17) = ( -0.200749E-03, -0.402501E-02)
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NM(17) = ( -0.695486E-04, -0.936217E-02)
NC(18) = ( -0.200749E-03,  0.402501E-02)
NK(18) = ( -0.201101E-01,  0.392473E 00)
NM(18) = ( -0.695486E-04,  0.936217E-02)
NC(19) = (  0.109622E-03,  0.240752E-01)
NK(19) = (  0.110626E-01,  0.225818E 01)
NM(19) = ( -0.453736E-04,  0.698871E-01)
NC(20) = (  0.109622E-03, -0.240752E-01)
NK(20) = (  0.110626E-01, -0.225818E 01)
NM(20) = ( -0.453736E-04, -0.698871E-01)
BASE( 1) - INPUT DISPL WVECT( 3) :      0.000
BASE( 2) - INPUT DISPL WVECT( 6) :      0.794
BASE( 3) - INPUT DISPL WVECT( 9) :      0.002
BASE( 4) - INPUT DISPL WVECT(12) :     -0.797
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SECTION III: AGRICULTURAL TRACTOR CHASSIS SUSPENSION SYSTEM FOR IMPROVED RIDE COMFORT

INTRODUCTION

Off-road vehicles, such as agricultural wheel tractors or rubber-tired earthmoving scrapers, have been designed to develop a predetermined drawbar pull or tractive effort. With the increased speed, size, and power output of these vehicles, the operator was subjected to increased levels of shock and vibration from the terrain and the attached implements. To reduce operator fatigue from shock and vibration, ergonomic considerations have become important design criteria for vehicle development. Although improvements in operator stations and seat suspension have increased convenience and comfort, further development of total vehicles was needed to extend the productivity of man-machine combinations.

Cadou and Bowser (1978) stated that the greatest need for improvement was the reduction of shock vibration transmitted to the operator and in the relative motion between the operator and the control devices for the vehicle. The International Standards Organization (ISO) has proposed a standard to limit the whole-body human vibration exposure from the vehicle-terrain interaction. Designing an off-road vehicle under basic automotive suspension theory or ISO Standards became difficult because the vehicle must provide the base support for the implement to regulate the implement depth and attitude during operation and control the implement position during transport.

The work for this dissertation evolved from a high-speed moldboard plow project that was carried out at Iowa State University. Wainwright (1976) suggested that development work should be done to improve ride comfort of the tractor for high-speed farming operations. During the testing of the plow, the operators experienced objectionable levels of vibration while traversing the field with the tractor-plow system. Most of the vibration on the tractor-operator station was caused by terrain with profile amplitudes of less than 150 millimeters and frequency components of less than 20 Hertz (Hz).

The purpose of this dissertation is to simulate the design and development of alternate vehicle configurations of agricultural tractors to achieve enhanced ride comfort at increased speeds. An exploratory concept of a chassis suspension system for an agricultural wheel tractor is described, which, it is hoped, will lead to further alternate concept generation and experimentation by the research community. The ride performance of the chassis suspension is evaluated by a computer simulation procedure, and the results are described in this dissertation.

DESIGN CONSIDERATIONS FOR OFF-ROAD VEHICLE RIDE COMFORT

To increase the travel speed of an agricultural tractor for improved productivity and to reduce operator exposure to vibration, the dynamic behavior of the vehicle must be modified to improve the operator ride comfort. Up to now, the vehicle travel speed has been limited by tolerance of the operator to a rough ride rather than the capability of the vehicle. The vehicle ride quality depends upon the terrain surface roughness, the travel speed, and the vehicle geometry and suspension characteristics.

The most objectionable levels of acceleration caused by the terrain and implement loads as experienced by the operator are in the frequency range of two to ten Hz (Soehne, 1965). To limit the resonance and discomfort to the human body, the level of vibration on the operator should be in the frequency range of one to two Hz. For the vehicle to have this ride quality characteristic, the following design factors, as outlined by Radforth (1978), must be considered and evaluated: (1) cab and seat location; (2) large tires; (3) arm suspension system; (4) springs and dampers; (5) wheel travel; and (6) unsprung to sprung mass ratio.

The following three basic suspension systems could possibly be used to attenuate the vibration modes of off-road vehicles: (1) seat; (2) cab; (3) chassis. In addition, these suspension systems may use passive, active, or semi-active attenuation elements. All three suspension types have advantages and disadvantages as vibration attenuators, and each has its corresponding performance characteristics.

For the agricultural equipment industry to market a tractor with improved operator ride comfort, several generations of engineering prototypes are required to evaluate the design parameters in both the laboratory and field. To minimize the development time from concept to final product, analytical computer-aided design techniques may be used for analyzing and evaluating the changes in design parameters in terms of the vibration frequency content and to provide repeatability of any number of test conditions.

DEVELOPMENT OF THE SUSPENSION SYSTEM

Within the next 10 years, farming operations may be based on tractor working speeds of 13 to 16 km/hr (Reichenberger, 1980). In all probability, an alternative suspension means, besides a seat suspension, will be required to improve the operator ride comfort. On the basis of the review of ride comfort means, a chassis isolation system was selected for further investigation as a means to attenuate ground induced motion between the axles and the chassis. Several desirable features of the concept were identified as:

1. The suspension will attenuate the bounce and pitch motion of the tractor.
2. The suspension will allow the chassis, cab, and operator platform to be included as the sprung mass. The sprung to unsprung mass ratio will be greater than the ratio for a cab suspension.
3. The suspension ought not to impart a sense of "insecurity" to the operator as might a cab suspension (Stayner, 1974). This phenomenon arises because some operators do not like the large relative movement between the cab and tractor at certain times.
4. With this suspension, the seat suspension will not need to provide a large relative movement, in excess of 130mm, between the seat and the operator controls. Most current seat suspension need a large relative movement because the vertical natural frequency of the tractor is nearly

coincident with the natural frequency of the seat suspension (Stayner et al., 1975).

5. The suspension will reduce the loads transmitted directly to the chassis structure, cab, and operator platform.

To gain an understanding of the concept, the TEREX TS-24 tractor-scraper unit's leading arm suspension was analyzed. This particular suspension system has been successful in improving the operator ride comfort (Cadou and Bowser, 1978).

All necessary data on the geometry of the vehicle and its suspension plus the weight distribution for the fully loaded vehicle have been reported in the patent disclosure (Mason, 1976).

The following four load conditions were analyzed to evaluate the suspension concept:

1. Front axle weight at the vehicle's static equilibrium position.
2. Shock load equal to two times the static front axle weight, imposed at the wheel by a square bump.
3. Load equal to the static front axle weight plus the tractive effort, imposed at the wheel by a square bump.
4. Load equal to the static front axle weight plus the braking effort, imposed at the wheel by a square bump.

The vector polygon analysis technique from engineering mechanics was used to compute the load transmitted through the wheels to the suspension ride cylinders and the arm pivot pins. The results of this analysis are presented in Figure 1.

A leading arm suspension configuration, similar to the TEREX except for a different orientation of the ride cylinders, was evaluated. The results of this analysis are presented in Figure 2. Next, the TEREX leading arm suspension was transformed into a trailing arm suspension and evaluated. The results of this analysis are presented in Figure 3. Finally, a leading arm suspension, similar to the TEREX but with a different vertical location of the arm pivot joint was analyzed. These results are shown in Figure 4.

Comparison of the four arm suspension configurations has shown that the TEREX suspension is somewhat better than the other leading arm suspensions and the trailing arm suspension. One may observe from Figures 1 and 4, that the arm pivot location determined the load that was transmitted to the chassis frame.

As a general rule, most off-road vehicles use a leading arm suspension for the front axle and a trailing arm suspension for the rear axle(s) as shown in Figure 5. The one primary design criterion for an arm suspension is that the arm pivot pin must be located below the axle centerline and as near as possible to the groundline. This pivot location has a direct effect on the stiffness of the frame and the size of the springing and damping mechanism. Also, the vibration attenuation mechanism is oriented in a direction to maximize the damping of the induced motion. The orientation may be altered, however, because of structural strength considerations and available attachment locations on the axle and the chassis frame.

The following objectives were specified to evaluate the suspended tractor design:

1. The ride comfort of the suspended tractor will meet the exposure and fatigue-decreased proficiency criteria as specified in the ISO Standard 2631 on whole-body vibration and the SAE Standard J1013 for testing agricultural seat suspensions.
2. The suspension will use only passive springing and damping elements.
3. The suspension will permit axle displacements equal to ± 100 mm in relation to the static equilibrium position of the tractor.
4. The suspension will permit the conventional three-point hitch mechanism to control the ground-engaging implement position and to regulate the implement depth and attitude during operation at working speeds from 13 to 16 km/hr.
5. The suspension will be used on a two wheel drive tractor.

The next step in the development process was to configure a suspended two-wheel drive agricultural tractor. The proposed tractor configuration is shown in Figure 6. A trailing arm concept was selected to isolate the chassis from the rear axle because it provided the most favorable conditions for driveline angularity and foreshortening and for attachment to the chassis frame. In addition, no modifications would be needed for the three-point implement hitch mechanism to operate in the draft and position control modes. The trailing arm suspension for the rear axle is shown in Figure 7.

The front axle suspension was designed to incorporate the features of both the leading and trailing arm concepts. Referring to Figure 5, one may observe that the leading arm concept allows the wheel to spring upward and rearward as it climbs over a bump; while the trailing arm concept allows the wheel to swing upward and forward as it climbs out of a depression. In Figure 8, the two arm concepts are superimposed to prescribe the motion of a wheel with a total displacment of 200 mm. The trailing arm suspension pivots around the point M to prescribe the wheel movement as the circular arc T. The leading arm suspension pivots around the point P to prescribe the wheel movement as the circular arc L. A line may then be passed through point E_1 and be tangent to the circular arcs, T and L, at the points E_3 and E_2, respectively. This straight line segment, E_3-E_1-E_2, prescribes the movement of the tractor front wheel.

To prescribe the straight line motion of the front wheels, a cam mechanism or a four-bar linkage can be used for the suspension. For initial consideration, a four-bar linkage mechanism was chosen as shown in Figure 9. This linkage must allow a point on the coupler link AB to pass through three prescribed positions E_1, E_2, and E_3 while the input link MA rotates through the angles θ_{12} and θ_{23} in a counterclockwise direction. The synthesis technique to coordinate the three point-positions of the coupler link and three positions of the input link is described by Hain (1967) and Soni (1974).

With this synthesis technique, many trial graphical solutions were required to arrive at an optimum suspension linkage design. The final suspension linkage considered for ride analysis is shown in Figure 9. The input link rotates at a constant speed through a total angular displacement of 15 degrees where angle A_3MA_1 equals 7.5 degrees and angle A_1MA_2 equals 7.5 degrees. These angles and the fixed reference centers M and Q on the tractor chassis were arrived at by hand optimization with the trial graphical configurations. Points E_1, E_2, and E_3 prescribe the position of the front wheel on the coupler link, the front axle. Linkage MA_1B_1Q represents the suspension position when the tractor was at its equilibrium position. Linkage MA_2B_2Q represented the suspension position when the front wheel was on a 100 mm bump and the chassis was level. Linkage MA_3B_3Q represented the suspension position when the front wheel was in a 100 mm depression and the chassis was level. Points C_1, C_2, and C_3 are defined as the moving instantaneous centers for the links MA and QB at the three linkage positions, respectively. An instantaneous center is defined as a point on one link about which some other link is rotating or a point common to two links having the same linear velocity magnitude and direction (Soni, 1974). This, in essence, allows the points E_1, E_2, and E_3 to be rotating about points C_1, C_2, and C_3, respectively. This, in turn, allows the first suspension to behave like a leading arm suspension with variable geometry and with the pivot pin below the center of the wheel. The leading arm suspension for the front axle is shown in Figure 7.

The agricultural wheel tractor with the chassis suspension system is shown in Figure 7. The suspended tractor is shown in Figure 10b at its static equilibrium position. Figure 10a shows the tractor encountering a depression with the front axle and a bump with the rear axle. Figure 10c shows the tractor encountering a bump with the front axle and a depression with the rear axle. In each case, the tractor chassis is held level so that the orientation of the front axle suspensions may be illustrated. In addition, the points C_1, C_2, and C_3 correspond to the instantaneous centers in Figure 9.

Rubber mount, or mechanical coil or torsion springs and hydraulic shock absorbers, or hydro-pneumatic spring and damping cylinders can be used as part of the front and rear suspension to attenuate the ground-induced motion. The rear suspension elements are mounted between the axle and the chassis frame so that the bounce and pitch movements experienced by the tractor will cause the springs and shock absorbers to telescope. Also, the springs and shock absorbers on the front axle suspension are mounted so that their line of action will be parallel to the line of movement for the wheels. The springs and shock absorbers will telescope when the tractor experiences bounce and pitch movements. Finally, rubber jounce bumpers can be located on the front and rear axle suspensions to limit the movement between the chassis frame and the suspension arms.

DESIGN EVALUATION BY COMPUTER SIMULATION.

At the design concept stage, analytical computer simulation programs are especially useful to evaluate agricultural tractor ride characteristics. This design approach, as shown in Figure 11, requires the quantitative description of the following ride analysis components: (1) description of a truly respresentative track or terrain to excite the vehicle; (2) description of the vehicle as a system of components; (3) calculation of the vehicle dynamic response; and (4) conversion of the vibrational characteristics on the operator to human-stress parameters.

Terrain inputs and excitations

For a valid physical or subjective assessment of ride comfort, the tractor must travel over a surface that can be accurately specified and easily duplicated at numerous test sites. The disadvantage in using actual field surfaces is that they change with varying soil and climatic conditions and repeated travel over them. Therefore, a nonyielding test surface is more desirable provided that it can generate representative vibrational excitations on the tractor as would the actual farm surface.

To provide a standard excitation source for evaluating seat suspensions, Matthews (1964, 1966) sought information on the intensity and characteristics of the ride vibration produced during normal farm tractor usage. From this investigation, two test tracks have been developed to encompass the range of agricultural terrain profiles and the ride vibration frequency spectra. The rough track (35 meters long) simulates a severely uneven, plowed field, while the smooth track (100 meters long) simulates a gently undulating, unpaved farm road. These two test tracks with their station-elevation coordinates have been included as part of the proposed ISO Standard 5007 for evaluating farm tractor seat suspensions and ride comfort, either experimentally or analytically.

The ISO 5007 smooth track was used as the excitation source for the tractor and tractor-implement system models because it is used extensively by the American and European tractor manufacturers and research communities. The basic method of representing the track is shown in Figure 12. The track is specified relative to the X_G, Y_G coordinate system by specifying the stations at which the track elevations are given. The station-elevation coordinates for the right track are shown in Figure 13. A set of cubic polynomial spline functions was fitted to connect the elevation of station i with the elevations of stations i+1, i+2, i+3; for example, stations 16, 17, 18, and 19. Provision is made for specifying track elevation at discrete time increments for each wheel, for example station 1 for the rear wheel. ISO Standard 5007 requires that the tractor travel over this track at a specified speed of 12.5 ± 0.5 km/hr. The time and frequency components of the right track for this specified speed are shown in Figure 14 and 15, respectively.

Simulation technique to model vehicle response

To evaluate ride response, simulation techniques must use a mathematical representation of the dynamic behavior of the tractor, tires, cab and seat suspension. The typical model will consist of a series of mathematical expressions that describe the response of a tractor traversing a specified terrain.

One analytical technique available to model the dynamic behavior of a tractor-implement system is with the generalized mechanical systems simulation programs. These programs have the capability to automatically formulate the equations of motion, constraints, and input functions and to simulate the dynamic response of any two- or three-dimensional, discrete system of constrained, rigid bodies.

One available simulation system is the Integrated Mechanisms Program (IMP), which is capable of analyzing both two- and three-dimensional, motion-constrained, rigid body, closed-loop kinematic chain mechanisms (Sheth and Uicker, 1972). To formulate the desired system, the program uses a problem-oriented language that consists of definition, data, request, control, delete, and graphics statements (Uicker, 1974). The user defines the topological connectivity of the system with the necessary numerical data to describe its relative orientation and location. The program then performs a topological analysis to recognize the number of links, the number and order of the kinematic looops, and other characteristics solely determined by the connectivity of the mechanism. The independent kinematic loops are selected so that the number of joints in each loop is minimum.

The program has the following simulation modes:

1. Kinematic Mode
2. Static Mode
3. Dynamic Mode

Upon request, any or all of the following entities may be determined by the program:

1. The positions, velocities, and accelerations of the links, joints, and points of interest on the links
2. The static and dynamic constraint forces, as well as the cyclic variations in the driving source characteristics;
3. The forces in the springs and dampers;
4. The graphical display of the system at specified time intervals;
5. The natural frequency and damping ratios;
6. The "small" oscillation transfer functions of the system motions resulting from the sinusoidally-varying motion, force, and torque inputs, and the vibrational mode shapes of the system. These two features were implemented as part of the dissertation research project for computer-aided vehicle dynamics studies of agricultural tractor-implement systems. The general form of the transfer function relating the system response to the input excitation in the frequency-domain has the following form

$$\frac{X(s)}{F(s)} = \sum_{i=1}^{m} \frac{N_i}{s-D_i} \qquad [1]$$

where

$X(s)$ = the response of the system variable in the frequency-domain;

$F(s)$ = the system force input excitation;

$N_i(s)$ = the complex modal numerator;

$D_i(s)$ = the complex system eigenvalues;

$s = j\omega$;

j = the symbol for complex notation.

The value of the subscript variable, i, ranged from one to m which was equal to two times the number of system free generalized coordinates (FGC).

The mathematical computer models

The IMP system was used to evaluate the effect of operator cab position and suspension system parameters on tractor ride characteristics. Nine tractor configurations were formulated according to the IMP modelling procedure. Each model had one of three operator cab positions and one of three suspension systems. The three operator cabs were positioned on the tractor:

1. at 455 mm ahead of the rear wheel centerline;
2. at 1270 mm ahead of the rear wheel centerline or at the midpoint of the wheelbase;
3. at 455 mm behind the front wheel centerline.

The three suspension systems were:

1. unsprung front and rear axles;

2. unsprung rear axle and sprung front axle;

3. sprung front and rear axles.

These nine tractor model configurations are shown in Figure 16. In addition, these same nine tractor model configurations were formulated with a semi-mounted moldboard plow, a single-axle transport trailer, and a tandem-axle transport trailer.

The fore-aft and lateral motion, as well as the vertical motion, are important in the evaluation of operator ride comfort. Thus, three-dimensional computer models should be used for this evaluation. Due to computing cost restrictions, two-dimensional models were developed to evaluate the effects of design changes only on the fore-aft and vertical operator motion.

Three of the nine IMP tractor-plow models are shown in Figures 17, 18, and 19 while two of the 18 IMP tractor-trailer models are shown in Figure 20. One can envision the remaining six tractor-plow models, the remaining 16 tractor-trailer models, and the nine tractor models based on the configurations in Figure 15. To constrain the movement between the links, the following two joint types were used:

1. Prismatic joint to allow translation;

2. Revolute joint to allow rotation.

Thus, the formulated mathematical models have the following system degrees-of-freedom:

1. Three degrees-of-freedom (two translations and one rotation) for the chassis;

2. Three degrees-of-freedom (two translations and one rotation)

for the cab;

3. A rotational degree-of-freedom for the moldboard plow;
4. A rotational degree-of-freedom for the single-axle or tandem-axle trailer;
5. A specified rotational degree-of-freedom for the three-point implement hitch to provide implement position control;
6. A rotational degree-of-freedom for the front axle suspension system linkage as shown in Figure 21;
7. A rotational degree-of-freedom for the rear axle suspension system linkage as shown in Figure 22;
8. Two specified translational degrees-of-freedom to input the terrain excitation to the front and rear tractor wheels;
9. One or two specified translational degrees-of-freedom to input the terrain excitation to the transport trailer wheel(s).

As indicated by the degrees-of-freedom, the tractor is considered to consist of two components: the cab and the chassis. The seat and operator are considered to be part of the cab. The chassis does not propel itself forward, therefore, an additional degree-of-freedom need not be added into the model.

The tractor, plow, and trailer tires were represented as a set of vertical prismatic joints and fore-aft prismatic joints. The orientation of these joints allowed the tractor chassis to have bounce and fore-aft motions. Each of these joints allowed a translational spring and damper to act within it. The revolute joints allowed the tractor chassis, the plow, and the trailers to have a rotational or pitching motion.

A set of specified vertical prismatic joints were used to input the time-varying track excitation to the tractor and trailer wheels. At a constant velocity, the ISO 5007 right smooth track was passed under each model, and the joints were displaced at an amplitude equal to the track elevation at that particular instant. The track input was phased according to each model's wheel spacing. No input excitation was used at the plow tail wheel.

The cab isomount pads are represented as a set of vertical prismatic joints and fore-aft prismatic joints. The orientation of these joints allowed the cab to have bounce and fore-aft motions. Each prismatic joint allowed a translational spring and damper to act within it. The fore-aft and vertical springs and dampers had equal spring and damping rates. The revolute joints allowed the tractor cab to have a rotational or pitching motion.

Vehicle system parameter determination

The mathematical tractor and tractor-implement models implemented for analysis require data, such as geometric dimensions, inertial properties, etc., to describe the tractor chassis and cab, the implement hitch system, the tires, the cab iosmount pads, the front and rear suspension systems, the plow, and the trailers.

The conventional tractor model represented a John Deere 4020 agricultural wheel tractor. The vehicle geometric data and the measured inertial data were supplied by Deere and Company engineering personnel.

The geometric and inertial data for the midchassis and forward cab position tractor models were estimated from the geometric and inertial data for the conventional tractor model.

The cab model represented the mathematical cab model used by Roley (1975) in his simulation work, and the cab geometric and inertial data were used directly. The cab isomount pad model represented the Barry Corporation 22004-4 Series Neoprene isomount pads, which are used to suppress transmitted noise to the operator's cab on agricultural equipment. The spring and damping rates were supplied by the Barry Wright Corporation. The vertical and longitudinal spring rates were assumed to be equal in magnitude while the vertical and longitudinal damping rates were assumed to be 0.05 times the vertical critical damping rate.

The front and rear tractor models represented Goodyear 7.5-16 (F-1) and 18.4-34 (R-1) agricultural tires, respectively. With the respective load-displacement curves, the vertical tire spring rates were determined from the static equilibrium vehicle axle weights. The rear tractor tire longitudinal spring rate is assumed to be 1.1 times greater than the vertical spring rate with no drawbar implement load and 1.5 times greater than the vertical spring rate with a drawbar implement load. These longitudinal spring rates were estimated from passenger car tire data. The front tractor tire and the plow tire were assumed to have zero longitudinal spring stiffness because they are considered free-rolling (Crolla, 1980a). The tractor rear tire vertical and longitudinal damping rates were assumed to be 0.07 times the vertical critical damping rate. The spring and damping rates for the plow tail wheel were assumed to be equal rates for the front tractor tire.

The plow model represented an Allis-Chalmers Series 2000 semi-mounted moldboard plow with three 454-mm bottoms. The geometric and inertial data for the plow were supplied by the Allis-Chalmers Corporation engineering personnel. For this simulation study, the plow was assumed to be operated at 12.5 km/h in a heavy clay soil at a cutting depth of 203 mm. The plow draft forces were applied at the centers of reaction of the moldboards, which are taken to be 27 percent of the way from the tip to the rear of the moldboard (Berenji, 1973). The draft force applied at each moldboard was 6.4 kilonewtons. The plow suction forces were applied to the center of reaction of the moldboard, which are taken to be 25 percent of the way up from the bottom of the moldboard. The suction force applied at each moldboard was 45 percent of the draft force. The plow draft and suction forces were held constant during the simulated time period.

The trailer models represented tipping trailers that are marketed by several German manufacturers. The geometric and inertial data were estimated from a German technical article (Kutzbach, 1978) and two product sales brochures for single-axle and tandem-axle trailers (Mengele und Sohne Maschinenfabrik und Eisengiesserei Gmbh, 1980; Gebrueder Welger, 1978). The single-axle trailer weighed 1200 kg and carried a payload of 5300 kg. The tandem-axle trailer weighed 1500 kg and carried a payload of 6500 kg. For this simulation study, the two trailers were assumed to be pulled at 12.5 km/h and 25 km/h over a hard soil surface. With the load-displacement curves for a Goodyear 12.5L-15 farm implement tire, the vertical spring rates were determined from the static equilibrium trailer axle weights. The vertical damping rate was assumed to be 0.07 times the vertical critical damping rate. The tires have zero longitudinal stiffness and damping because they are free-rolling.

The rolling resistances for the tractor wheels, the plow tail wheel, and the trailers' wheels were calculated in accordance to ASAE Standard D230.2 "Agricultural Machinery Management Data". For the tractor-plow models, the dimensionless ratio C_n for firm soils was assumed, while for the tractor-trailer models, the dimensionless ratio C_n for hard soils was assumed.

Suspension system parameter determination

The stiffness and damping rates for the tractor suspension were determined from automotive suspension design theory equations (Cole, 1972; Mola, 1974). The computed rates were used as only a first step in the development process. Ultimately, field and laboratory tests would be necessary to attain the optimum rates for ride comfort.

As a starting point in ride design, the tractor ride frequency was assumed to equal one Hz. The ride frequency, f_{ride}, was affected by the ride spring rate and the magnitude of the static-sprung mass acting on it. Mathematically, this may be expressed by the following relationship

$$f_{ride} = (K'/2\pi M) \qquad [2]$$

where

K' = the ride spring rate;

M = the static sprung mass of the tractor.

In turn, the ride spring rate, K', may be expressed mathematically as the following expression

$$K' = (k_S \cdot k_T)/(k_S + k_T) \qquad [3]$$

where

K_S = the suspension spring rate;

K_T = the tire spring rate.

Based on automotive road tests, the rear suspension ride rate should be approximately 20 percent greater than the front suspension ride rate.

Shock absorbers are required to dampen the ground-induced motion. Optimum damping had been achieved when the shock absorber provided an equal amount of damping for the motions of the sprung mass and motions of the unsprung mass. Based on the motion of the sprung mass, the critical damping, C_C, may be expressed by the following relationship

$$C_C = \sqrt{4\ K'\ M} \qquad [4]$$

where

K' = the ride spring rate;

M = the sprung mass

Based on the motion of the unsprung mass, critical damping may be expressed as the following relationship

$$C_C = \sqrt{4\ k_{HOP}\ m} \qquad [5]$$

where

$k_{HOP} = k_S + k_T$, the rate of the suspension and tire springs in parallel;

m = the unsprung mass.

For good ride comfort, these two critical damping rates should be as near to equal as possible. One should observe that this criterion will be difficult to meet for the rear suspension due to the large unsprung mass of the tractor rear axle and wheels. Any changes in the suspension characteristics, m, M, k_S, k_T, will cause motions of either the sprung or unsprung masses to be overdamped or underdamped.

With this theory, the suspension design parameters were calculated for the three tractor configurations with the front and rear axle suspension systems. The three tractor configurations with only a front axle suspension system have the same front suspension parameters as the fully suspended tractor configurations. No implement drawbar loads were considered as part of the unsprung rear axle mass.

A wide variety of suspension parameters may be evaluated to arrive at an optimum set of suspension parameters to improve the operator ride comfort. Because of computer budget constraints, only five suspension parameter changes were considered:

1. The rear suspension ride rate was 20 percent greater than the front suspension ride rate. The front and rear suspension damping rates were one-fourth of the sprung mass critical

damping rates, respectively. These parameters were used as a starting point for suspension design (Cole, 1972; Mola, 1974).

2. The rear suspension ride rate was 20 percent greater than the front suspension ride rate. The front and rear suspension damping rates were one-eighth of the sprung mass critical damping rates, respectively.
3. The rear suspension ride rate was 40 percent greater than the front suspension ride rate. The front and rear suspension damping rates were one-fourth of the sprung mass critical damping rates, respectively.
4. The rear suspension rate was determined with tire stiffness rate for the implement load acting on the tractor. The rear suspension ride rate was 20 percent greater than the front suspension ride rate. The front and rear suspension damping rates were one-fourth of the sprung mass critical damping rates, respectively.
5. The rear suspension ride rate was 20 percent greater than the front suspension ride rate. The front suspension damping rate was one-fourth of the sprung mass critical damping rate while the rear suspension damping rate was one-fourth of the unsprung mass critical damping rate.

A designation code had been developed to describe the model configuration according to its cab position, suspension type, and variation of the suspension parameters. This designation code is presented in Table 1. For example, the code, TCBD, represents a tractor with a conventional cab position, suspended front and rear axles, and the following suspension parameters. The rear suspension ride rate is 20 percent greater than the front suspension ride rate, and the front and rear suspension damping rate is one-eighth of the sprung mass critical damping rate.

The suspension parameters for the three suspended tractor configurations were calculated from the inertial data for the John Deere 4020 tractor and its components. The suspension parameters for the conventional cab position tractor configurations are presented in Table 2. The suspension parameters for the mid-chassis cab position tractor configurations are presented in Table 3. The suspension parameters for the forward cab position tractor configurations are presented in Table 4.

Design evaluation

The IMP program was used to compute the system transfer functions as a first step in evaluating the ride comfort of the tractor system models. A set of transfer functions for each design variable were computed by IMP for each specified input joint of the model and in each global direction. Sixty-six models were evaluated: three unsprung tractor models; three unsprung tractor-plow models; six suspended front axle tractor-plow models; 15 fully suspended tractor models; 15 fully suspended tractor-plow models; six suspended front axle tractor-trailer models; and six fully suspended tractor-trailer models. The three unsprung tractor models, three unsprung tractor-plow models, and six unsprung tractor-trailer models were used as a base for evaluating the ride comfort of the 21 suspended tractor models, 21 suspended tractor-plow models, and 12 suspended tractor-trailer models.

All system models were assumed to be linear so that the IMP transfer function - frequency-domain technique could be used to reduce the computational numerical analysis costs. The system transfer functions represent the dynamic characteristics of each tractor, tractor-plow, and tractor-trailer model about its static equilibrium position.

The three tractor-plow system models that are shown in Figures 17, 18, and 19, and the two tractor-trailer system models that are shown in Figure 20, were used to illustrate the IMP simulation process. The Appendix contains the actual computer output listings to formulate and initiate the analysis, and the analysis results for the conventional tractor-plow model.

The IMP simulation process may further be elaborated by considering the tractor-plow model shown in Figure 16 and its statement listing in the Appendix. The first statement gives a representative name to the system, while the second statement specifies the ground link as TRACK. The next group of statements defines the tractor chassis topology, provides the numerical data for the link coordinate frames, defines a point of interest on the chassis and assigns its coordinates, and assigns the masses and inertias to the chassis. Similar sets of statements describe the rear tractor tires, the front tractor tires, the tractor cab, the implement hitch, and the plow. The next statement requests the static equilibrium position of the system, while the remaining statements request the positions of the joint motions and points and the constraint force at the equilibrium position, the natural and damped frequencies of the system, and the transfer functions of the system with motion input excitations. The last statement, EXECUTE/HOLD, caused the requested analysis to be performed.

The first part of the results analysis output was the title, providing the general information about the system and the type of analysis performed. The system was in the static mode because the statement, FIND/EQULIB, requested the static equilibrium position.

The next information was on the degrees-of-freedom of the system. The tractor-plow system had 10 degrees-of-freedom. With the statement DATA/POSITN, three position inputs to the system were specified for the joints, FSMP, RSMP, and APIN. Thus, there were three user specified generalized coordinates (SGCs). The remaining seven degrees-of-freedom were the IMP system selected generalized coordinates (FGCs). The FGCs were the constraint variables for the VRTR, RJNT, VPH, VFTR, RIPD, CJNT, and FIPD joints.

The next statement, PRINT/FREQ, requested the program to calculate the natural and damped frequencies, the damping ratios, and the decay rates of the system. The damping factors corresponded to the real parts of the eigenvalues and the damped frequencies corresponded to the imaginary parts of the eigenvalues. Since there were seven FGCs, there were seven natural frequency components. One may observe which natural frequency was associated with the corresponding FGC.

The next statement, PRINT/POSITN (ALL), requested the constraint variable values and the points of interest at the static equilibrium position.

The next statement, PRINT/DYNAM/MOTION/ACCEL/SEAT/, requested the linearized transfer of the system about its static equilibrium position. The system was excited by the SGCs. The acceleration components for the transfer function were computed for the point, SEAT, which was located on the link, CAB. One may observe that the program computed three sets of transfer functions for the point, SEAT; X-components, Y-components, and Z-components. The transfer function components for the specified freedom, APIN, will not be used to compute the acceleration at the seat point, SEAT, of the tractor-plow system.

The entire flow of analysis with this tractor-plow system has now been completed.

The transfer functions represent the dynamic characteristics of the tractor, the tractor-plow, and the tractor-trailer systems. In turn, the transfer functions were combined with the input excitations to express the dynamic response of the system. This approach is illustrated in Figure 23.

For the IMP simulation process, a post-processor program was developed to calculate the dynamic response of the system. The flow diagram for this program is shown in Figure 24. this program was interfaced with the IMP simulation program and will be incorporated as a segment of the IMP simulation program at a later date.

In the first phase of the program, the time domain input data was digitized so that it may be transformed to the frequency domain. The force and/or motion input data can be either generated from mathematical expressions or measured from field or laboratory experiments. The input motion may be displacement, velocity, and/or acceleration.

For the tractor ride simulation analysis, the right ISO 5007 smooth track profile was digitized as the input excitation data to the front and rear tractor wheels, the wheel for the single-axle trailer, and the wheels for the tandem-axle trailer. Provision was made for calculating the track elevation at discrete time intervals and phasing the track under each wheel at a particular time. The time domain input data was then transformed to frequency domain.

For the tractor ride simulation analysis, the ISO 5007 smooth track right profile was digitized at the discrete time interval Δt of 0.0125 sec. With this sampling rate, the highest frequency that can be defined was 40 Hz. The highest frequency was referred to as the Nyquist frequency f_c that, for this case, was equal to $1/(2\Delta t)$, actually $(1.0/2(0.0125) = 40$ Hz) (Ramirez, 1975; Bendat and Piersol, 1971).

This Nyquist frequency value was selected for two reasons:

1. The Fast Fourier Transform (FFT) was used to transform the time series data to a frequency domain series. Input for the FFT subroutine must be an integer power, PWR, of two number of data points, N

   $$N = 2^{PWR} \qquad [6]$$

   The variable, PWR, was selected to be 11, hence, 2048 points were required. With this number of data points and the 0.0125 sec time interval, 85 meters (m) could be traversed during a program run. If PWR is equal to 12, 4096 data points would be required. However, 4096 data points would encompass the track profile but were not used because of the higher computer core useage charges.

2. All rigid body modes of the tractor system models were excited.

Next, the time series of the track profile was detrended to delete any linear trends in the data. Detrending was accomplished by the expression

$$X_K = X_{ok} - X \qquad [7]$$

where

X_k = the detrended track profile elevation;

X_{ok} = the original track profile elevation;

X = the mean of the track profile elevations

The time series data was window tapered to avoid the problem of leakage when using the FFT. A cosine taper data window was used to shape the input data as shown in Figure 25 and as follows

$$X_k = X_{k(old)} \cdot W_k \qquad [8]$$

where

$$W_k = 0.5\ [1 - \cos(\pi(k - 1)/2N)] \qquad k = 1, 2, \ldots, R$$

$$W_k = 1 \qquad k = R+1, \ldots, N-R$$

$$W_k = 0.5\ [1 - \cos(\pi(k - 1)/2N)] \qquad k = N-R+1, \ldots, N$$

$$R = N/10$$

Once the input data has been detrended and window tapered, the real time series, TPY, is combined into a complete series, Y. Then, the Discrete Fourier Transform of Y was computed by the well-known FFT algorithm developed by Cooley and Tukey (Ramirez, 1975; Bendat and Piersol, 1971). This algorithm gained its speed by using the repeating properties of the Discrete Fourier Transform, DFT,

$$Y_m = \frac{1}{N} \sum_{k=0}^{N-1} TPY_k\ e^{-j2\Pi mk/N} \qquad [9]$$

The values of the subsript variables, k and m, ranged from zero to N-1 number of data points.

Now, the program calculated the dynamic response of the system from the input and the transfer functions. Using Equation [1], the acceleration response for the SEAT point, X(s), may be expressed as

$$X_j(s) = \left[\sum_{i=1}^{m} \left(\frac{\omega^2 NM_i + \omega\ NC_i + NK_i}{s-D_i} \right) + \sum_{k=1}^{n} B_{jk} \right] s^2 Y_k(s) \qquad [10]$$

where

NM_i = the complex modal mass numerators for the system;

NC_i = the complex modal damping numerators for the system;

NK_i = the complex modal stiffness numerators for the system;

D_i = the complex conjugate eigenvalues or denominators for the system;

$s^2Y(s)$ = the acceleration frequency components of the track profile input at each wheel (the SGC variables, FSMP and RSMP). The acceleration input excitations were derived from the track displacement input by multiplying the displacement input by s^2.

B_{jk} = the base motion inputs for the SGC variables;

$s^2 = -\omega^2$;

$s = j\omega$;

ω = the discrete frequency components, rad/sec.

The acceleration input excitations were derived from the track displacement input by multiplying the displacement input by s^2. The value of the subcript variable, i, ranged from one to two times the N number of system FGCs. The value of the subscript variable, j, ranged from one to three, and corresponded to the accelerations in the X, Y, and Z global coordinate directions. The value of the subscript variable, k, ranged from one to the n number of system SGCs.

The acceleration response for the variable SEAT for a conventional tractor model is shown in Figure 26. The numerical results were printed out by the program but are not presented.

The program also has the capability to calculate the displacement and velocity response of the system, but this capability was not used in the ride analysis.

At this point, the user may terminate the execution of the program or request the program to calculate one or all of the following analysis options:

1. The stationary Power Spectral Density (PSD) of the design variable(s). The PSD is calculated by using the following equation

$$PSD_i\ (\omega_k) = \frac{0.2t}{0.875\ N} \left| X_i^*(\omega_k) \cdot X_i(\omega_k) \right| \qquad [11]$$

where

$PSD_i(\omega_k)$ = the Power Spectral Density of the ith mode at the kth discrete frequency component;

t = the time increment, sec;

N = the number of data points for the FFT algorithm;

$X_i(\omega_k)$ = the Fourier transform of the ith input mode at the kth discrete frequency component;

$X_i(\omega_k)$ = the complex conjugate of $X_i(\omega_k)$;

ω_i = the ith discrete frequency component.

The kth frequency is given by

$$f_k = \frac{k}{tN} \qquad [12]$$

The subscript variable, k, ranged from one to N number of data points for the FFT algorithm. This technique was not used for ride analysis.

2. The Root Mean Square (RMS) acceleration ride number for agricultural tractor seat ride evaluation.

The RMS acceleration values were calculated by integrating the area under the PSD acceleration curve and taking the square root of the area. An ISO 5007 and SAE J1013 weighted RMS value was calculated for the vertical and horizontal seat vibrations. The SAE weighting curves for horizontal and vertical accelerations are shown in Figures 27 and 28. The frequency weighted RMS values were calculated by multiplying frequency increment areas of the PSD values by the appropriate weighting factor with Simpson's Rule. The RMS acceleration curves for the tractor-plow system are shown in Figure 23. The RMS value was computed over a 20 Hz frequency range. The numerical results of the weighted accelerations were also printed out by the program but are not presented.

The ISO Standard 2631 fatigue-decreased proficiency curves (1-, 4-, and 8-hour) were superimposed on the RMS acceleration plots. These proficiency curves are shown in Figures 29 and 30.

Mathematically, the mean square, $E(X^2)$, may be found by

$$E(X^2) = \int_{-\infty}^{\infty} \mathrm{PSD}\ (\omega)\ d\omega \qquad [13]$$

where

ω = the discrete frequency values.

For a finite length time period T seconds long, having a total of N samples, the mean square may be calculated as

$$E(X^2) = \int_{-N/2}^{N/2} PSD\ (f)\ df \qquad [14]$$

where

$f = \omega/2\pi$, the circular frequency components.

For the ride analysis, the RMS acceleration ride number may be expressed as

$$RMS = \sum_{k=1}^{N} W_R^{\ 2} \left| X_k^{\ *} \cdot X_k \right| \Delta f \qquad [15]$$

where

W_R = the SAE J1013 weighting factor;

X_K = the kth acceleration frequency component;

$X_K^{\ *}$ = the complex conjugate of X_K;

Δf_k = the kth circular frequency component.

The value of the subscript variable, k, ranged from one to N number of data points for the FFT algorithm.

3. The time domain response of the design variable(s) was computed with the inverse Discrete Fourier Transform (DFT) of X by the FFT algorithm.

The time domain responses for the variable, SEAT, for the conventional tractor model are shown in Figure 26. The numerical results were also printed out by the program but are not presented.

The post-processor program has the capability to calculate the frequency response and the mode shape(s) of the system. The frequency response option uses the system eigenvalues and complex numerators (eigenvectors) to calculate the magnitude of X/Y and X/F and the phase angle θ for a given frequency range for each SGC. The magnitude and phase angle results are calculated with respect to each motion and force input excitation. Frequency versus magnitude and phase angle plots for a conventional tractor model are shown in Figures 31 and 32. Numerical results were printed out by the program but are not presented. The mode shape option uses the system eigenvalues, eigenvectors, and geometry to plot the system configuration with the excited system configuration superimposed on it. A mode shape plot is provided for each system degree-of-freedom for each natural frequency, respectively. The set of mode shape plots for a conventional tractor model are shown in Figure 33.

RESULTS AND DISCUSSION

The IMP analysis system was used to obtain the system transfer functional results for the 24 tractor, 24 tractor-plow, and 18 tractor-trailer models. In turn, the post-processor program used these system transfer function results and the ISO 5007 smooth track profile to compute the frequency-domain acceleration response of each model. The program computed the RMS acceleration ride number and the weighted RMS acceleration response based on SAE J1013 and ISO 2631 standards and plotted out the RMS acceleration response results.

For the design evaluation, the ISO 5007 smooth track was passed under the tractor, tractor-plow, and tractor-trailer models at a speed of 12.5±0.5 km/h as specified by the ISO 5007 standard. Further, the ISO 5007 smooth track was passed under the tractor-trailer models at a speed of 25.0 km/h as a means to simulate an on-road condition.

The RMS longitudinal and vertical acceleration ride numbers for the 24 tractor, and 24 tractor-plow models are presented in Figures 34 and 35, respectively. The RMS longitudinal and vertical acceleration ride numbers for the 18 tractor-trailer models are presented in Figures 36 and 37, respectively. To interpret the computer model designation, refer to Table 1 for the designation code. The RMS acceleration plots for the conventional tractor are presented in Figure 26. The remaining RMS acceleration plots for the models are presented in Figures 38 - 66.

On the basis of the RMS ride number and the RMS acceleration response plots, the following conclusions and observations were made:

1. The chassis suspension system reduced vibration levels and

improved operator ride comfort.

2. When the results for the three unsuspended tractor configurations alone were compared, the tractor with the conventional cab position provided the best overall ride comfort.
3. When the results for the three suspended front axle tractor configurations alone were compared, all three tractors with their suspension system parameters based on automotive suspension design theory provided approximately the same ride comfort.
4. When the results for the three fully suspended tractor configurations alone were compared, the tractor with the conventional cab position and its suspension system parameters based on automotive suspension design theory provided the best ride quality.
5. When the results for the three unsuspended tractor configurations with attached plows were compared, the tractor with the conventional cab position provided the best ride comfort.
6. When the results for the suspended front axle tractor configurations with attached plows were compared, the tractor with the forward cab position and its suspension parameters based on automotive suspension design theory provided the best ride comfort.
7. When the results for the fully suspended tractor

configurations with attached plows were compared, the tractor with the midchassis cab position and its suspension system parameters based on automotive suspension design theory provided the best ride comfort.

8. When the results for the three unsuspended tractor configurations with attached single axle trailers were compared, all three tractors provided approximately the same ride comfort at 12.5 and 25.0 km/h.
9. When the results for the three suspended front axle and the three fully suspended tractor configurations with attached single axle trailers were compared, the tractor with the forward cab position provided the best ride comfort at 12.5 and 25.0 km/h.
10. When the results for the three unsuspended tractor configurations with attached tandem axle trailers were compared, the tractor with the midchassis cab position provided the best ride comfort at 12.5 and 25.0 km/h.
11. When the results for the three suspended front axle tractor configurations with attached tandem axle trailers were compared, all three tractors provided approximately the same ride comfort at 12.5 and 25.0 km/h.
12. When the results of the three fully suspended tractor configurations with attached tandem axle trailers were compared, the tractor with the conventional cab position provided the best ride comfort at 12.5 and 25.0 km/h.

13. The moldboard plow provided an effective damping means to reduce the vibration levels imposed on the operator, but the vibration excitations due to the plow-soil interaction were not considered in this study.
14. The transport trailer provided a means to increase the vibration levels imposed on the operator.
15. The single axle and the tandem axle trailer provided about similar vibration levels to be imposed on the operator.
16. When the transport speed with the trailers was doubled, the vibration levels imposed on the operator were increased.

For an agricultural tractor, the bounce and pitch modes of vibration are nearly always coupled. The degree of coupling is variable and is dependent on the tractor wheelbase, the tire stiffness, the suspension system parameters, the centers of gravity and inertia of the tractor and the attached implement. The location of the pitch center is an important consideration for ride comfort, as shown in Figure 67. If the operator is located at the pitch center, the operator would experience little vibration despite the fact that the tractor was pitching around him. On the other hand, the further the operator is located away from the pitch center, the worse will be the vibration levels experienced by him. On most tractors, the operator will be located always above and usually away from the pitch center. Therefore, the pitching motion will cause the operator to move in both the longitudinal and vertical directions.

Crolla (1980a, 1980b) has investigated the effect of a tractor pulling a trailer on operator ride vibration. It was shown that the tractor operator vibration levels were higher when pulling a trailer than for the tractor operating alone. Milroy (1976) investigated the differences in operator ride vibration for a tractor alone and a tractor-trailer system and showed similar results. As the tractor-trailer system traversed the track, the vertical input excitations at the tires caused the trailer to pitch, which, in turn, was input to the tractor hitch and caused an amplified bucking motion at the operator seat position. This motion was the direct result of the low hitching point on the tractor and the high center of gravity and large inertial properties of the trailer.

Crolla (1976) also investigated the effect of a tractor pulling a moldboard plow. He found that the operator ride vibration in the pitch mode was heavily damped owing to the action of the soil forces acting on the plow. The vibration levels were lower when plowing than when the tractor was operating alone.

From the computer simulation analyses, the ride vibration levels were generally higher when the tractors were attached to transport trailers than when the tractors were operated alone or attached to moldboard plows. In addition, the ride vibration levels were generally lower when the tractors were attached to plows than when operated alone. These phenomena were observed in the RMS acceleration ride numbers that were computed for the tractor alone and the tractor-implement system configurations.

On the basis of the RMS acceleration ride numbers computed for the tractor and tractor-implement configurations and the results presented by Crolla, the following conclusions and observations were made:

1. For the tractor alone configurations, the tractor with the conventional cab position provided the best ride comfort because the operator was located approximately at the pitch center of the tractor. The chassis suspension system further improved the operator ride comfort.
2. For the tractor-plow configurations, the tractor with the forward cab position and a suspended front axle and the tractor with a midchassis cab position and the fully suspended chassis provided the best ride comfort.
3. For the tractor-trailer configurations, the tractors with the forward cab position and the suspended front axle or the fully suspended chassis provided the best ride comfort.
4. For the tractor-plow and tractor-trailer configurations, the pitch centers of the tractors were toward the front of the tractors. Hence, the ride comfort for the tractors with midchassis and forward cab positions was better because the operator was positioned nearer to the pitch centers of these tractors.

When all the tractor configurations and the tasks of operating alone and pulling an attached implement were considered, the optimum tractor configuration for improved ride comfort was found to be the tractor with the forward cab position and a suspended front axle. This tractor configuration would be quite similar to the tractor-scraper unit described by Cadou and Bowser (1978).

If further reduction of the operator ride vibration levels with a chassis suspension system is to be investigated, the simulation models must be formulated to include: (1) the nonlinear spring and damper characteristics of the tires, the cab isolation system, and the chassis and seat suspension systems;(2) the frictional forces acting in the suspension system; (3) the mathematical relationships for the implement hitch control system, the power train torque-speed characteristics, the soil-tire interface, and the soil-implement interface. Also, additional terrain profiles and implement loading conditions should be used to more fully evaluate the suspension system. Finally, the tractors should be configured to have the appropriate tire sizes and the weight distribution of the mainframe structure and the powertrain components. These complex models may be too costly to be analyzed in a university computing environment.

Optimization techniques can be used to obtain the optimal suspension parameters for improved ride comfort. One suitable technique that can be incorporated into the IMP program is the parameter estimation method (Chrostowski et al., 1978). Also, finite element analysis techniques may be combined with the constrained rigid body analysis technique to obtain more nearly optimal suspension parameters (Baum et al., 1977).

In this computer simulation study, no economic or manufacturing aspects and constraints or vehicle handling and stability characteristics were considered. It will be the ultimate goal of the tractor manufacturer, however, to provide operator ride comfort at the minimum cost. It may be probable that one of the other suspended tractor configurations, or even a totally different tractor configuration for a totally new suspension concept, will be considered. The manufacturer may want to consider the use of an active or a semi-active chassis suspension system for improved ride comfort.

SUMMARY

The development of a chassis suspension system for an agricultural tractor has been described for improving the operator ride comfort. Twenty-four tractor, 24 tractor-plow, and 18 tractor-trailer models were formulated with the IMP analysis program to evaluate the effect of a suspension system and its parameters and the effect of the position of the operator cab. RMS ride numbers and RMS acceleration plots provided the basis to evaluate the effect of the suspension system and cab position on operator ride comfort. The IMP simulation and post-processor programs provided the natural frequencies, the frequency response magnitude and phase angle plots, and the mode shapes of the vibrational modes to gain further insight to evaluate the effect of ride comfort. The generalized mechanical systems simulation program, IMP, has proven to be a valuable analytical tool for evaluating the suspension parameters affecting operator ride comfort.

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TABLE 1. Tractor suspension system model designation code

T: TRACTOR
S: TRACTOR-PLOW SYSTEM
I: TRACTOR-SINGLE AXLE TRAILER SYSTEM
J: TRACTOR-TANDEM AXLE TRAILER SYSTEM

C: CONVENTIONAL CAB POSITION
M: MIDCHASSIS CAB POSITION
F: FORWARD CAB POSITION

U: UNSPRUNG FRONT AND REAR AXLES
F: SPRUNG FRONT AXLE ONLY
B: SPRUNG FRONT AND REAR AXLES

-: $K_{FS} = 1/8\ K_{RT}$; $K'_{RS} = 1.2\ K'_{FS}$; $C = 1/4\ C_C$

D: $K_{FS} = 1/8\ K_{RT}$; $K'_{RS} = 1.2\ K'_{FS}$; $C = 1/8\ C_C$

R: $K_{FS} = 1/8\ K_{RT}$; $K'_{RS} = 1.4\ K_{FS}$; $C = 1/4\ C_C$

S: $K_{FS} = 1/8\ K_{RT}$; $K'_{RS} = 1.2\ K'_{FS}$; $C = 1/4\ C_C$; $K_{RS} = f(K_{RTP})$

X: $K_{FS} = 1/8\ K_{RT}$; $K'_{RS} = 1.2\ K'_{FS}$; $C = 1/4\ C_C$; $m = m_{UR}$

where:

K_{FS} = front suspension spring rate

K_{RS} = rear suspension spring rate

K_{RT} = rear tire spring rate

K_{RTP} = rear tire spring rate/include implement load

K'_{FS} = front suspension ride rate

K'_{RS} = rear suspension ride rate

C = damping rate

C_C = critical damping rate

m = unsprung mass

m_{UR} = unsprung rear axle mass

TABLE 2. Suspension parameters for coventional cab position tractor model

	TCB	TCBD	TCBR	TCBS	TCBX
k'_{FS}	44.8 (255.7)	44.8 (255.7)	44.8 (255.7)	44.8 (255.7)	44.8 (255.7)
k'_{RS}	53.7 (306.8)	53.7 (306.8)	53.7 (356.1)	53.7 (306.8)	53.7 (306.8)
k_{FS}	50.5 (288.5)	50.5 (288.5)	50.5 (288.5)	50.5 (288.5)	50.5 (288.5)
k_{RS}	58.8 (335.7)	58.8 (335.7)	69.3 (335.7)	58.8 (335.7)	58.8 (335.7)
C_F	3.6 (20.35)	1.8 (10.17)	3.6 (20.35)	3.6 (20.35)	3.6 (20.35)
C_R	5.2 (29.6)	2.6 (14.8)	5.6 (31.9)	5.2 (29.5)	21.8 (124.34)

F = front suspension

R = rear suspension

k' and k: N/mm (lb/in); C: N·s/mm (lb·sec/in)

TABLE 3. Suspension parameters for midchassis cab position tractor model

	TMB	TMBD	TMBR	TMBS	TMBX
k'_{FS}	59.1 (337.5)	59.1 (337.5)	59.1 (337.5)	59.1 (337.5)	59.1 (337.5)
k'_{RS}	70.9 (405.0)	70.9 (405.5)	82.7 (472.5)	70.9 (405.0)	70.9 (405.0)
k_{FS}	69.2 (395.0)	69.2 (395.0)	69.2 (395.0)	69.2 (395.0)	69.2 (395.0)
k_{RS}	80.3 (458.7)	80.3 (458.7)	95.8 (547.3)	80.3 (458.7)	80.3 (458.7)
C_F	4.7 (26.9)	2.4 (13.5)	4.7 (26.9)	4.7 (26.9)	4.7 (26.9)
C_R	5.9 (33.7)	2.0 (16.9)	6.4 (36.4)	5.9 (33.4)	22.3 (127.3)

F = front suspension

R = rear suspension

k' and k: N/mm (lb/in); C: N·s/mm (lb·sec/in)

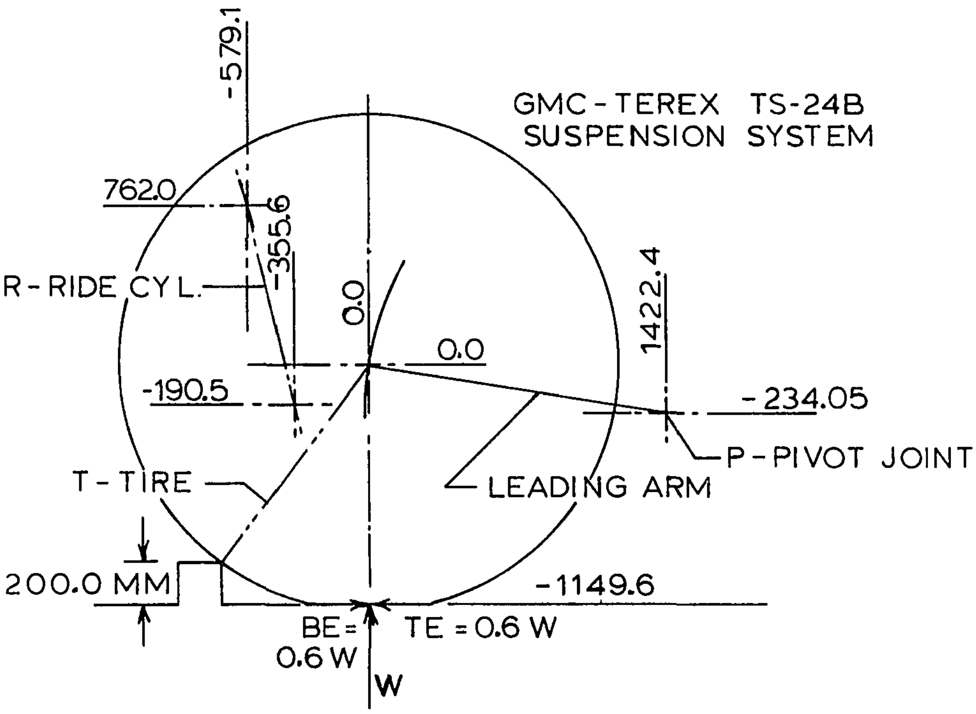
TABLE 4. Suspension parameters for forward cab position tractor model

	TFB	TFBD	TFBR	TFBS	TFBX
k'_{FS}	73.5 (419.5)	73.5 (419.5)	73.5 (419.5)	73.5 (419.5)	73.5 (419.5)
k'_{BS}	88.2 (503.4)	88.2 (503.4)	102.9 (587.3)	88.2 (507.4)	88.2 (503.4)
k_{FS}	85.8 (490.0)	85.8 (490.0)	85.8 (490.0)	85.8 (490.0)	85.8 (490.0)
k_{BS}	103.2 (589.1)	103.2 (589.1)	123.9 (707.37)	101.6 (579.9)	103.2 (589.1)
C_F	5.2 (29.5)	2.6 (14.8)	5.2 (29.5)	5.2 (29.5)	5.2 (29.5)
C_R	5.2 (29.6)	2.6 (14.8)	6.3 (36.2)	5.8 (32.9)	20.8 (118.6)

F = front suspension

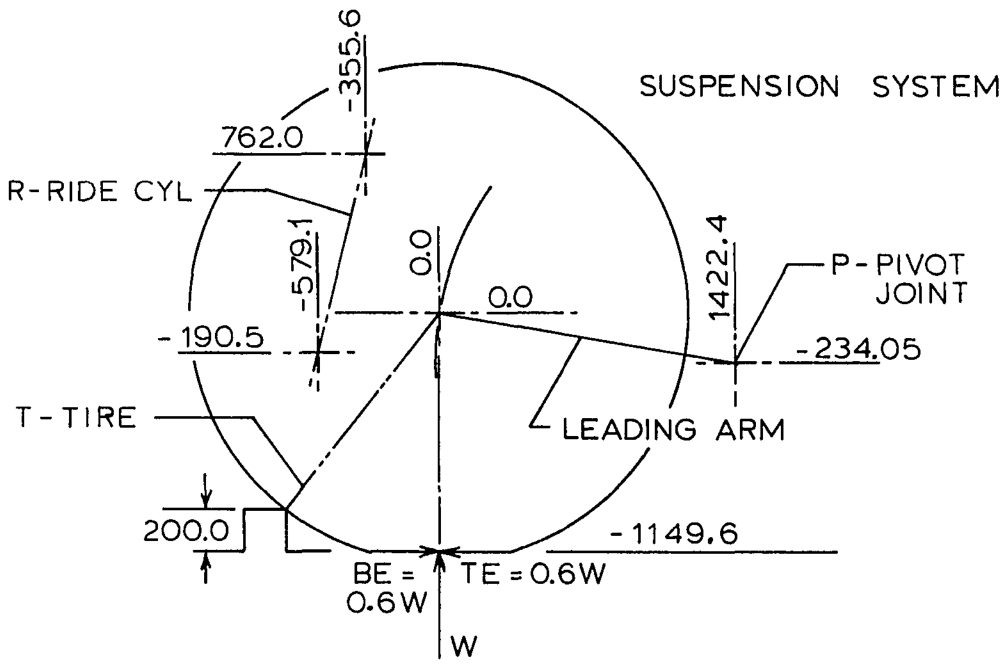
R = rear suspension

k' and k: N/mm (lb/in); C: N·s/mm (lb·sec/in)



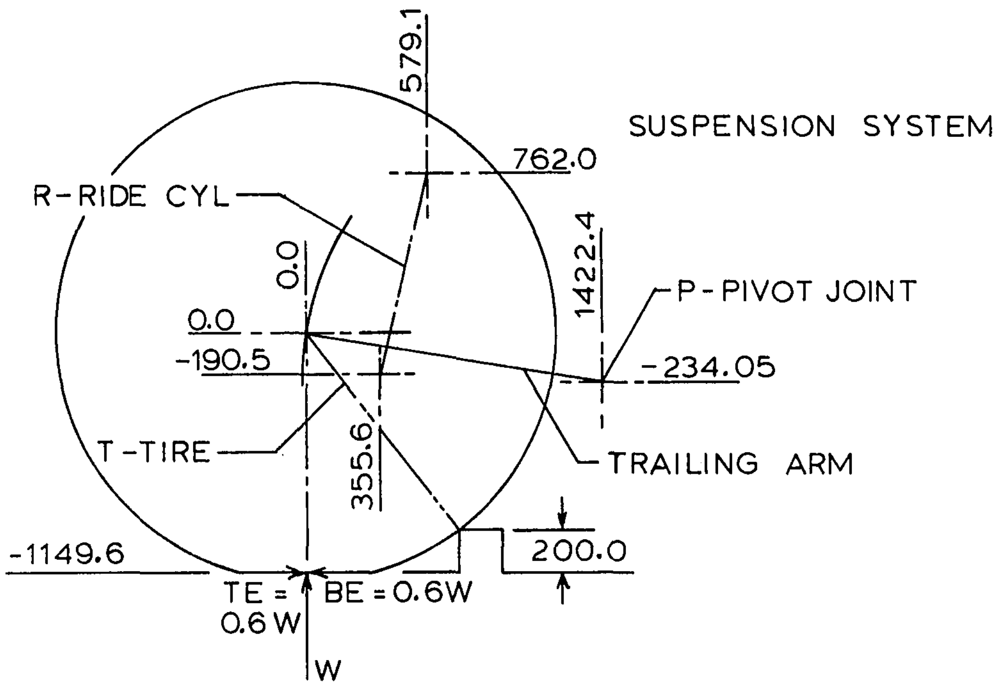
LOAD, NEWTONS	T	R	P
STATIC	422580	281125	91635
2-G BUMP	845160	629420	653890
RATED LOAD W/ TRAC EFFORT	492865	322495	146790
RATED LOAD W/ BRK EFFORT	492865	217965	378100

FIGURE 1. Force analysis of the TEREX suspension system



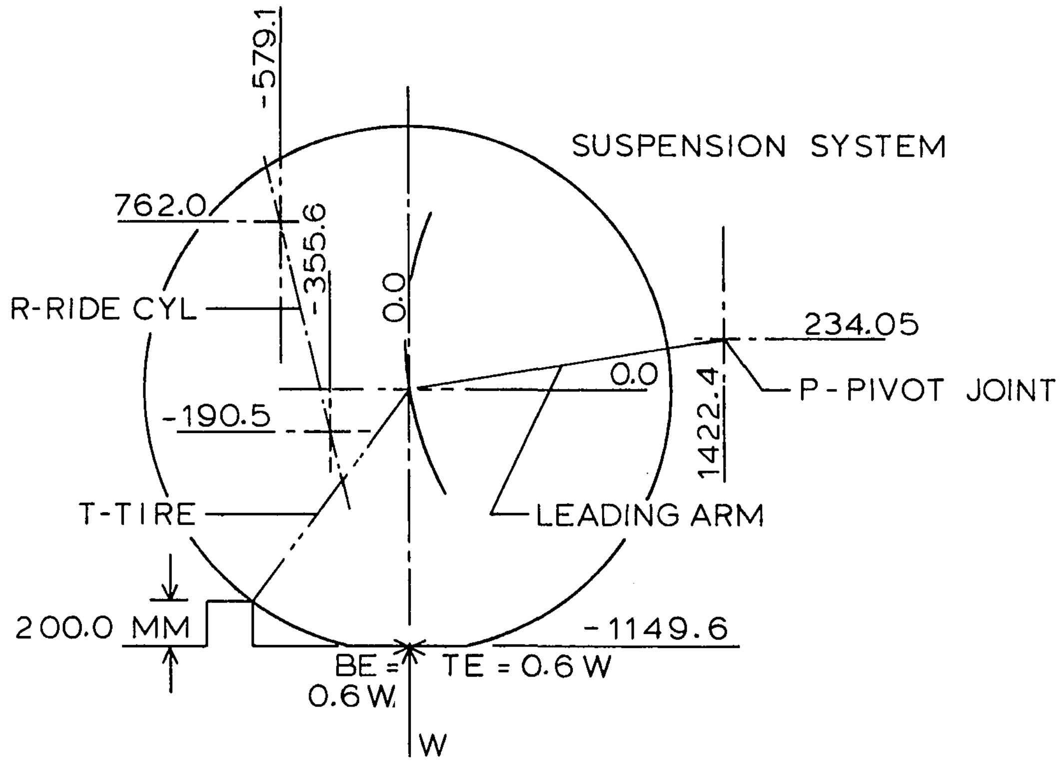
LOAD, NEWTONS	T	R	P
STATIC	422580	306925	146790
2-G BUMP	845160	544905	415910
RATED LOAD W/ TRAC EFFORT	492865	424805	349185
RATED LOAD W/ BRK EFFORT	492865	200170	311375

FIGURE 2. Force analysis of the leading arm suspension with the re-oriented ride cylinder



LOAD, NEWTONS	T	R	P
STATIC	422580	582715	200170
2-G BUMP	845160	785110	707265
RATED LOAD W/ TRAC EFFORT	492865	353635	191275
RATED LOAD W/ BRK EFFORT	492865	785110	556030

FIGURE 3. Force analysis of the trailing arm suspension



LOAD, NEWTONS	T	R	P
STATIC	422580	328060	133445
2-G BUMP	845160	461500	644990
RATED LOAD W/ TRAC EFFORT	492865	511545	150125
RATED LOAD W/ BRK EFFORT	492865	139005	400340

FIGURE 4. Force analysis of the leading arm suspension system with a higher pivot joint location

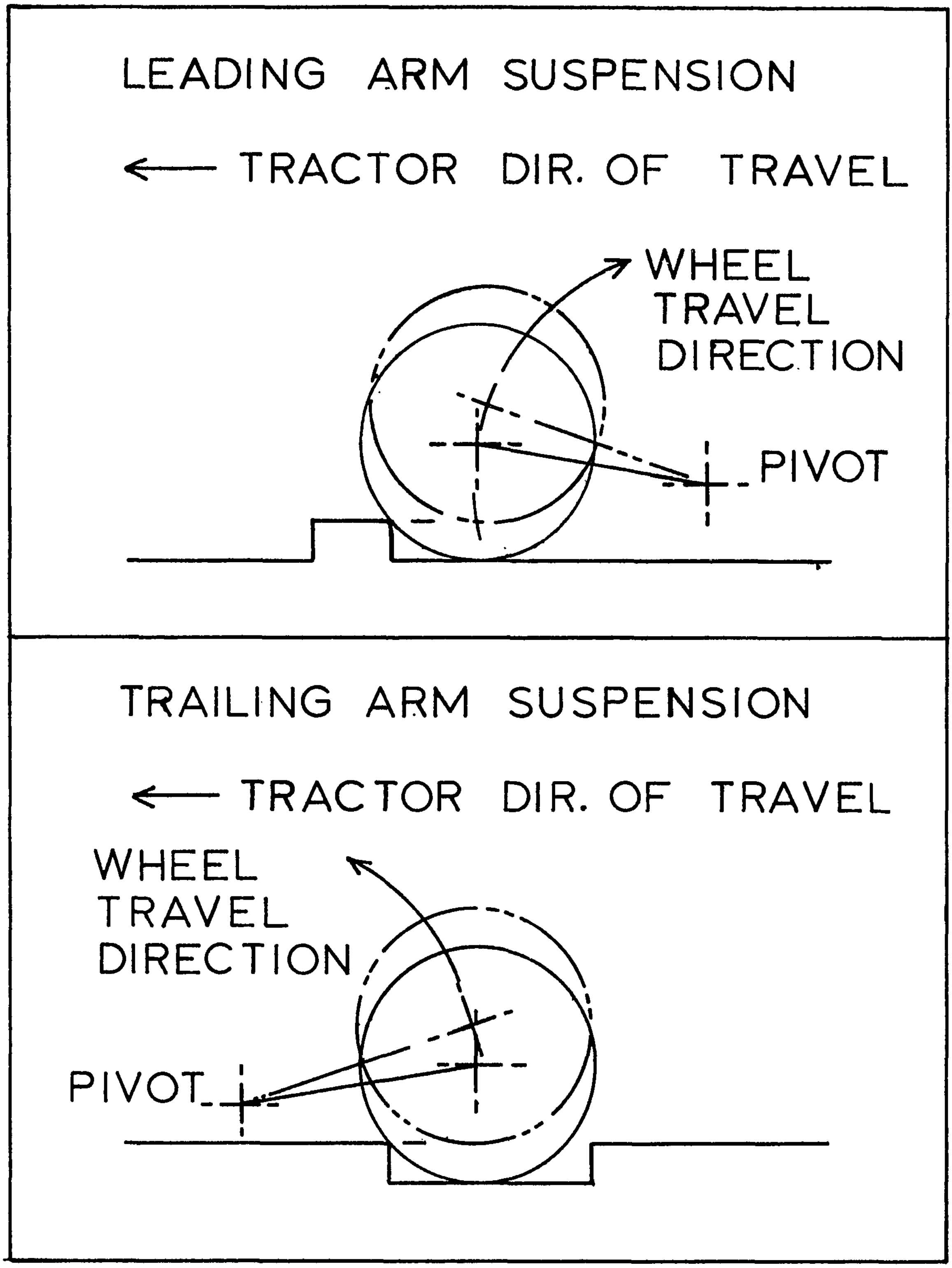


FIGURE 5. Arm suspension systems

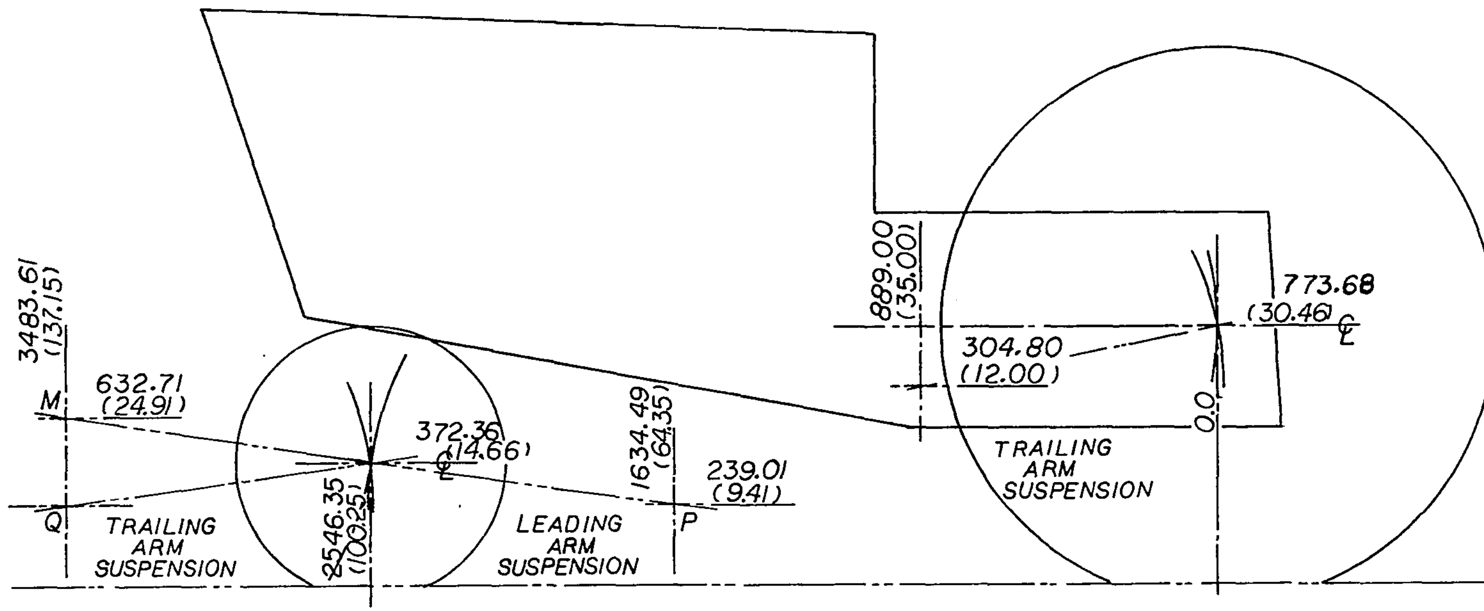


FIGURE 6. Suspended tractor configuration

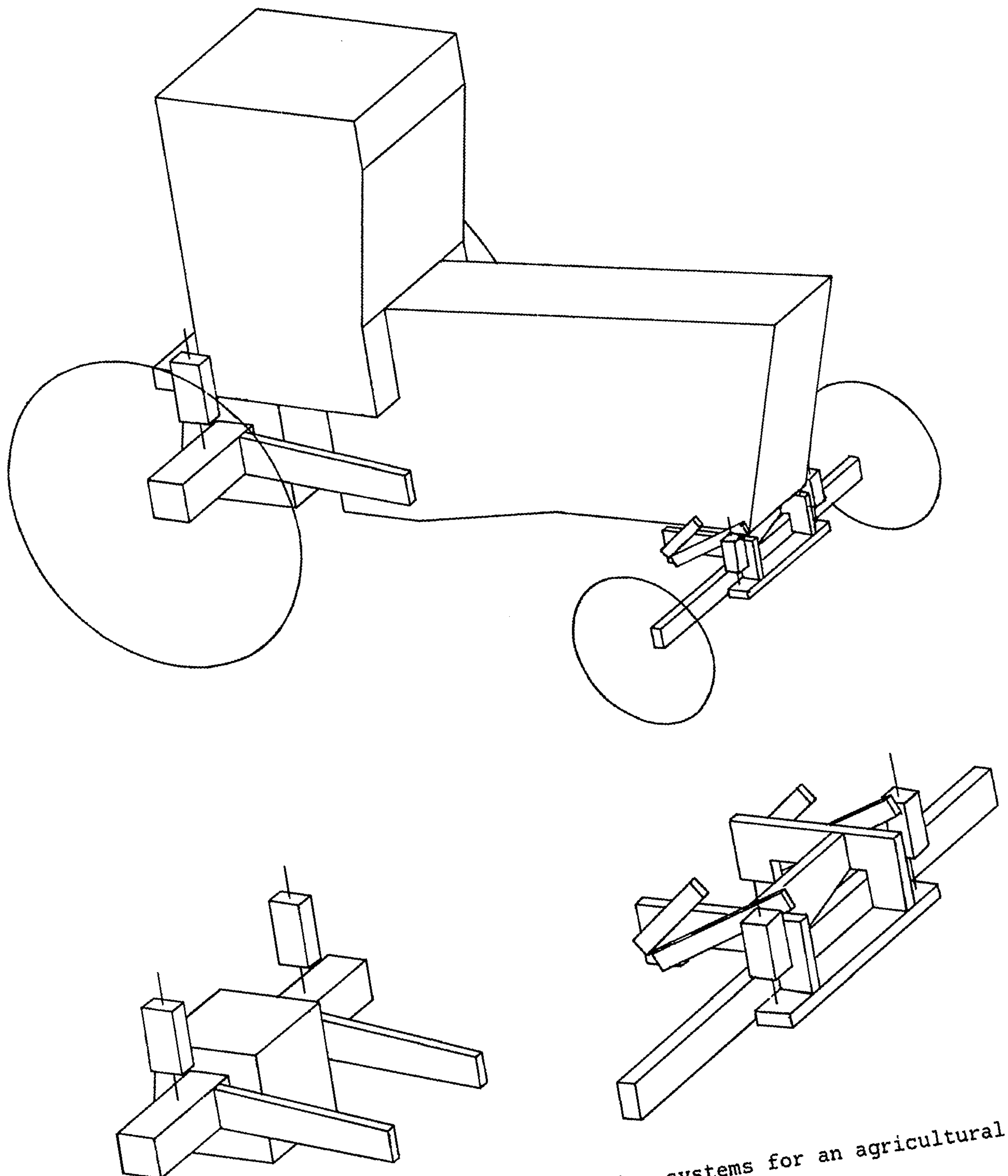


FIGURE 7. Front and rear axle suspension systems for an agricultural tractor

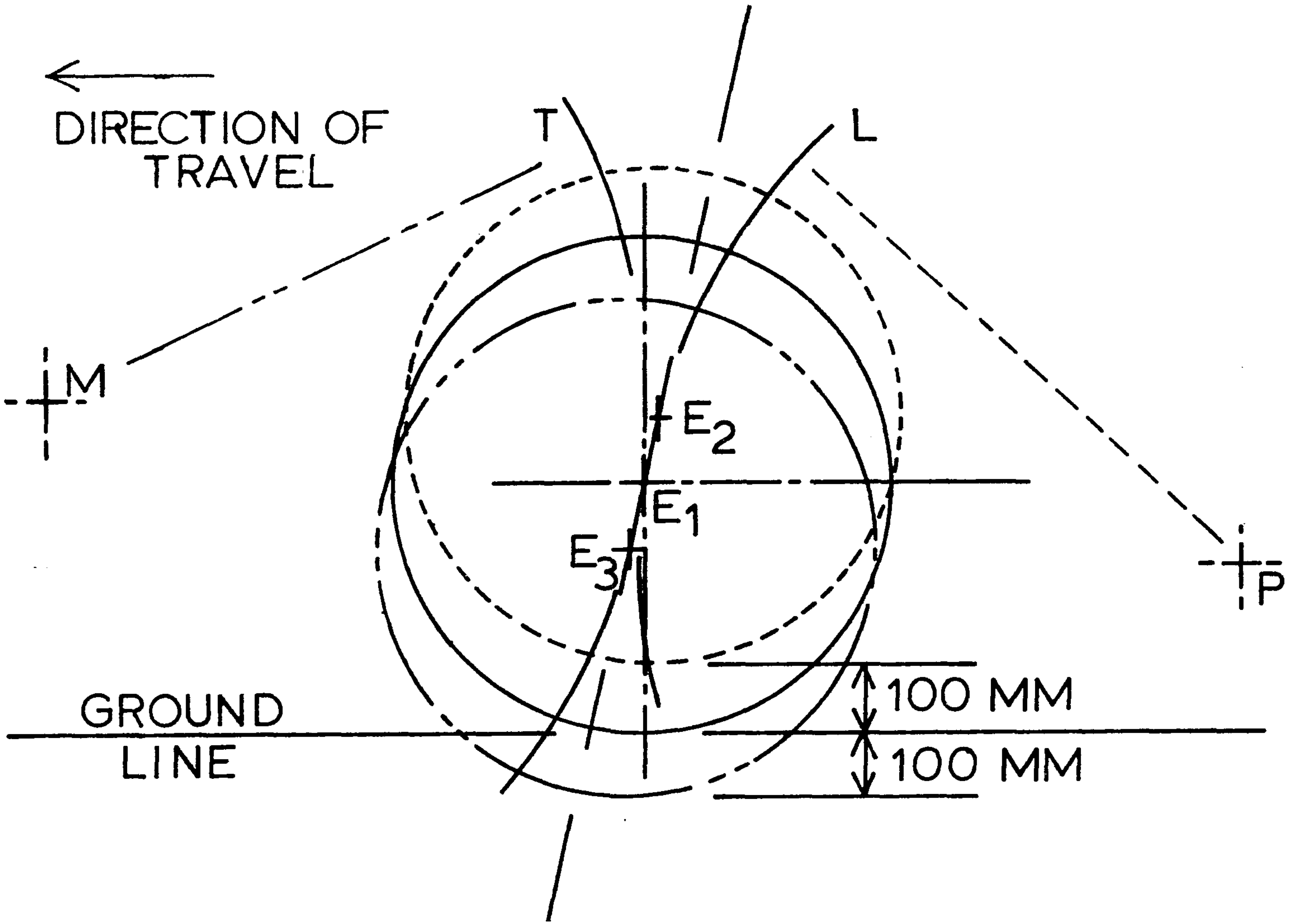


FIGURE 8. Motion of the wheel for the leading and trailing arm suspension systems

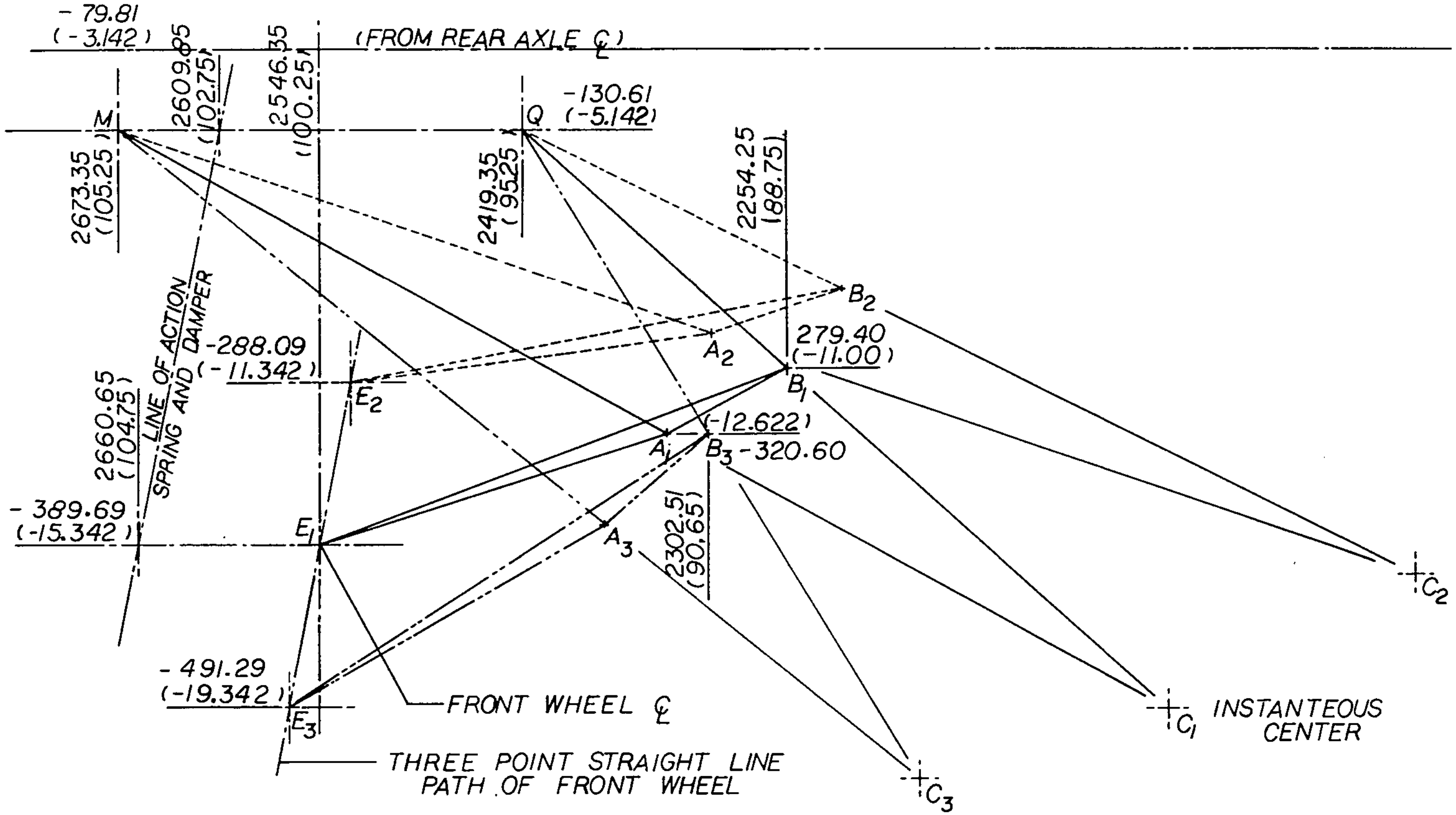


FIGURE 9. Point path layout for the front suspension system linkage

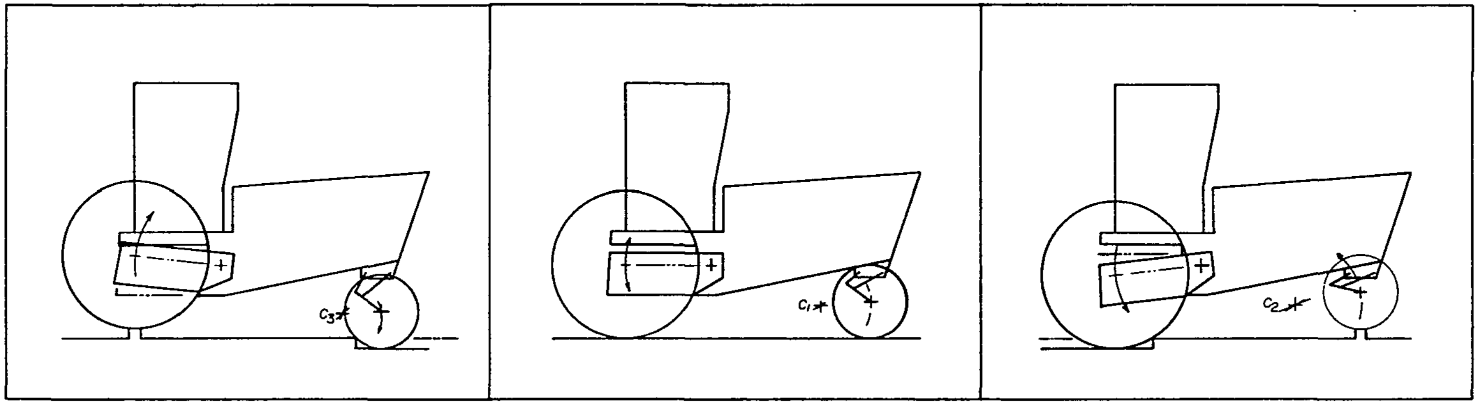


FIGURE 10. Motion of the front and rear axle suspension systems

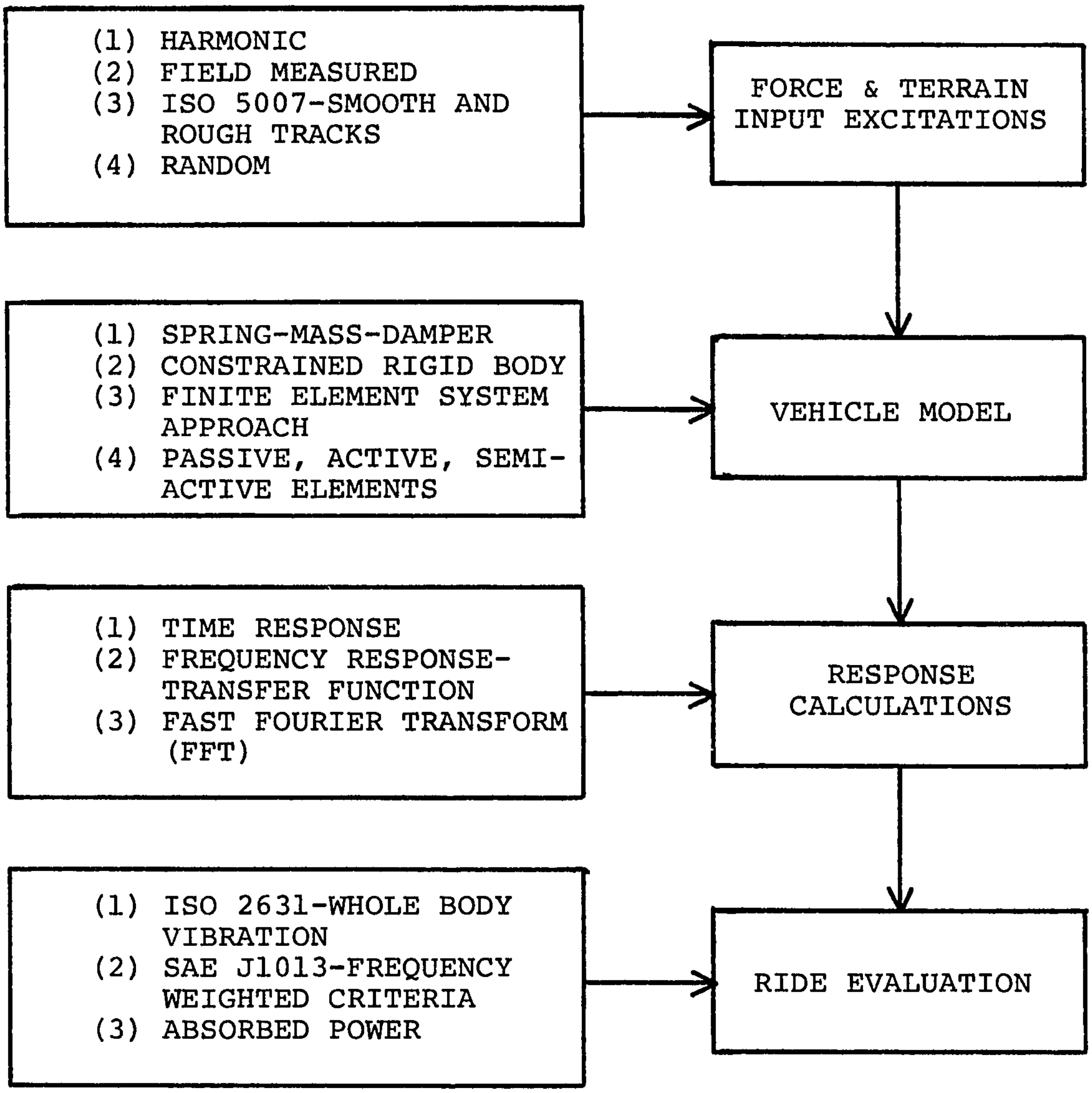


FIGURE 11. Flow process of the computer-aided design approach for tractor ride evaluation

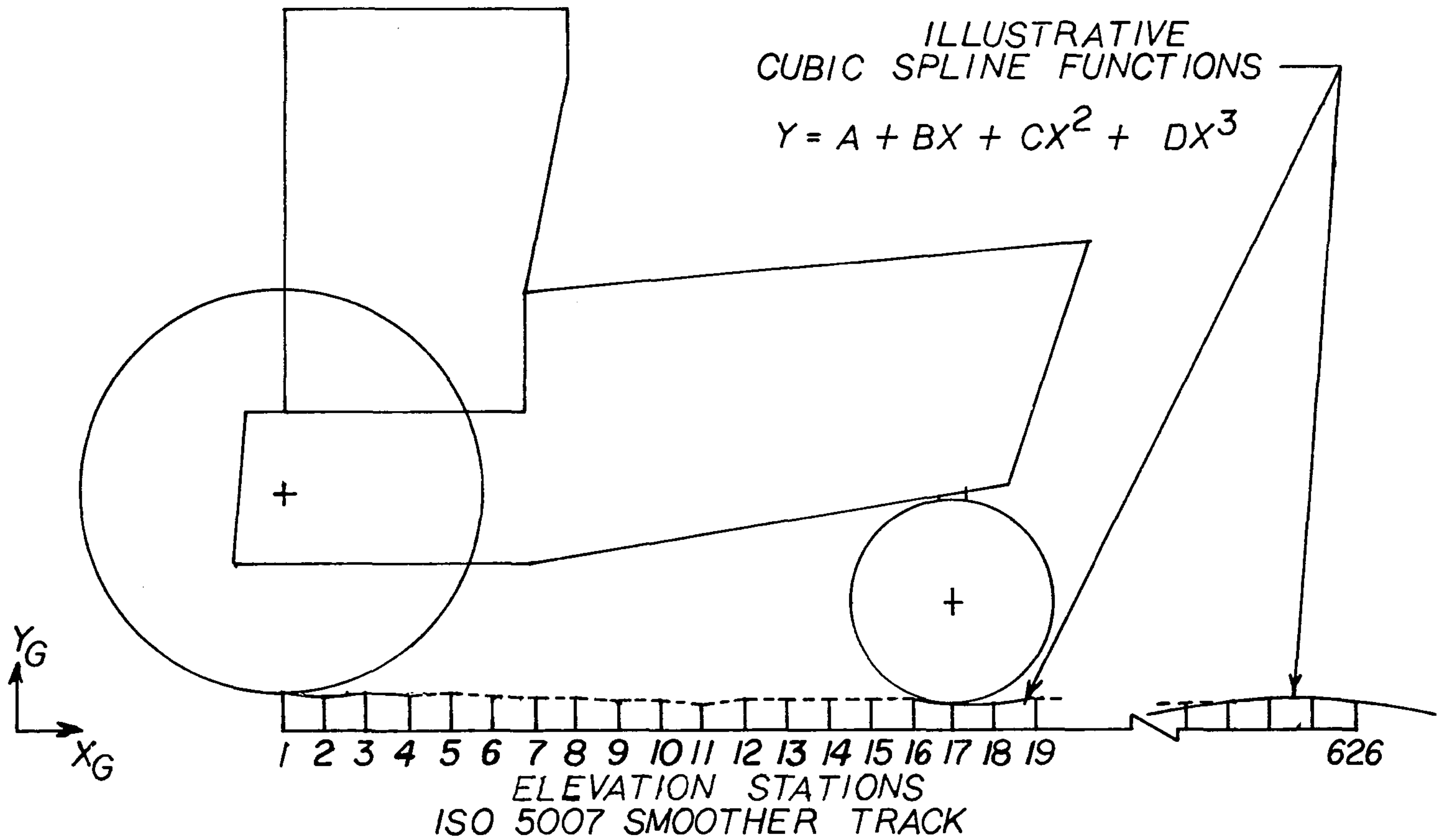


FIGURE 12. Terrain representation for the ISO 5007 smooth track

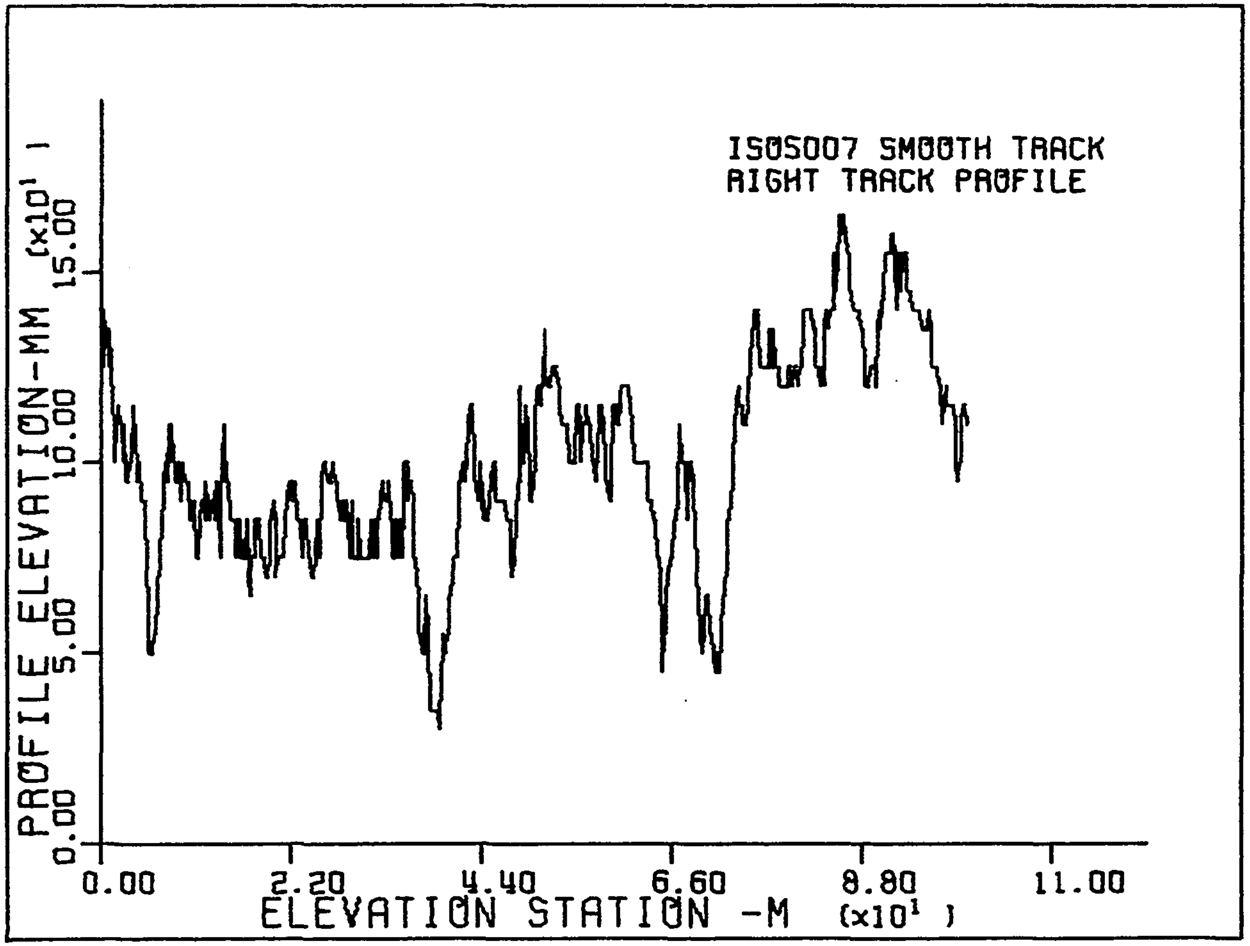


FIGURE 13. ISO 5007 right smooth track profile

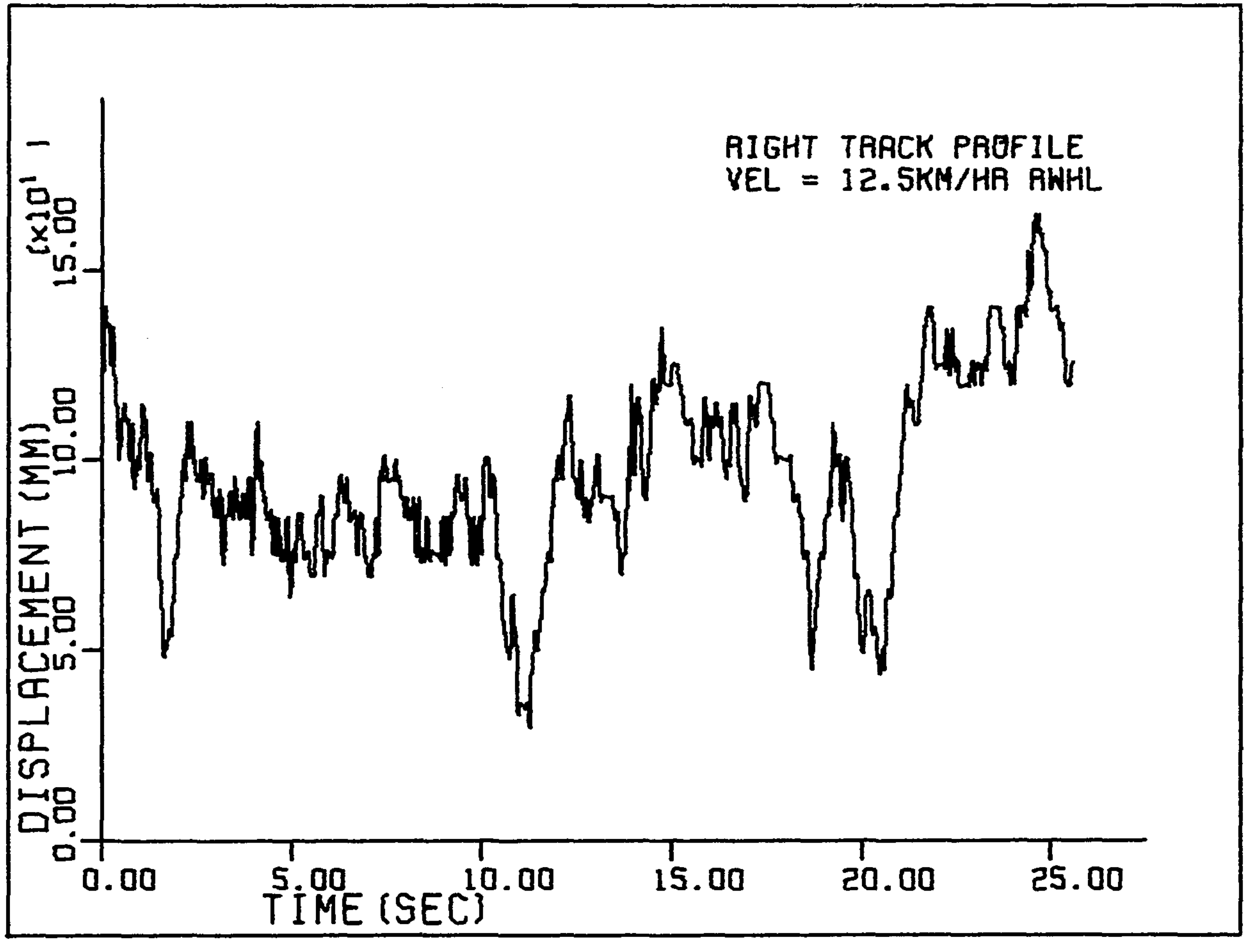


FIGURE 14. Displacement time components of the ISO 5007 right smooth track

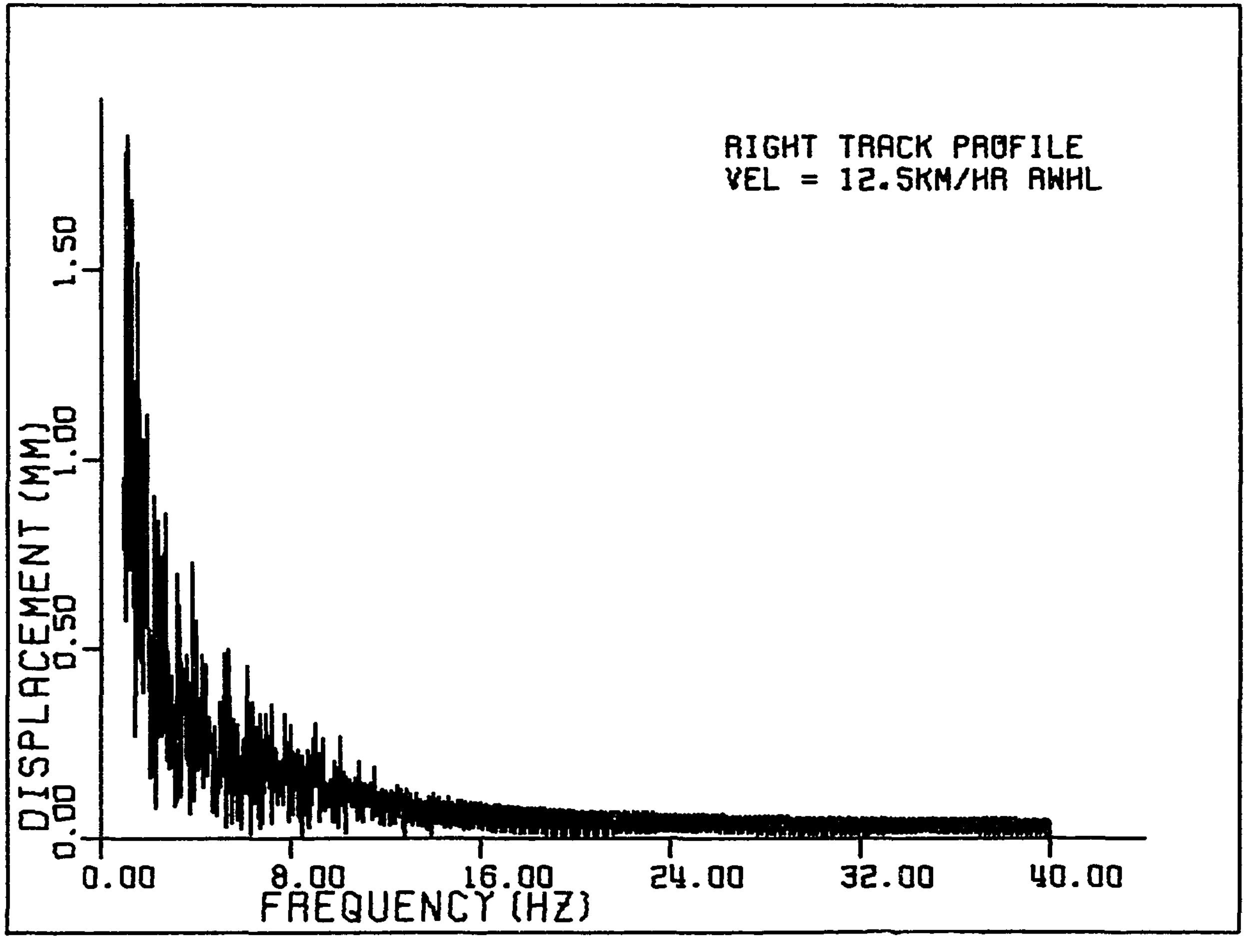


FIGURE 15. Displacement frequency components of the ISO 5007 right smooth track

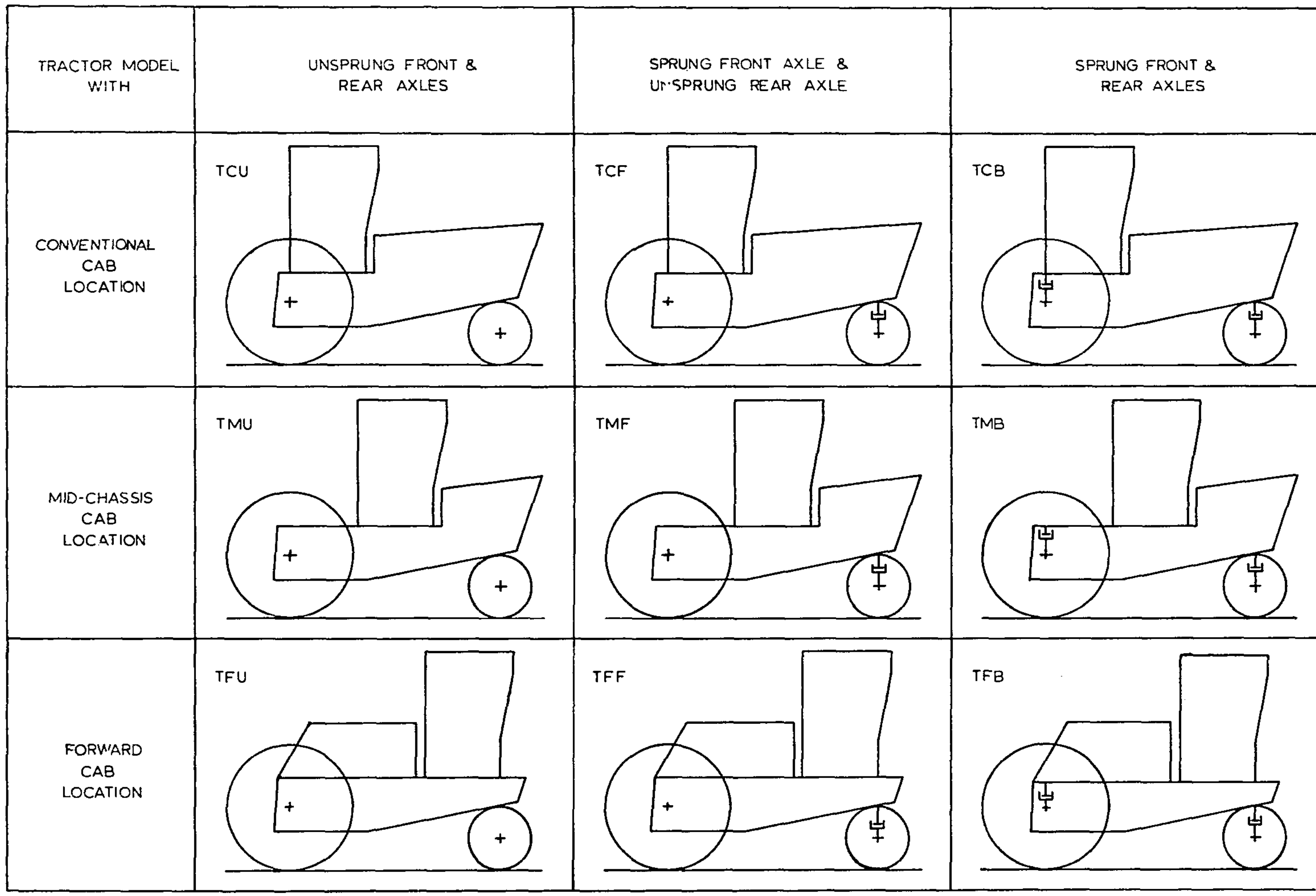


FIGURE 16. Tractor chassis, cab, and suspension system configurations

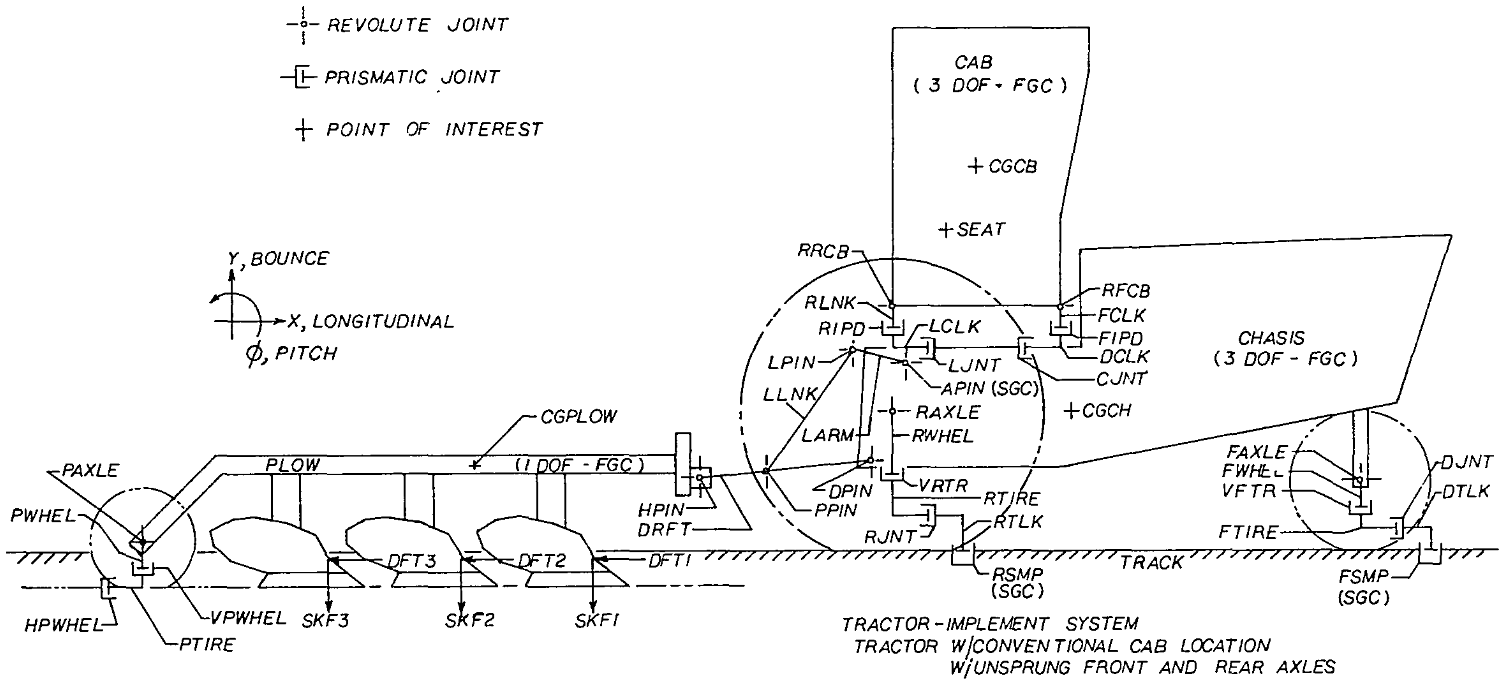


FIGURE 17. Conventional cab position tractor with plow model

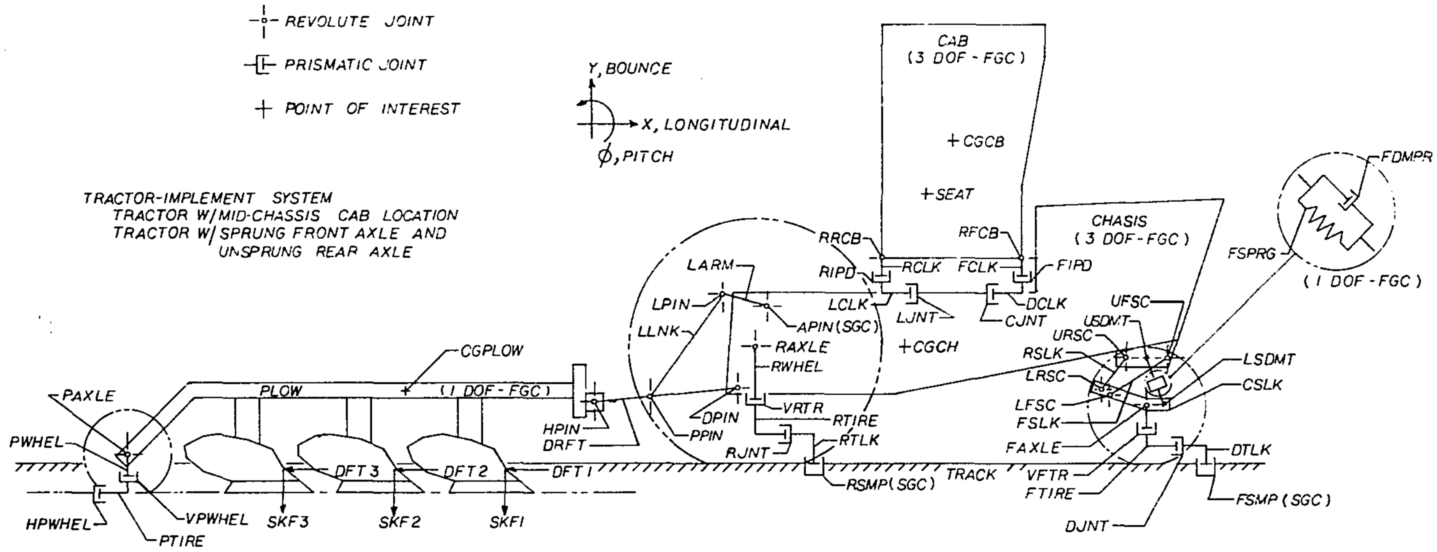


FIGURE 18. Midchassis cab position tractor with plow model

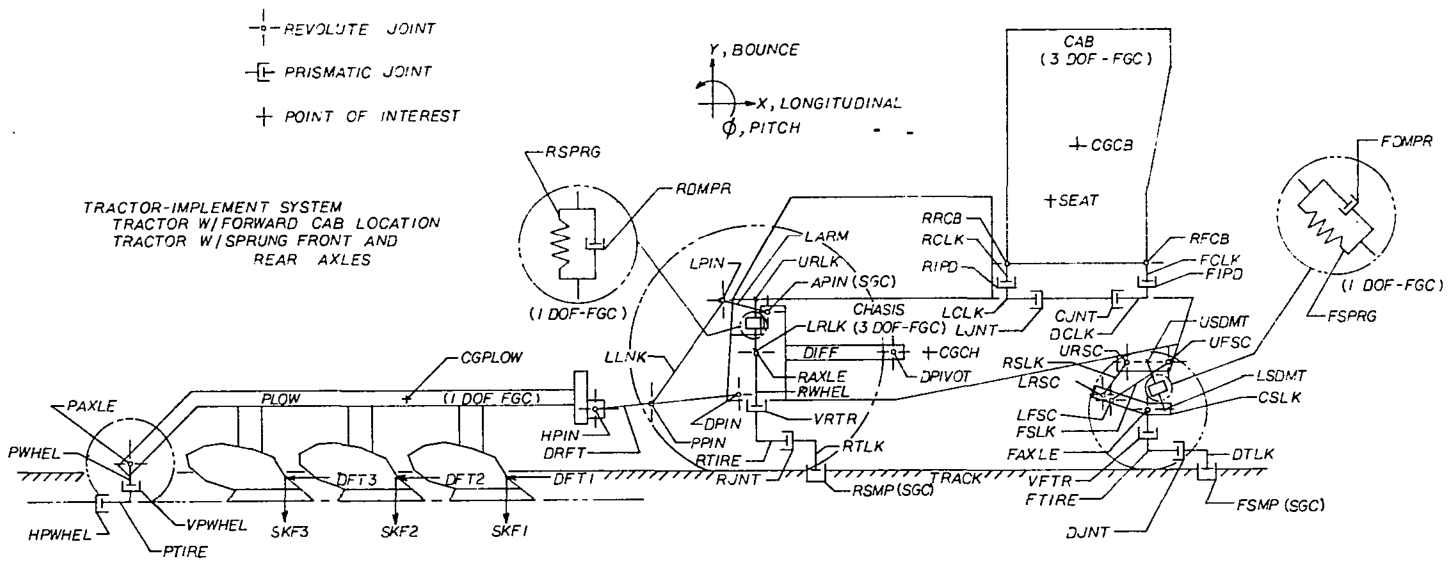


FIGURE 19. Forward cab position tractor with plow model

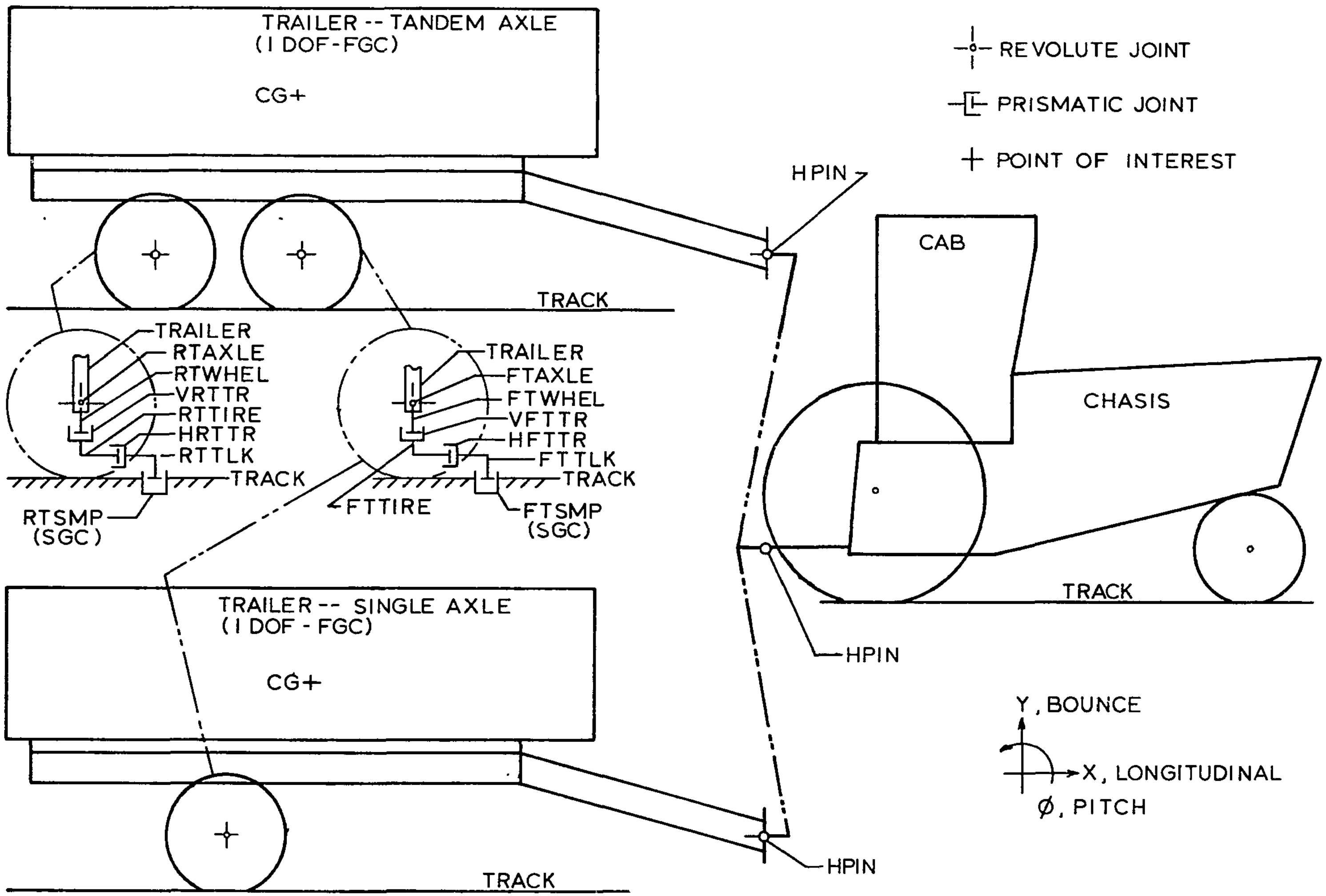


FIGURE 20. Conventional cab position tractor with single-axle and tandem-axle trailer models

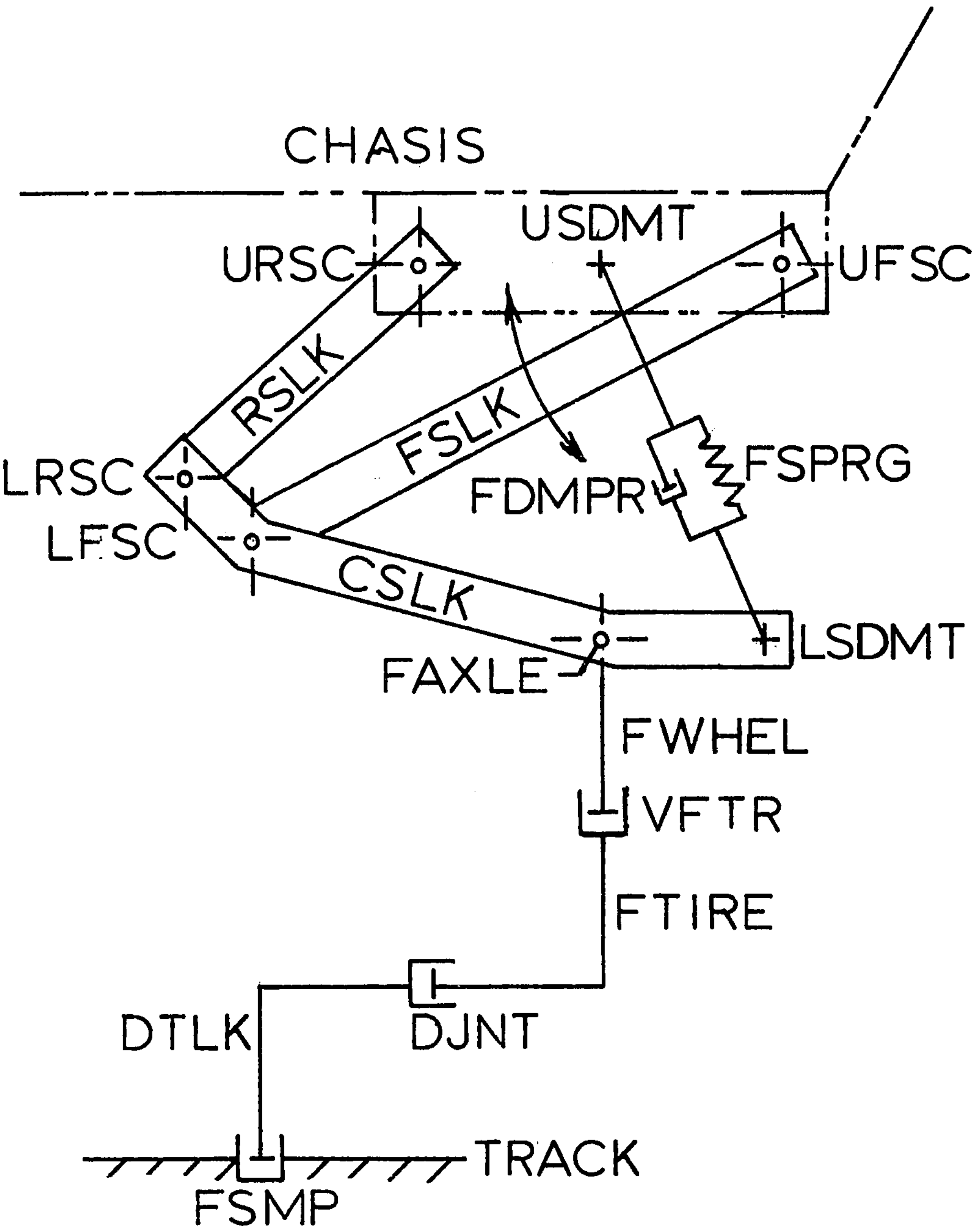


FIGURE 21. Front axle suspension system and wheel model

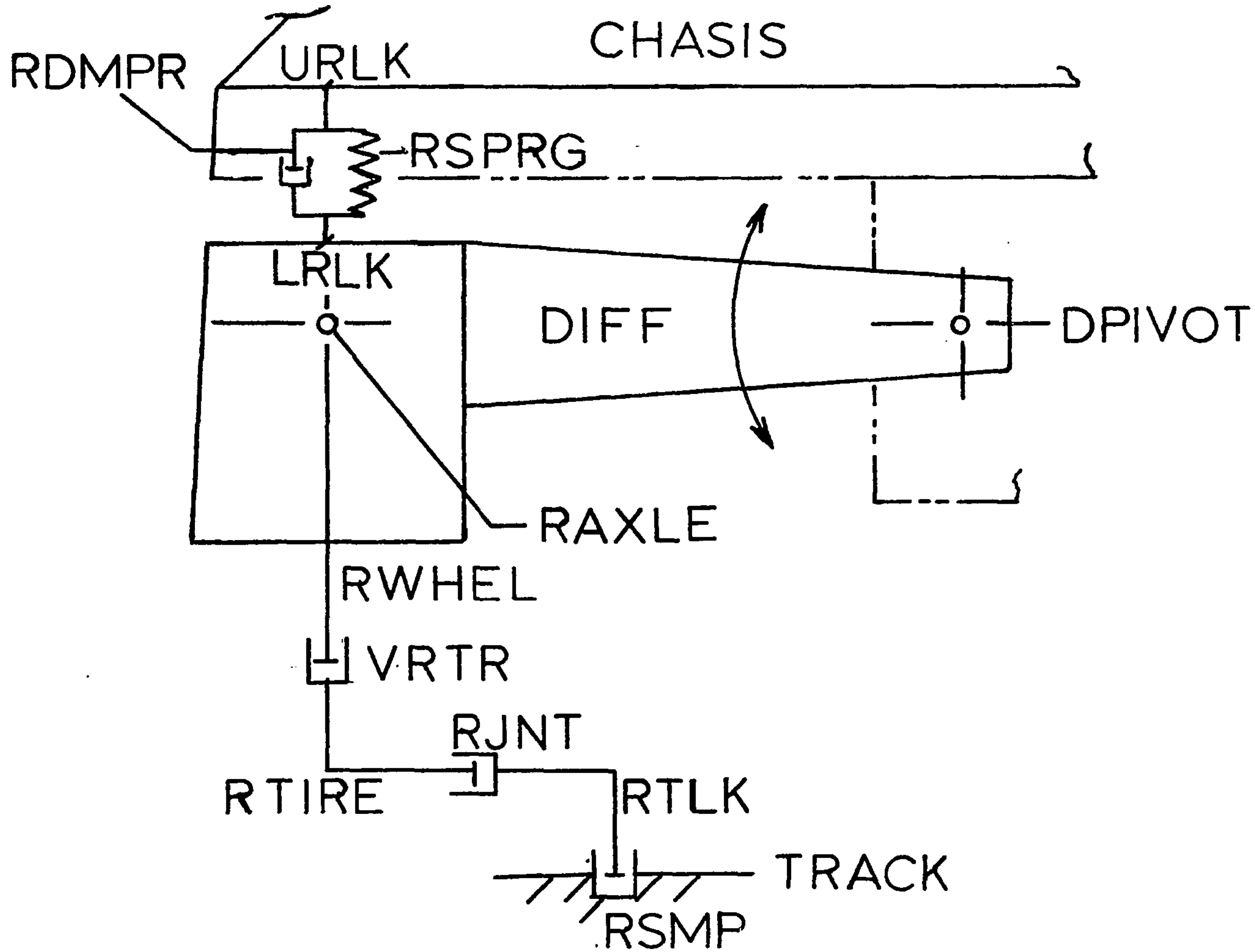


FIGURE 22. Rear axle suspension system and wheel model

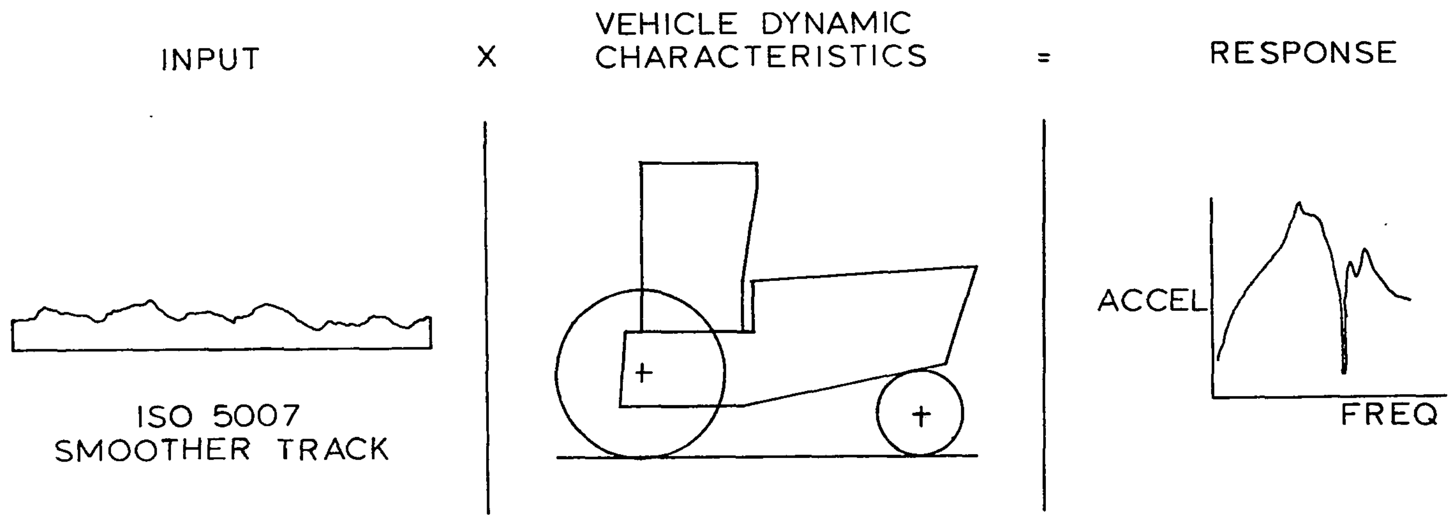


FIGURE 23. Vehicle response to terrain disturbances

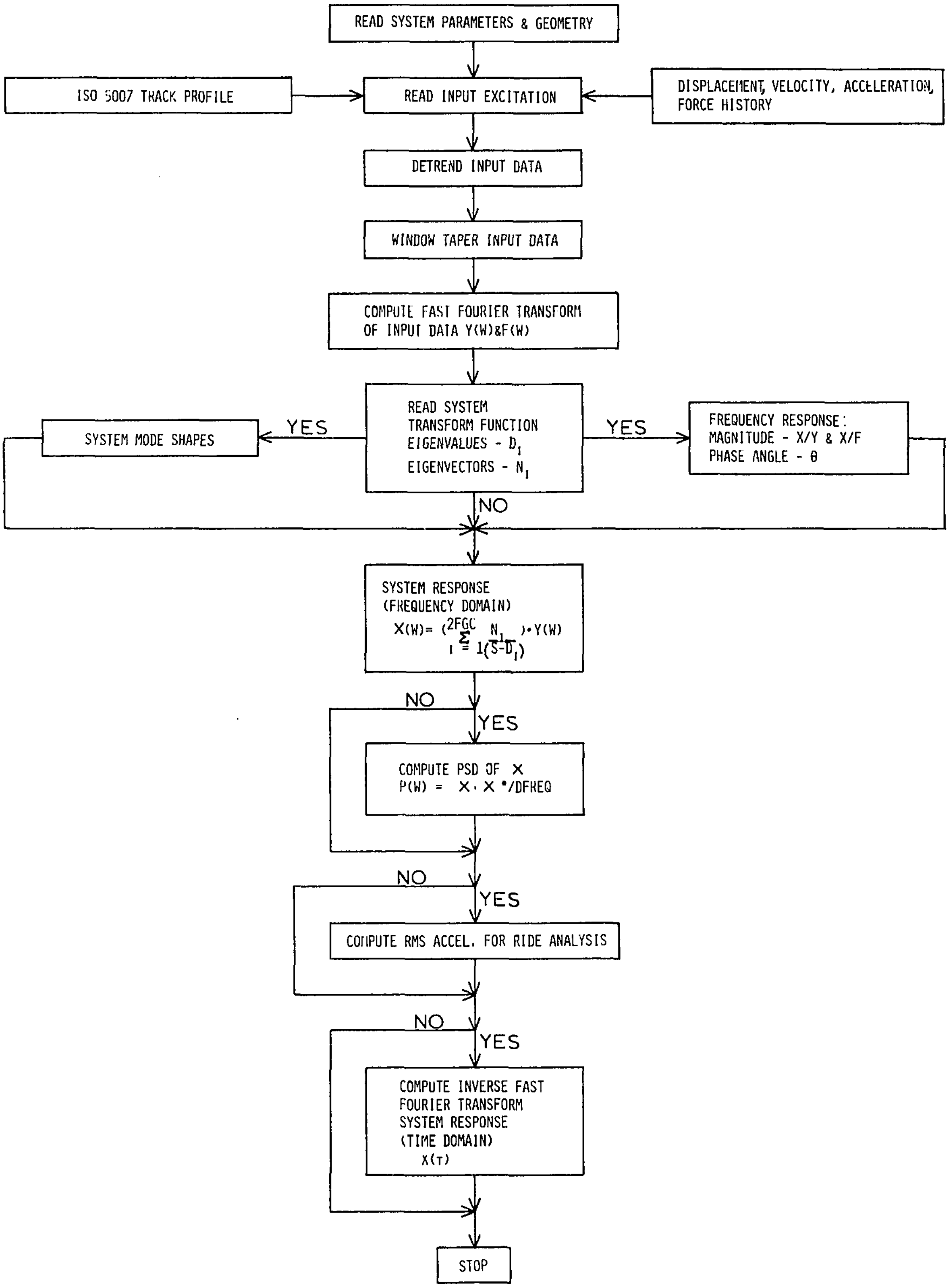


FIGURE 24. Flow process for the IMP post-processor program

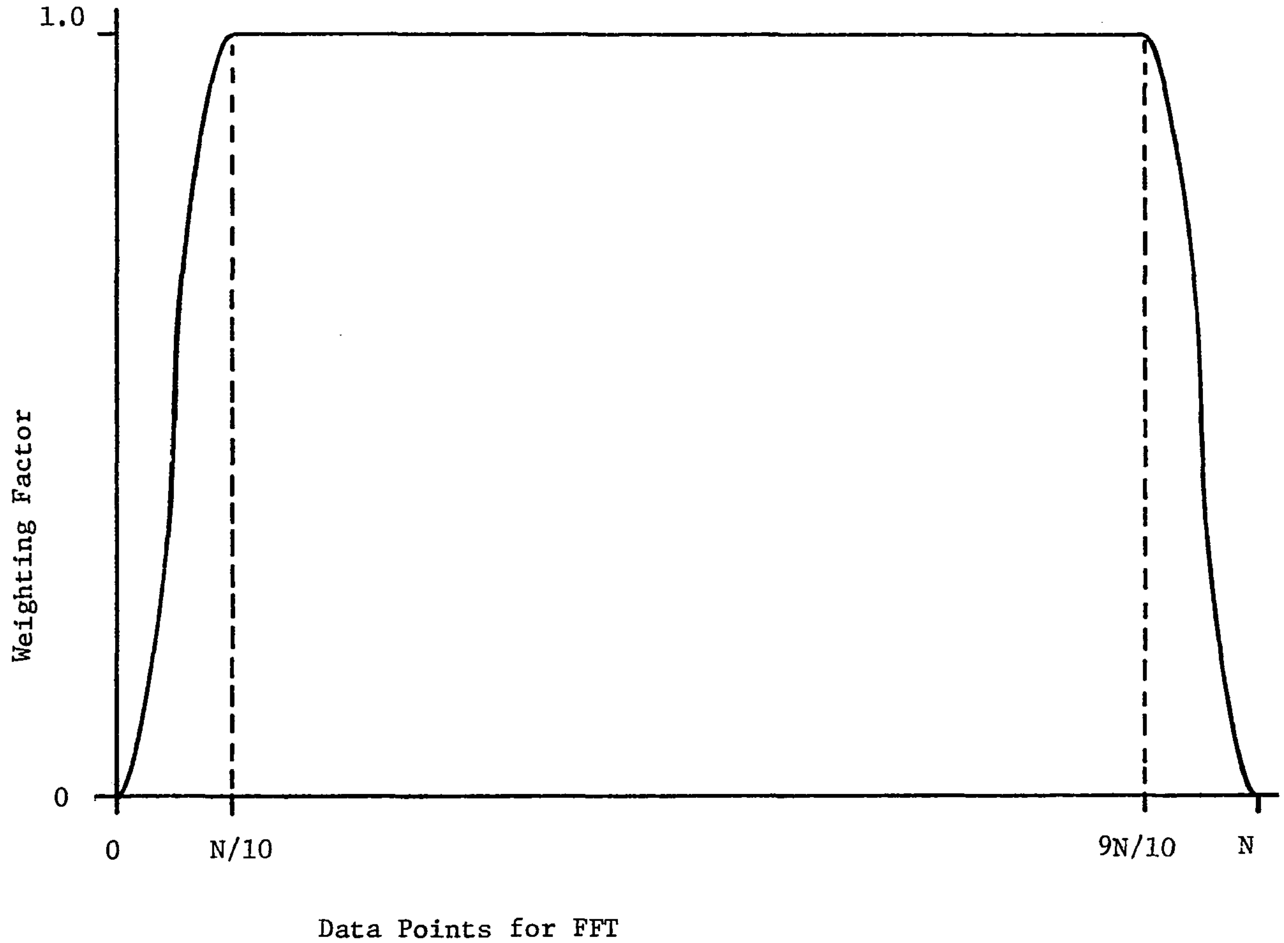


FIGURE 25. Cosine taper data window

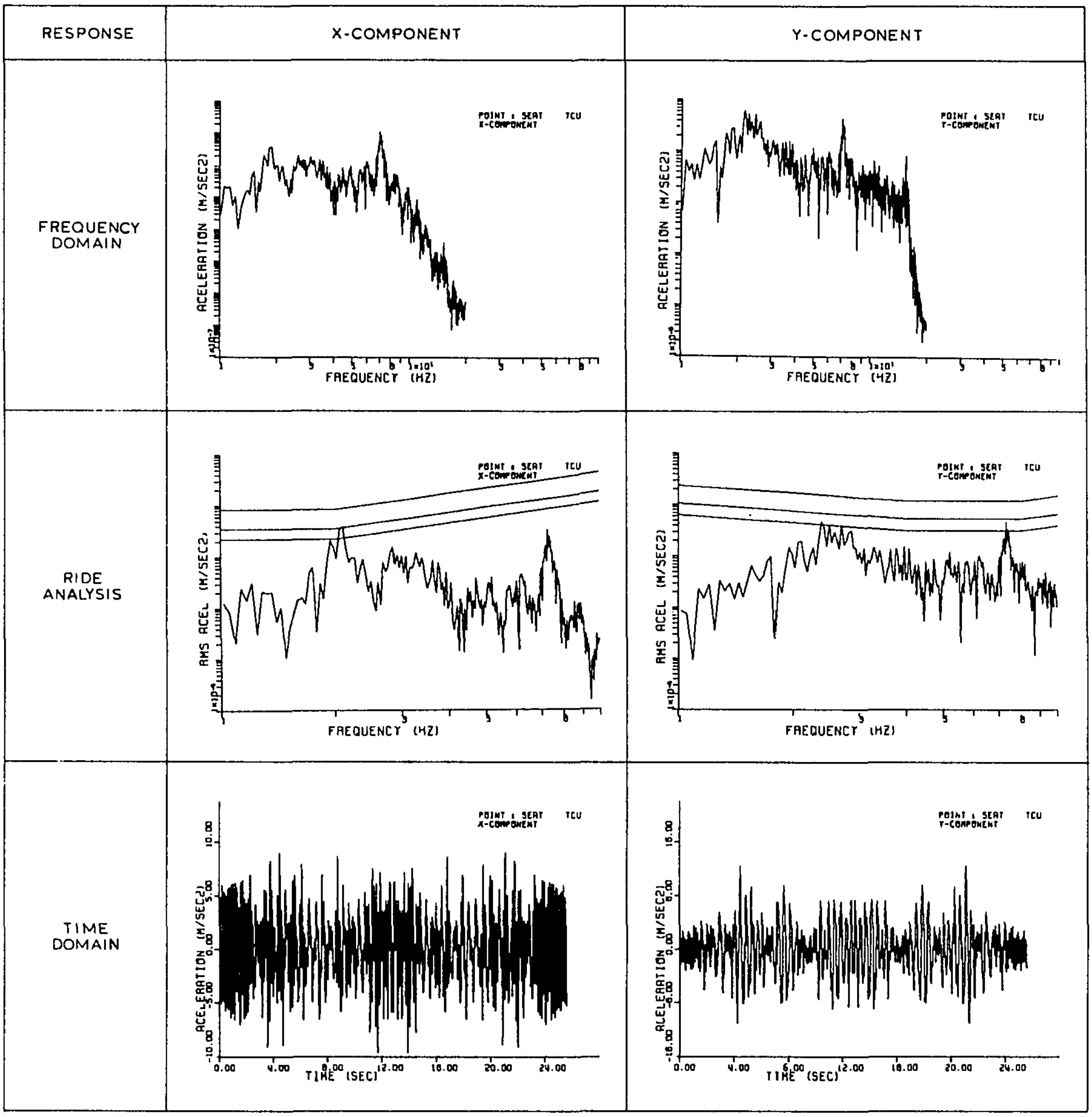


FIGURE 26. Acceleration response of the tractor seat point in the frequency- and time-domains

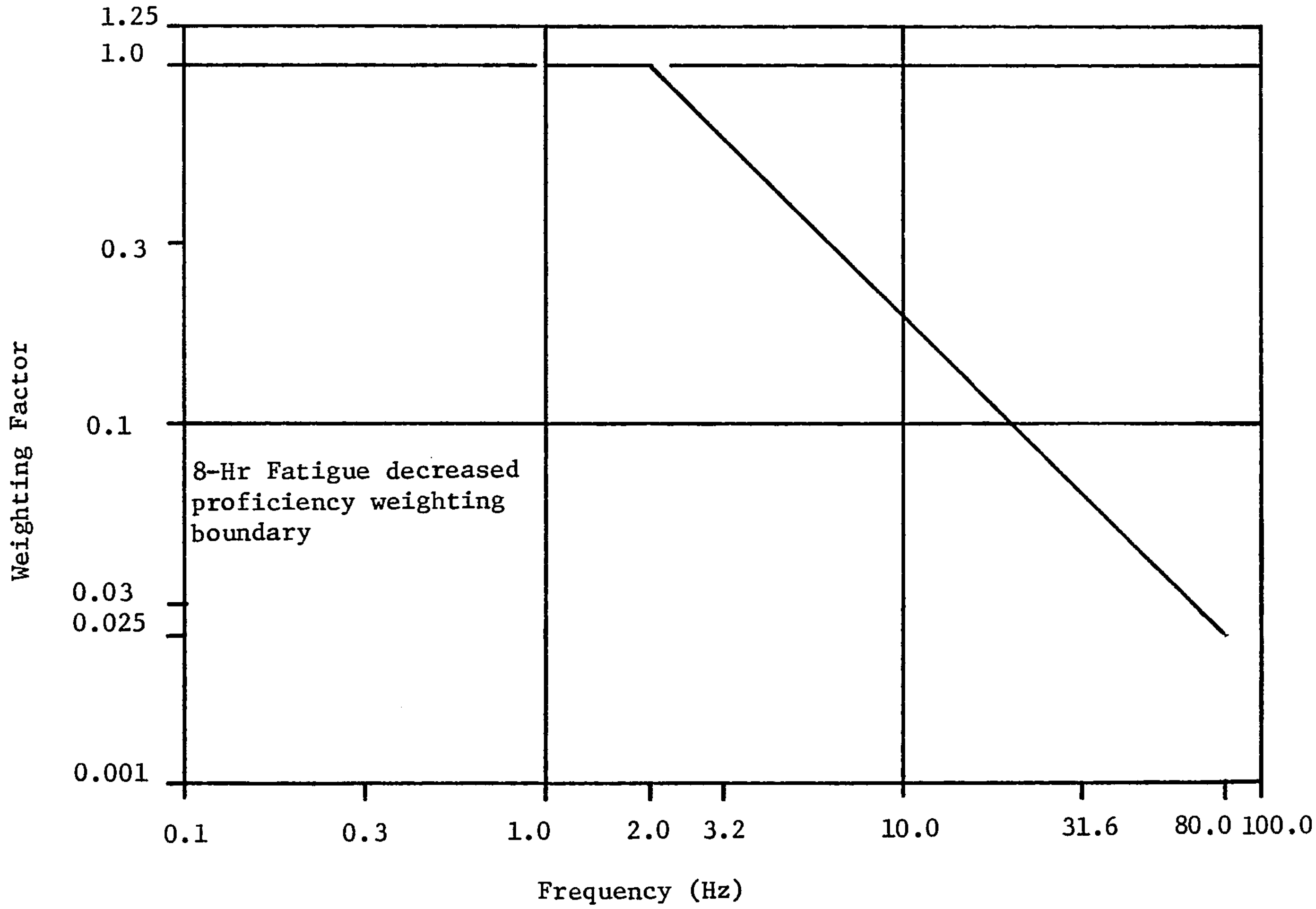


FIGURE 27. ISO frequency weighting for longitudinal and lateral accelerations

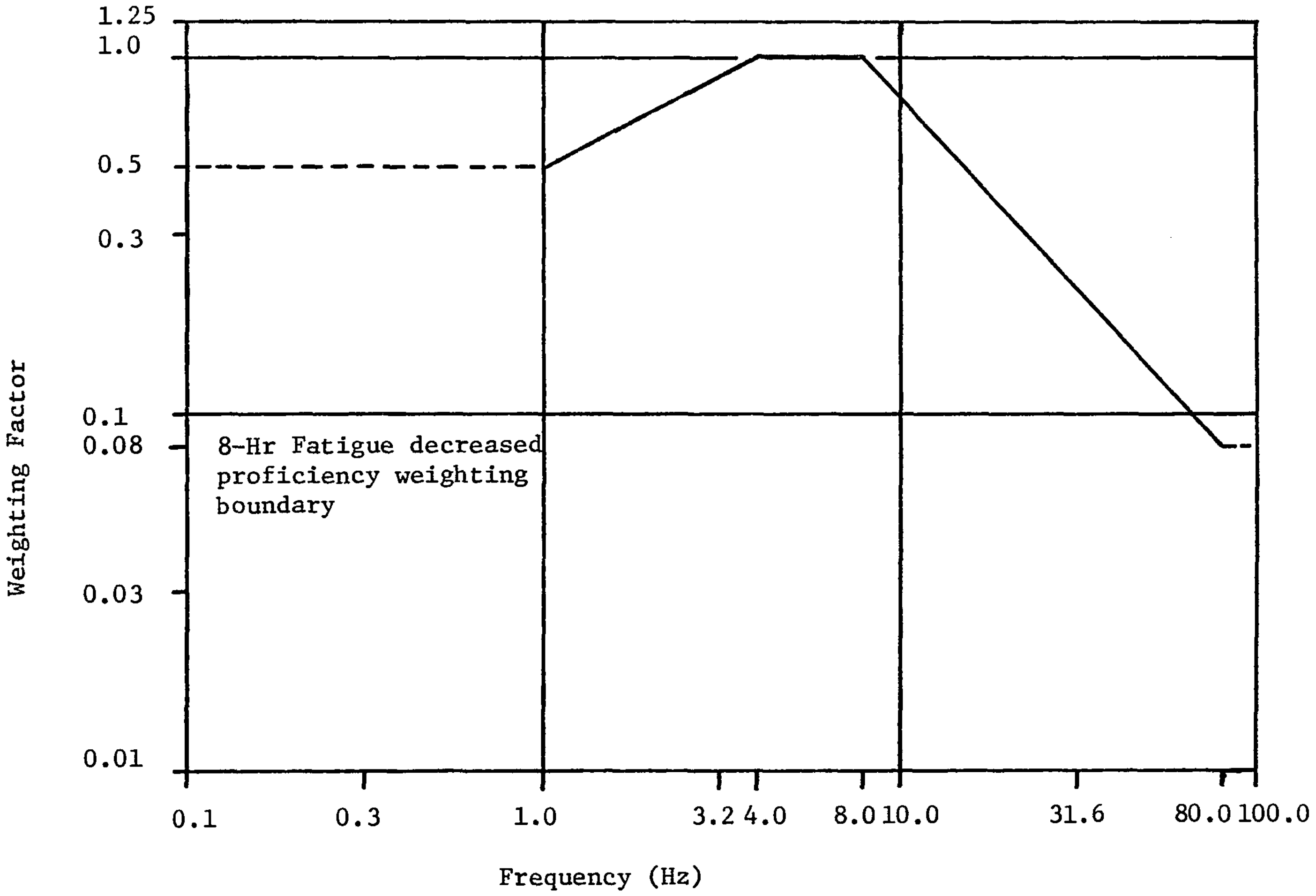


FIGURE 28. ISO frequency weighting for vertical accelerations

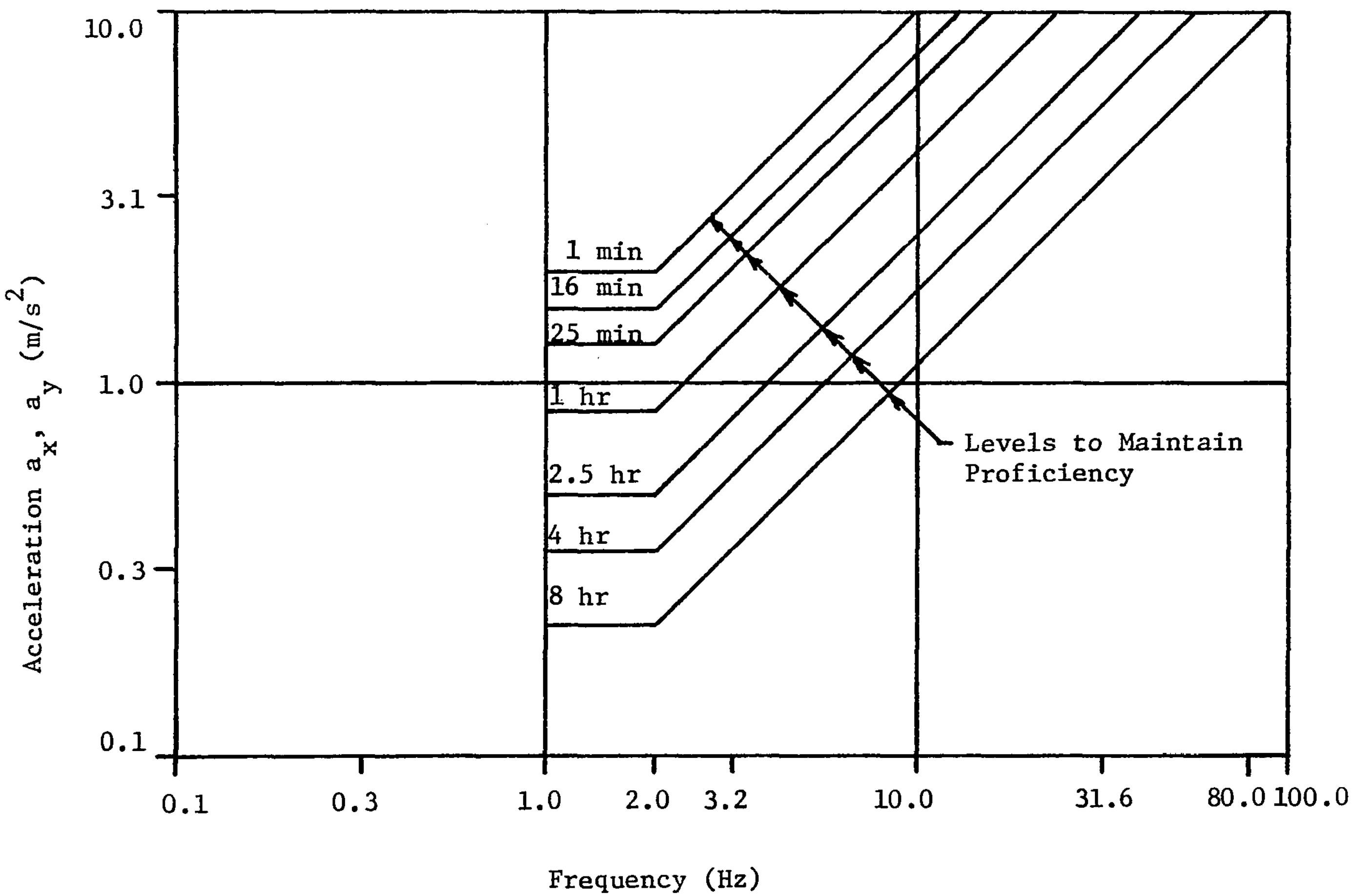


FIGURE 29. ISO endurance curves for longitudinal and lateral responses

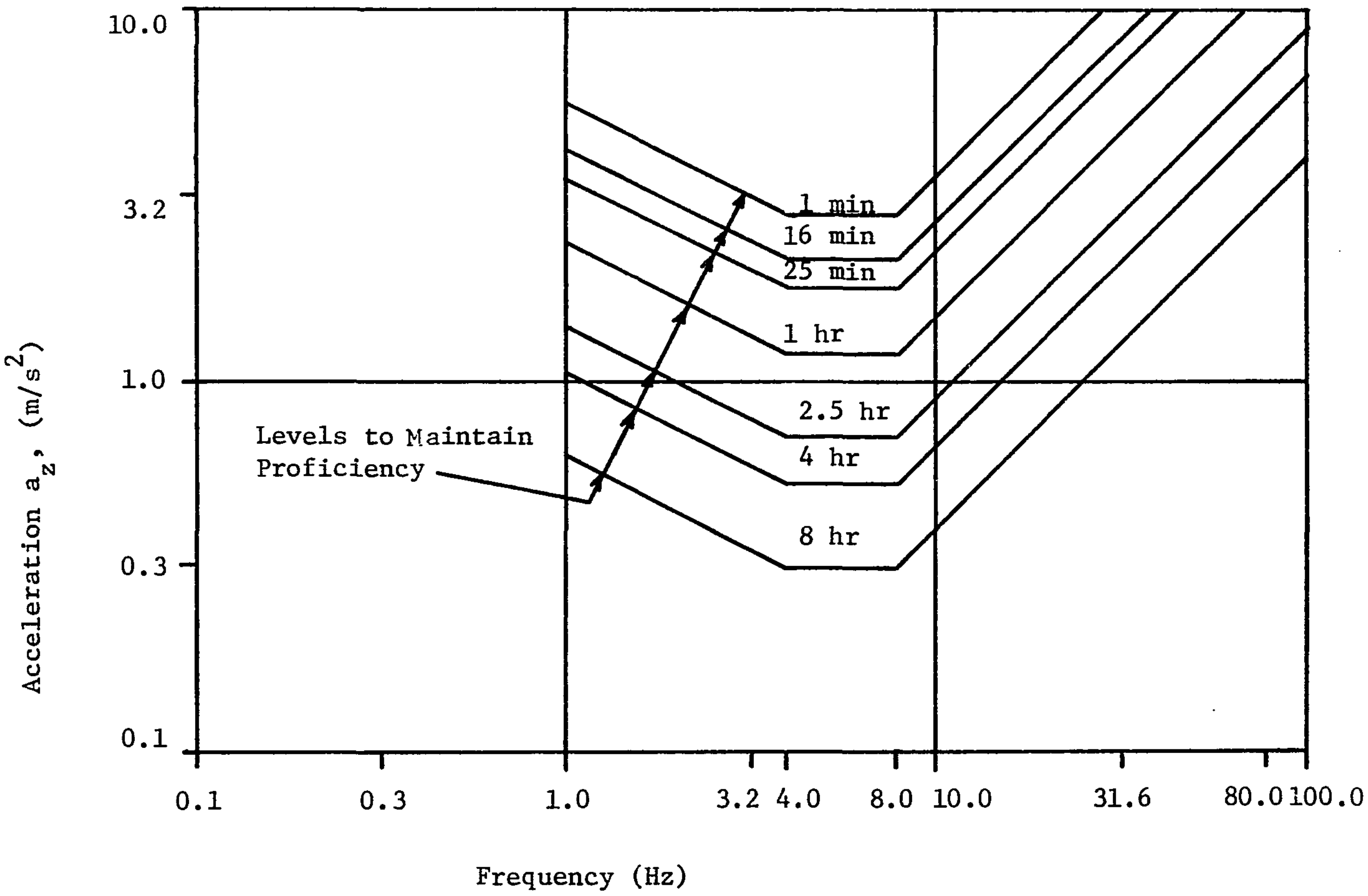


FIGURE 30. ISO endurance curves for vertical responses

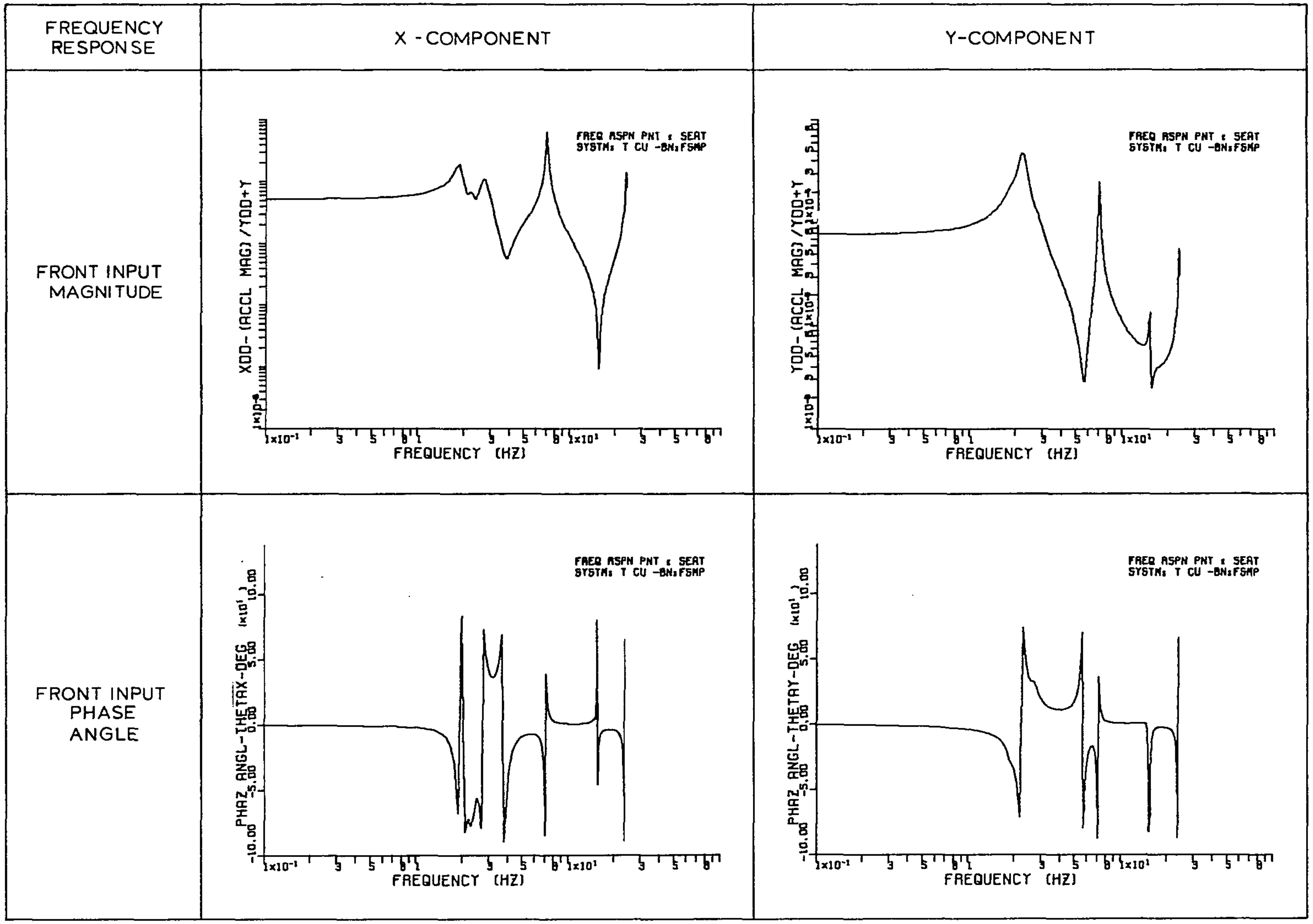


FIGURE 31. Frequency response for the tractor seat point with respect to the front wheel input excitation

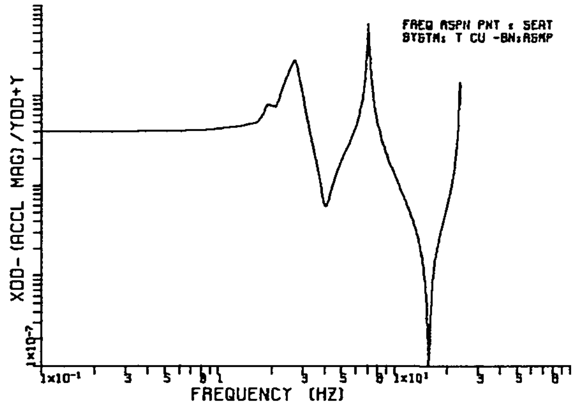
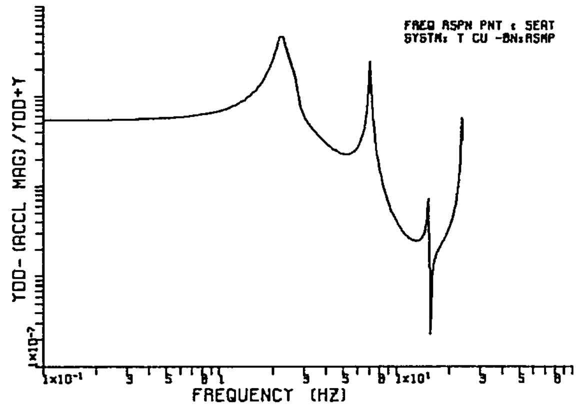
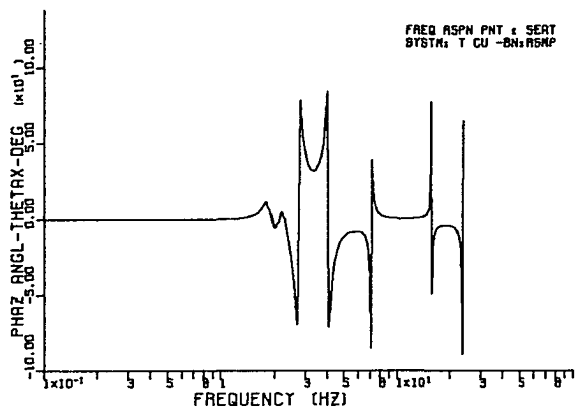
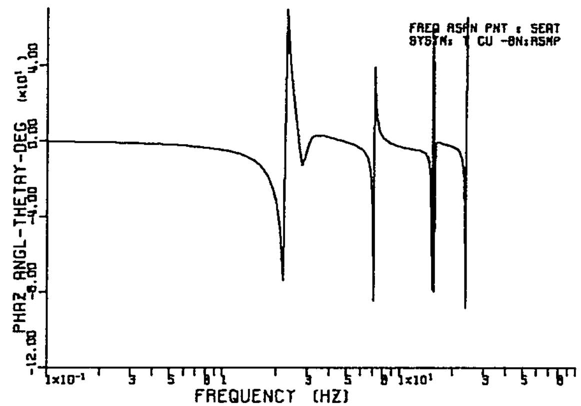
FREQUENCY RESPONSE	X-COMPONENT	Y-COMPONENT
REAR INPUT MAGNITUDE		
REAR INPUT PHASE ANGLE		

FIGURE 32. Frequency response for the tractor seat point with respect to the rear wheel input excitation

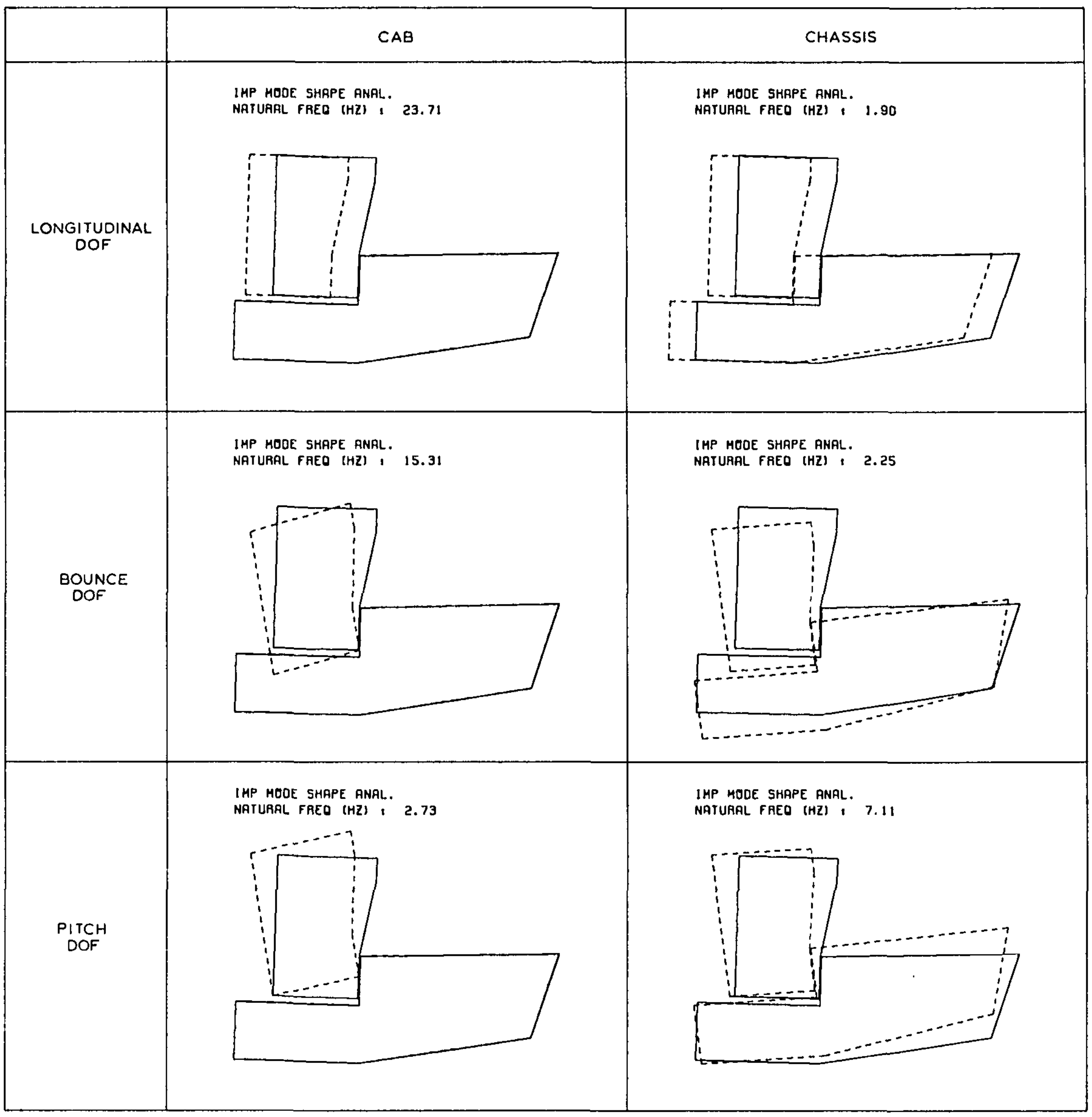


FIGURE 33. Modes of vibration for the conventional cab position unsprung tractor

	Tractor, T			Tractor-plow, S			Suspension Parameters
	Conventional cab, C	Mid-Chassis cab, M	Forward cab, F	Conventional cab, C	Mid-chassis cab, M	Forward Cab, F	
Unsprung	0.33 (TCU)	0.31 (TMU)	0.31 (TFU)	0.27 (SCU)	0.27 (SMU)	0.30 (SFU)	U
Sprung Front Axle Only	0.24 (TCF)	0.25 (TMF)	0.24 (TFF)	0.17 (SCF)	0.19 (SMF)	0.18 (SFF)	F
	0.47 (TCFD)	0.45 (TMFD)	0.47 (TFFD)	0.43 (SCFD)	0.44 (SMFD)	0.58 (SFFD)	FD
Both Axles Sprung	0.15 (TCB)	0.16 (TMB)	0.17 (TFB)	0.16 (SCB)	0.12 (SMB)	0.18 (SFB)	
	0.22 (TCBX)	0.24 (TMBX)	0.22 (TFBX)	0.16 (SCBX)	0.15 (SMBX)	0.16 (SFBX)	BX
	0.14 (TCBD)	0.16 (TMBD)	0.17 (TFBD)	0.45 (SCBD)	0.40 (SMBD)	0.67 (SFBD)	BD
	0.14 (TCBR)	0.17 (TMBR)	0.16 (TFBR)	0.44 (SCBR)	0.40 (SMBR)	0.66 (SFBR)	BR
	0.14 (TCBS)	0.16 (TMBS)	0.16 (TFBS)	0.44 (SCBS)	0.40 (SMBS)	0.66 (SFBS)	BS

RMS Acceleration Number (m/sec^2)

FIGURE 34. Longitudinal RMS acceleration ride numbers for the tractor and tractor-plow models

	Tractor, T			Tractor-plow, S			Suspension Parameters
	Conventional cab, C	Mid-Chassis cab, M	Forward cab, F	Conventional cab, C	Mid-Chassis cab, M	Forward cab, F	
Unsprung	0.72 (TCU)	0.78 (TMU)	1.00 (TFU)	0.58 (SCU)	0.62 (SMU)	0.82 (SFU)	U
Sprung Front Axle Only	0.61 (TCF)	0.66 (TMF)	0.63 (TFF)	0.53 (SCF)	0.53 (SMF)	0.44 (SFF)	F
	4.41 (TCFD)	3.68 (TMFD)	2.96 (TFFD)	4.34 (SCFD)	3.59 (SMFD)	2.92 (SFFD)	FD
Both Axles Sprung	0.33 (TCB)	0.41 (TMB)	0.61 (TFB)	0.26 (SCB)	0.27 (SMB)	0.45 (SFB)	
	0.79 (TCBX)	0.69 (TMBX)	0.81 (TFBX)	0.53 (SCBX)	0.37 (SMBX)	0.47 (SFBX)	BX
	4.39 (TCBD)	4.31 (TMBD)	4.44 (TFBD)	1.68 (SCBD)	1.19 (SMBD)	1.99 (SFBD)	BD
	4.18 (TCBR)	4.31 (TMBR)	4.22 (TFBR)	1.69 (SCBR)	1.19 (SMBR)	2.00 (SFBR)	BR
	4.18 (TCBS)	4.30 (TMBS)	4.38 (TFBS)	1.67 (SCBS)	1.19 (SMBS)	2.00 (SFBS)	BS

RMS Acceleration Number (m/sec^2)

FIGURE 35. Vertical RMS acceleration ride numbers for the tractor and tractor-plow models

		Tractor-Trailer (Single Axle), I			Tractor-Trailer (Tandem Axles), J			Suspension Parameters
		Conventional Cab, C	Mid-Chassis Cab, M	Forward Cab, F	Conventional Cab, C	Mid-Chassis Cab, M	Forward Cab, F	
Travel Speed 12.5 km/h	Unsprung	0.180 (ICU)	0.210 (IMU)	0.190 (IFU)	0.138 (JCU)	0.093 (JMU)	0.159 (JFU)	U
	Sprung Front Axle Only	0.134 (ICF)	0.103 (IMF)	0.001 (IFF)	0.115 (JCF)	0.120 (JMF)	0.120 (JFF)	F
	Both Axles Sprung	0.339 (ICB)	0.377 (IMB)	0.016 (IFB)	0.006 (JCB)	0.157 (JMB)	0.128 (JFB)	B
Travel Speed 25.0 km/h	Unsprung Axles	0.258 (ICU)	0.313 (IMU)	0.279 (IFU)	0.271 (JCU)	0.160 (JMU)	0.293 (JFU)	U
	Sprung Front Axle Only	0.194 (ICF)	0.154 (IMF)	0.002 (IFF)	0.200 (JCF)	0.213 (JMF)	0.218 (JFF)	F
	Both Axles Sprung	0.493 (ICB)	0.541 (IMB)	0.023 (IFB)	0.012 (JCB)	0.278 (JMB)	0.229 (JFB)	B

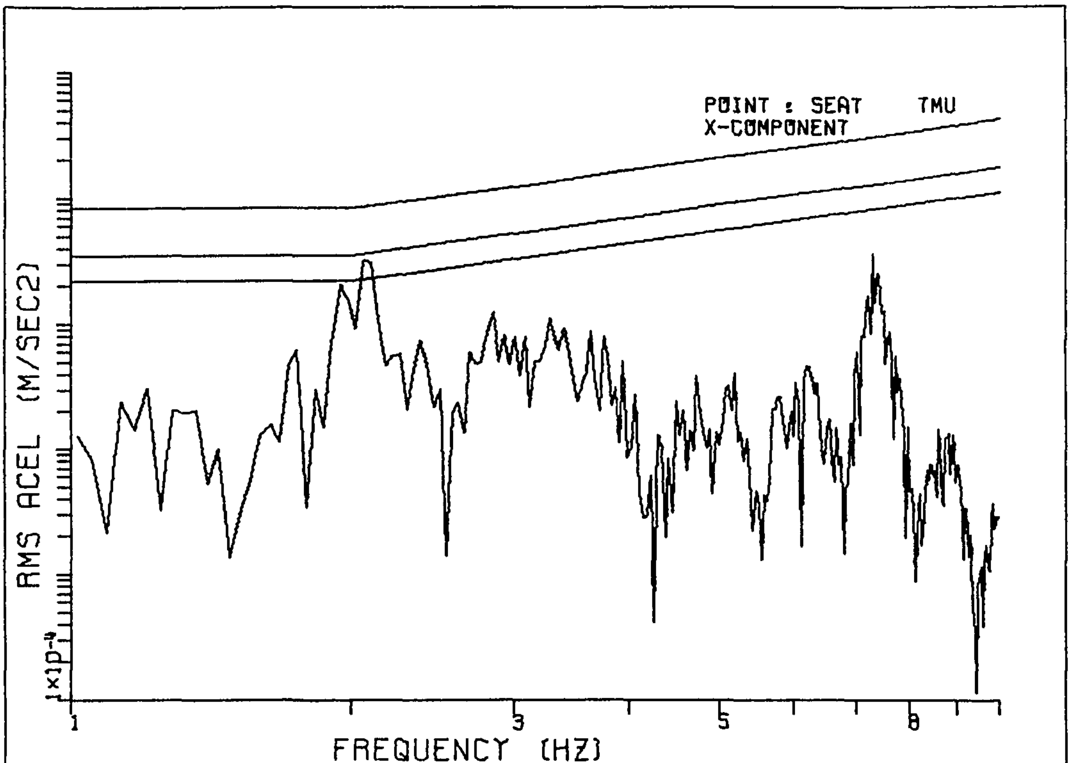
RMS Acceleration Number (m/s^2)

FIGURE 36. Longitudinal RMS acceleration ride numbers for the tractor-trailer models

		Tractor-Trailer (Single Axle), I			Tractor-Trailer (Tandem Axles), J				
		Conventional Cab, C	Mid-Chassis Cab, M	Forward Cab, F	Conventional Cab, C	Mid-Chassis Cab, M	Forward Cab, F		
Travel Speed 12.5 km/h	Unsprung	0.799 (ICU)	0.796 (IMU)	0.796 (IFU)	0.805 (JCU)	0.806 (JMU)	0.806 (JFU)	U	Suspension Parameters
	Sprung Front Axle Only	0.794 (ICF)	0.792 (IMF)	0.795 (IFF)	0.806 (JCF)	0.806 (JMF)	0.806 (JFF)	F	
	Both Axles Sprung	1.299 (ICB)	0.957 (IMB)	0.799 (IFB)	0.806 (JCB)	0.807 (JMB)	0.813 (JFB)	B	
Travel Speed 25.0 km/h	Unsprung Axles	2.263 (ICU)	2.258 (IMU)	2.255 (IFU)	2.260 (JCU)	2.257 (JMU)	2.261 (JFU)	U	
	Sprung Front Axle Only	2.258 (ICF)	2.258 (IMF)	2.258 (IFF)	2.261 (JCF)	2.261 (JMF)	2.261 (JFF)	F	
	Both Axles Sprung	3.000 (ICB)	2.701 (IMB)	2.271 (IFB)	2.261 (JCB)	2.262 (JMB)	2.285 (JFB)	B	

RMS Acceleration Number (m/s^2)

FIGURE 37. Vertical RMS acceleration ride numbers for the tractor-trailer models



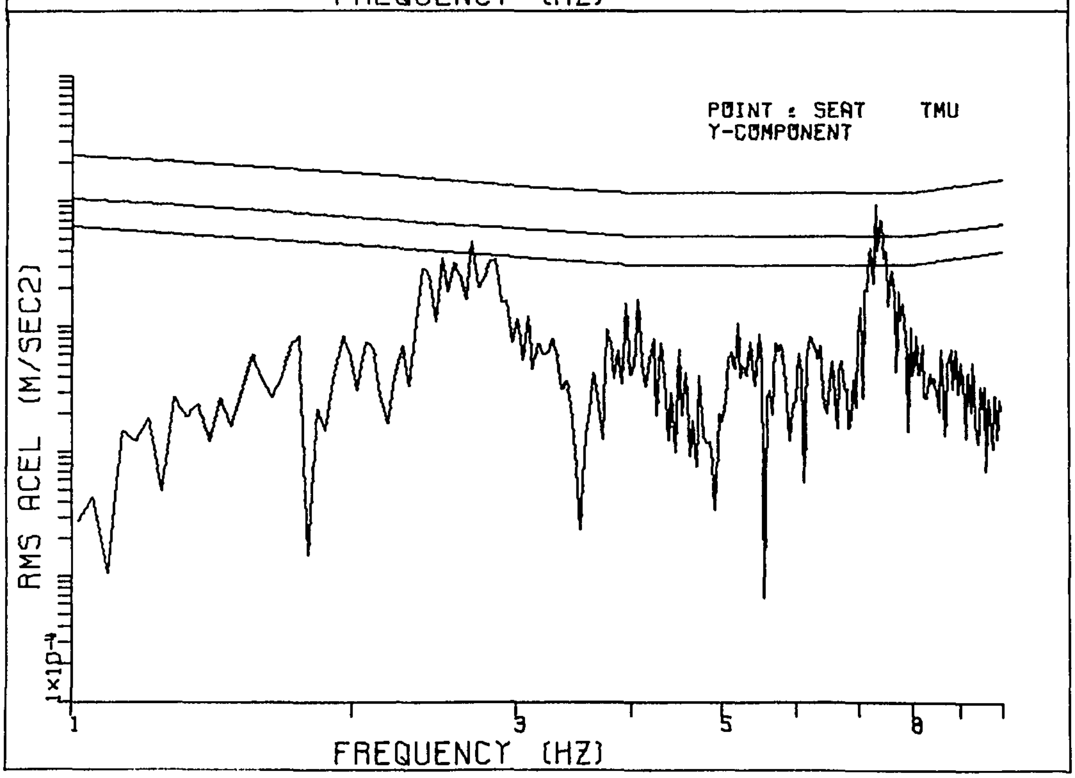


FIGURE 38. RMS acceleration responses for the unsuspended midchassis cab tractor model

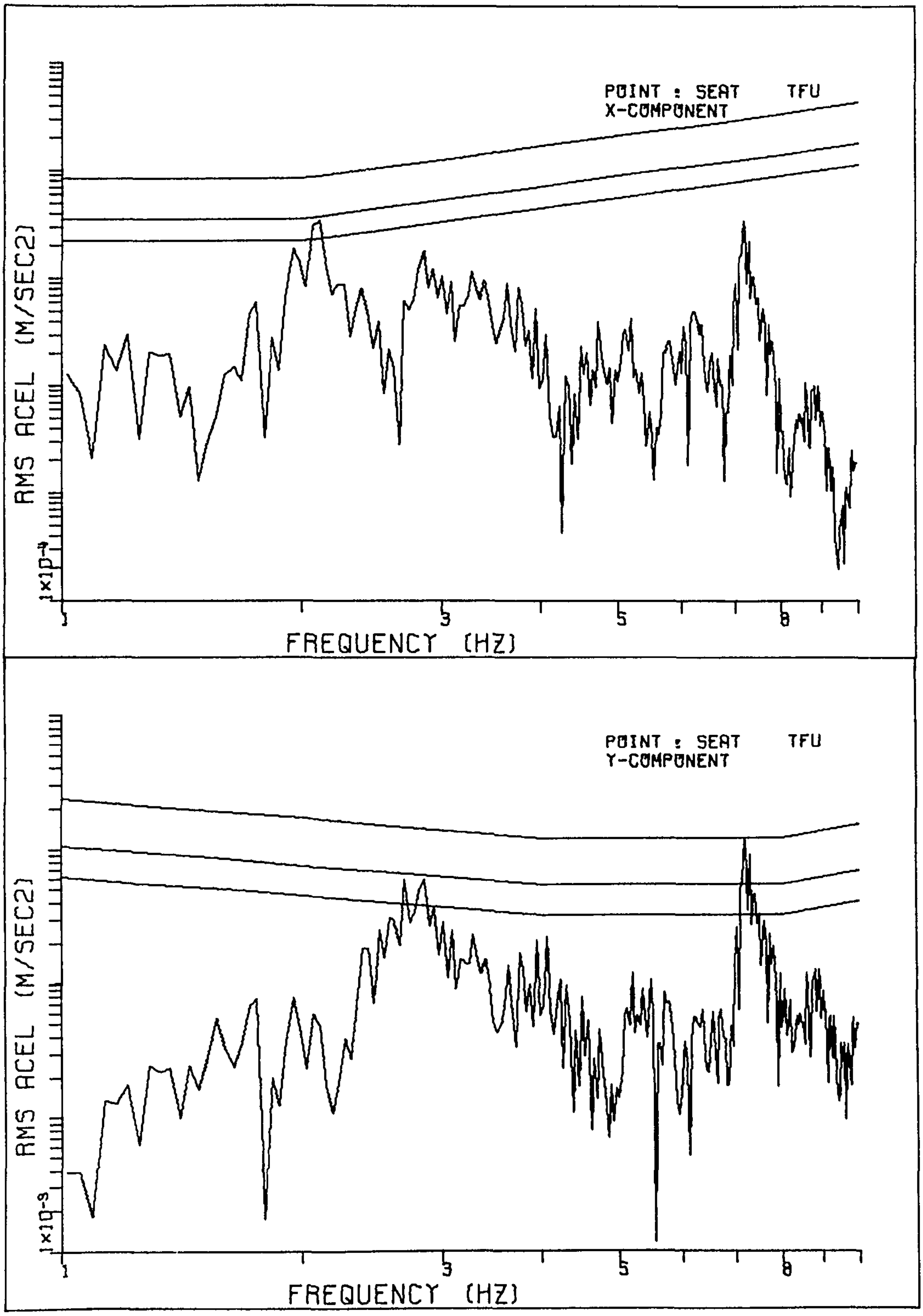


FIGURE 39. RMS acceleration responses of the unsuspended forward cab tractor model

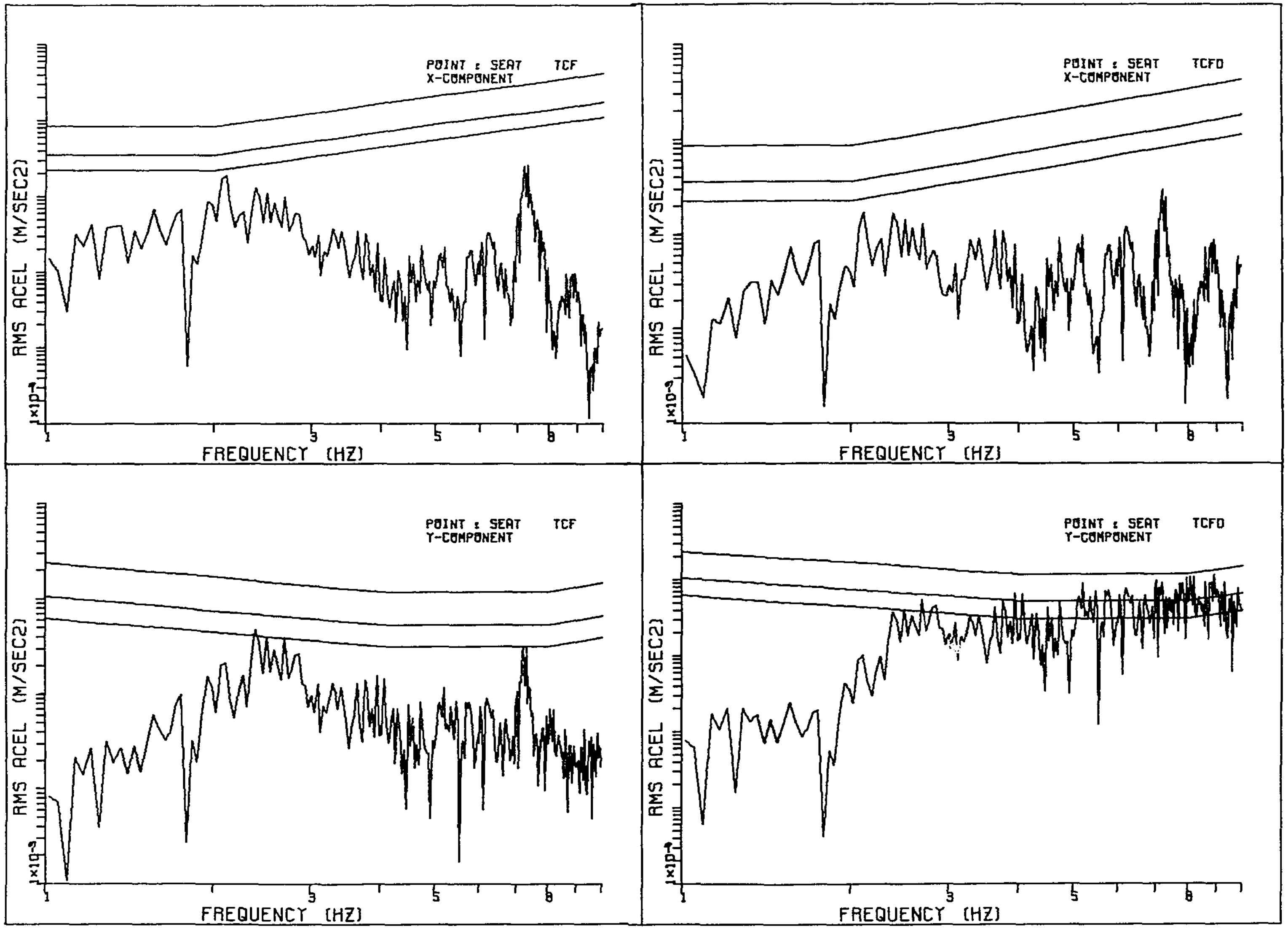


FIGURE 40. RMS acceleration responses of the suspended front axle conventional tractor models

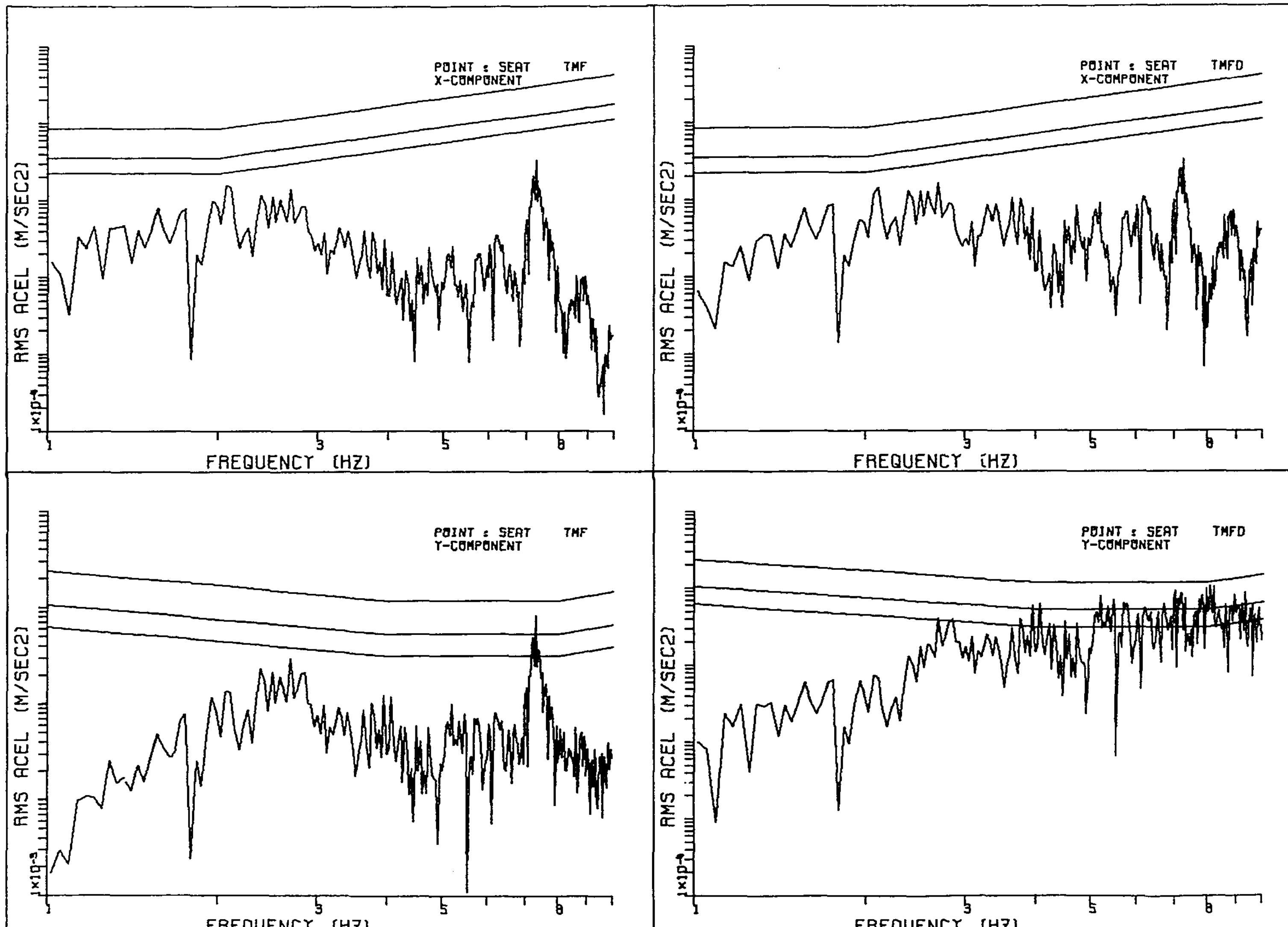


FIGURE 41. RMS acceleration responses of the suspended front axle midchassis cab tractor models

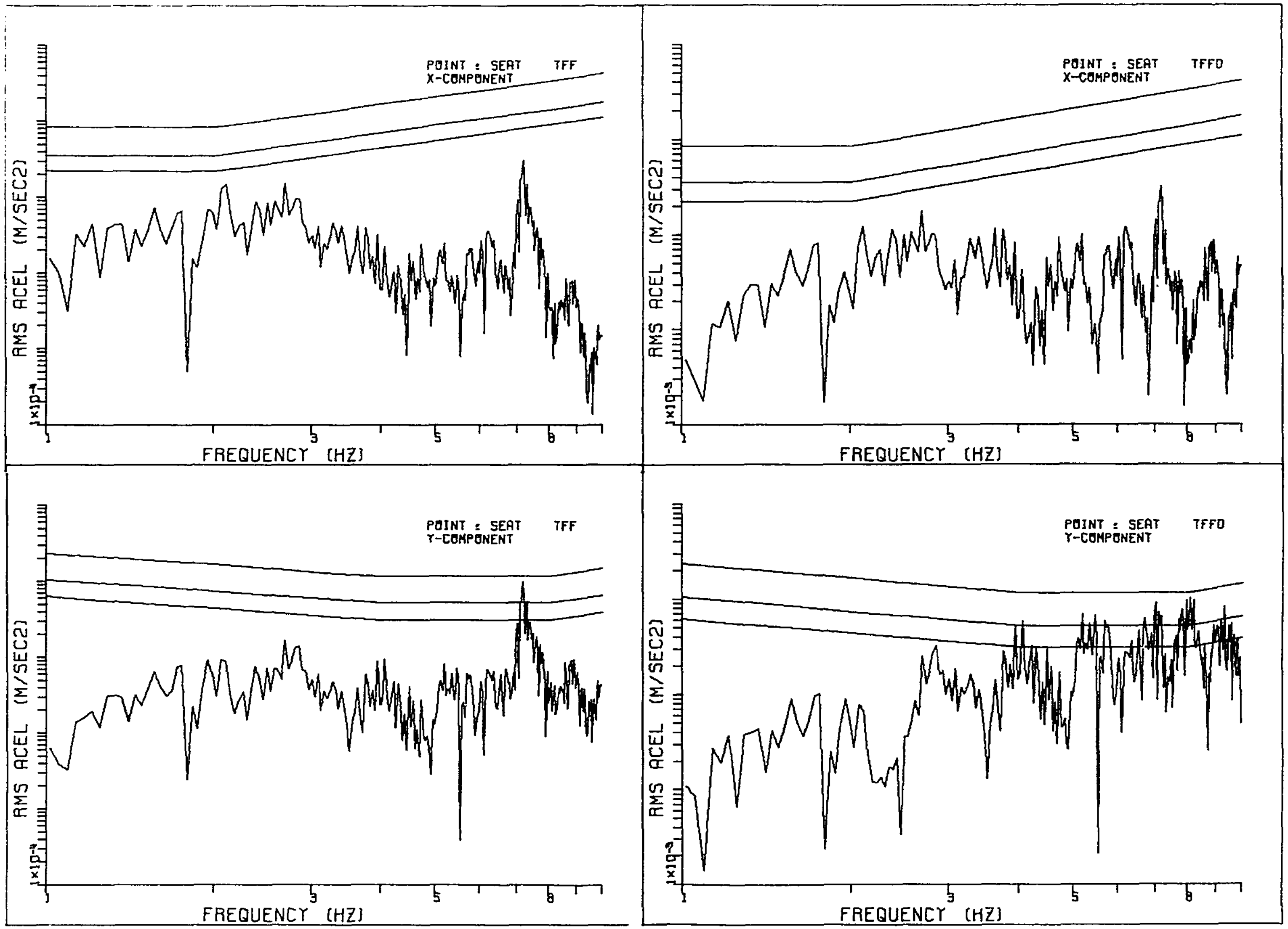


FIGURE 42. RMS acceleration responses of the suspended front axle forward cab tractor models

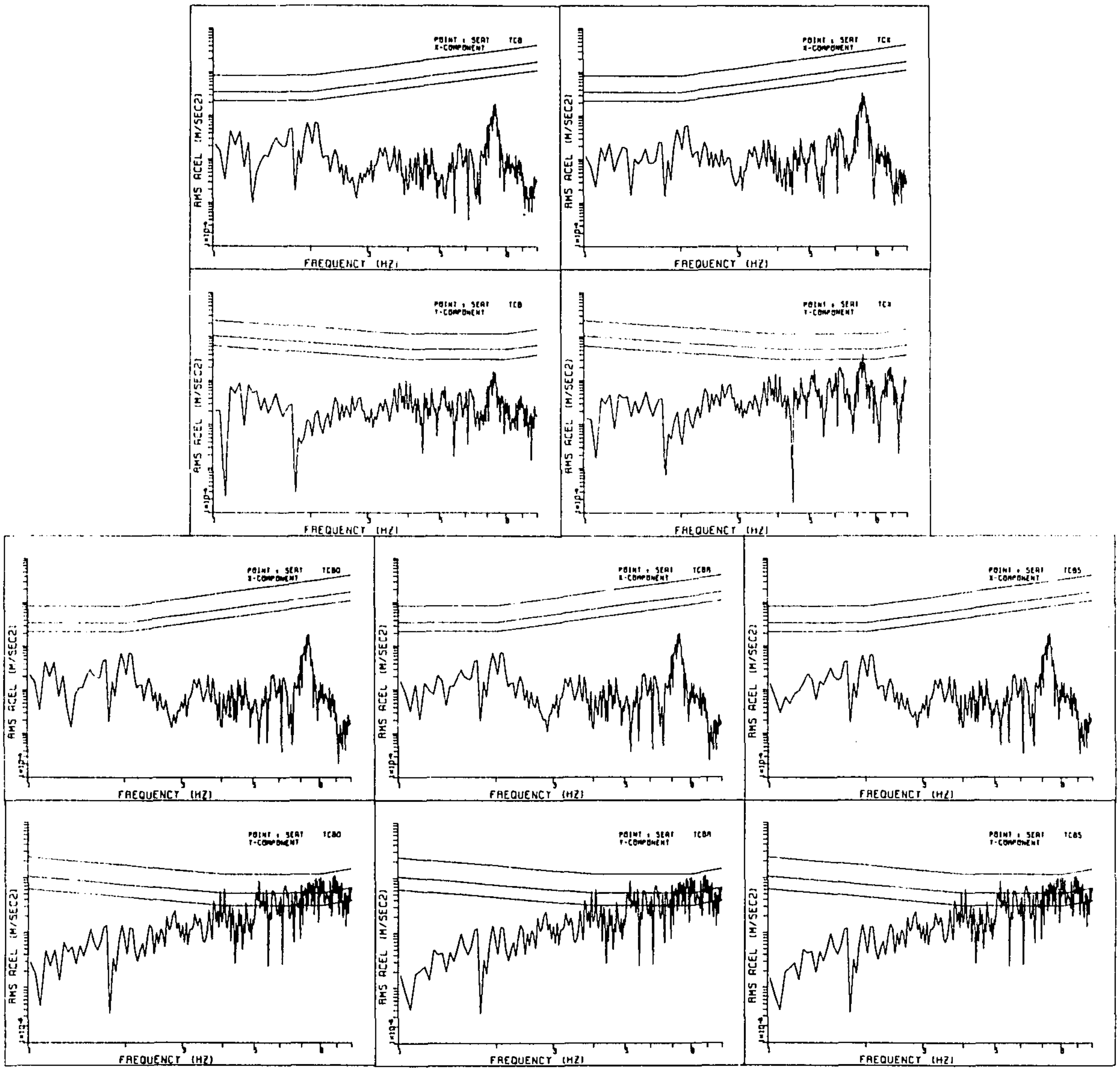


FIGURE 43. RMS acceleration responses of the fully suspended conventional tractor models

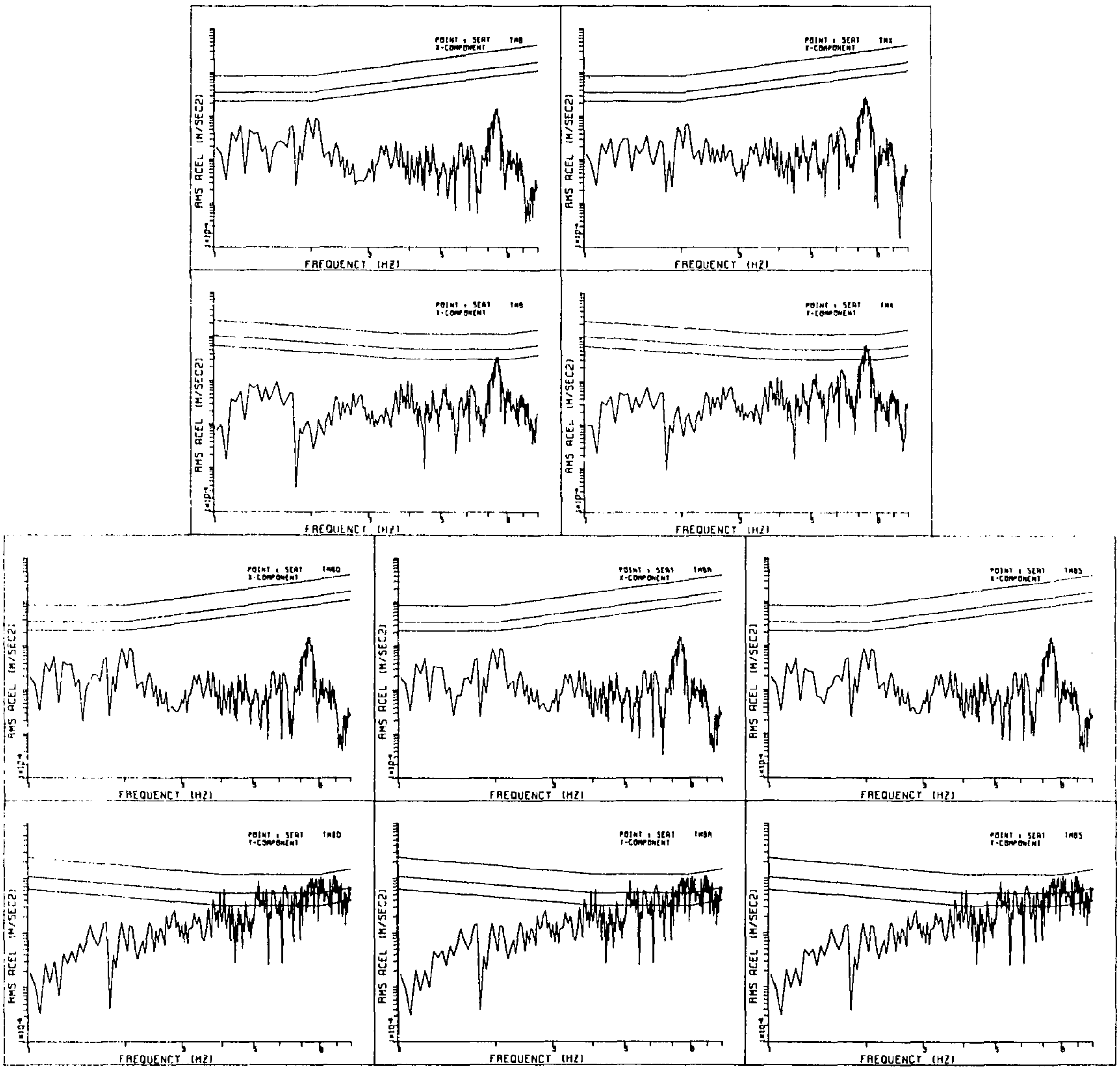


FIGURE 44. RMS acceleration responses of the fully suspended midchassis cab tractor models

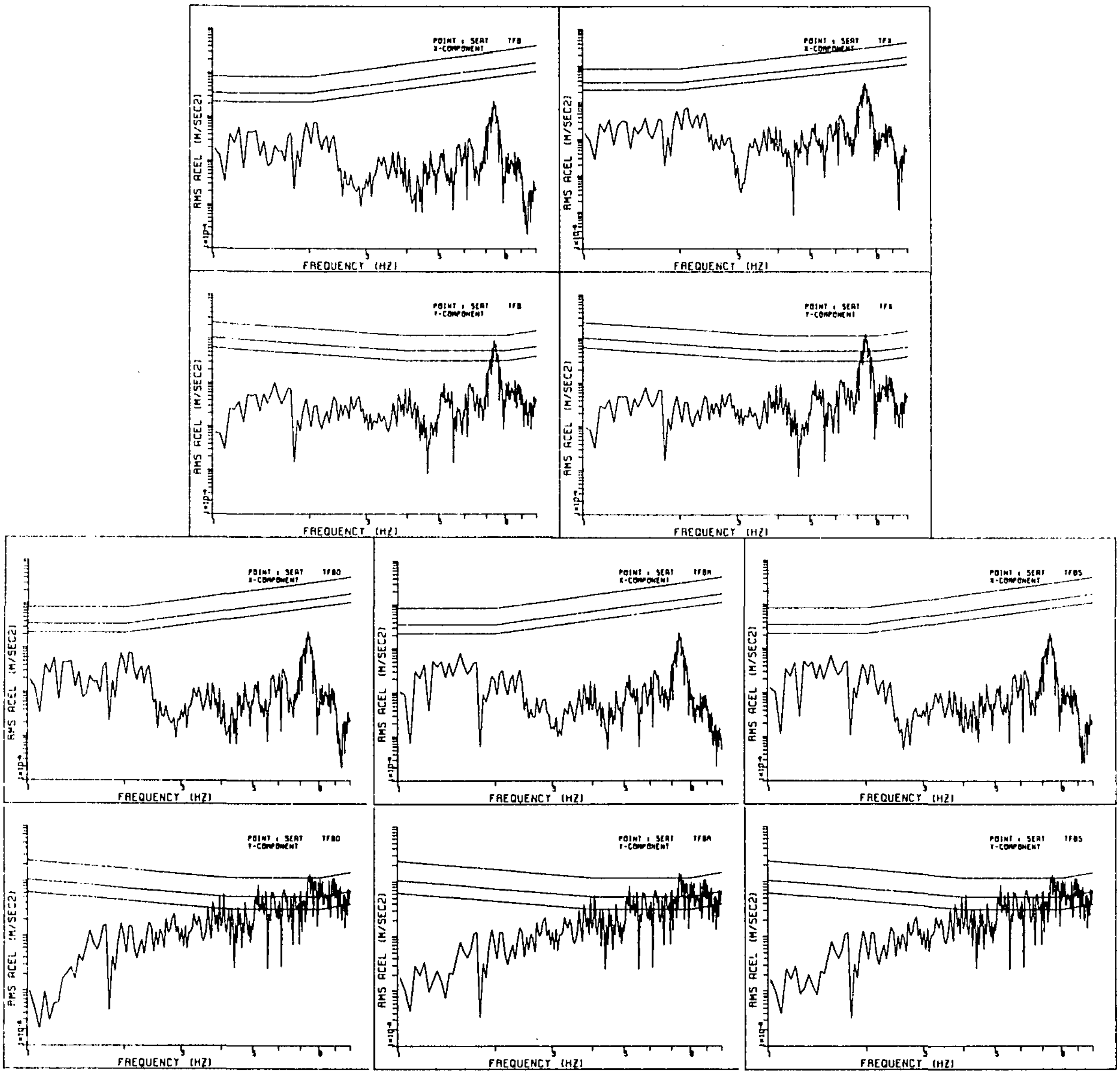


FIGURE 45. RMS acceleration responses of the fully suspended forward cab tractor models

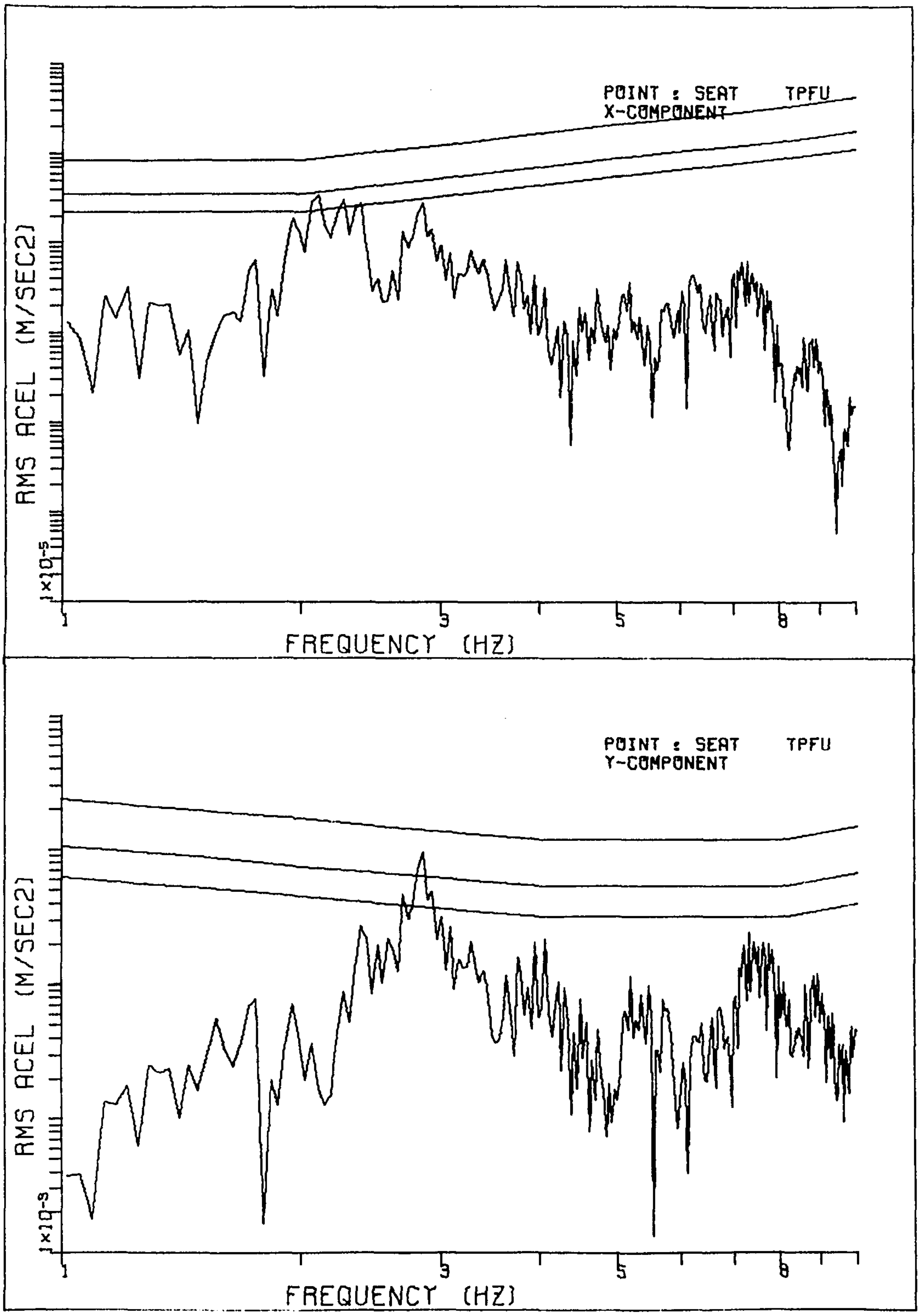


FIGURE 46. RMS acceleration responses of the unsuspended tractor-plow model

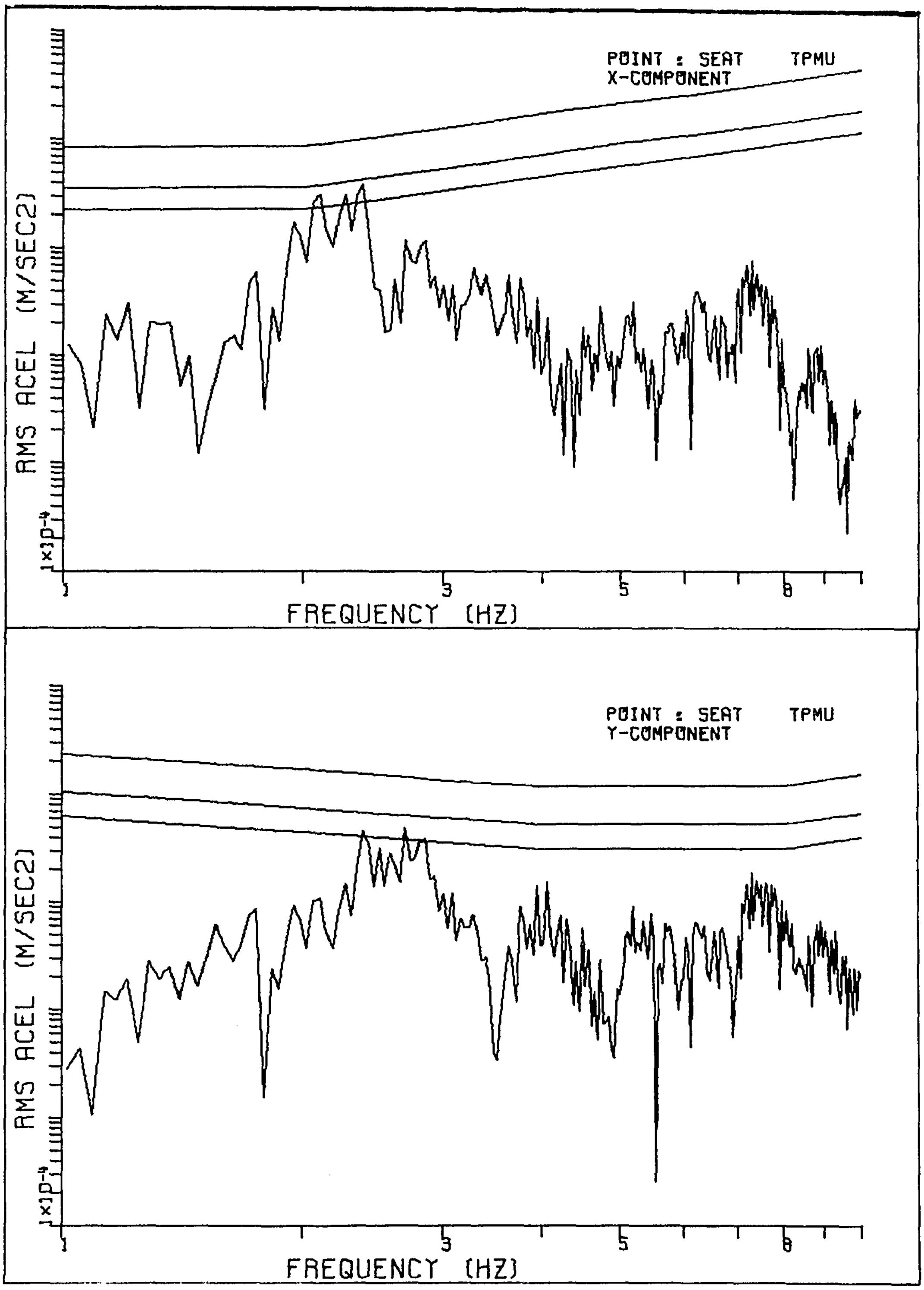


FIGURE 47. RMS acceleration responses of the unsuspended midchassis cab tractor-plow model

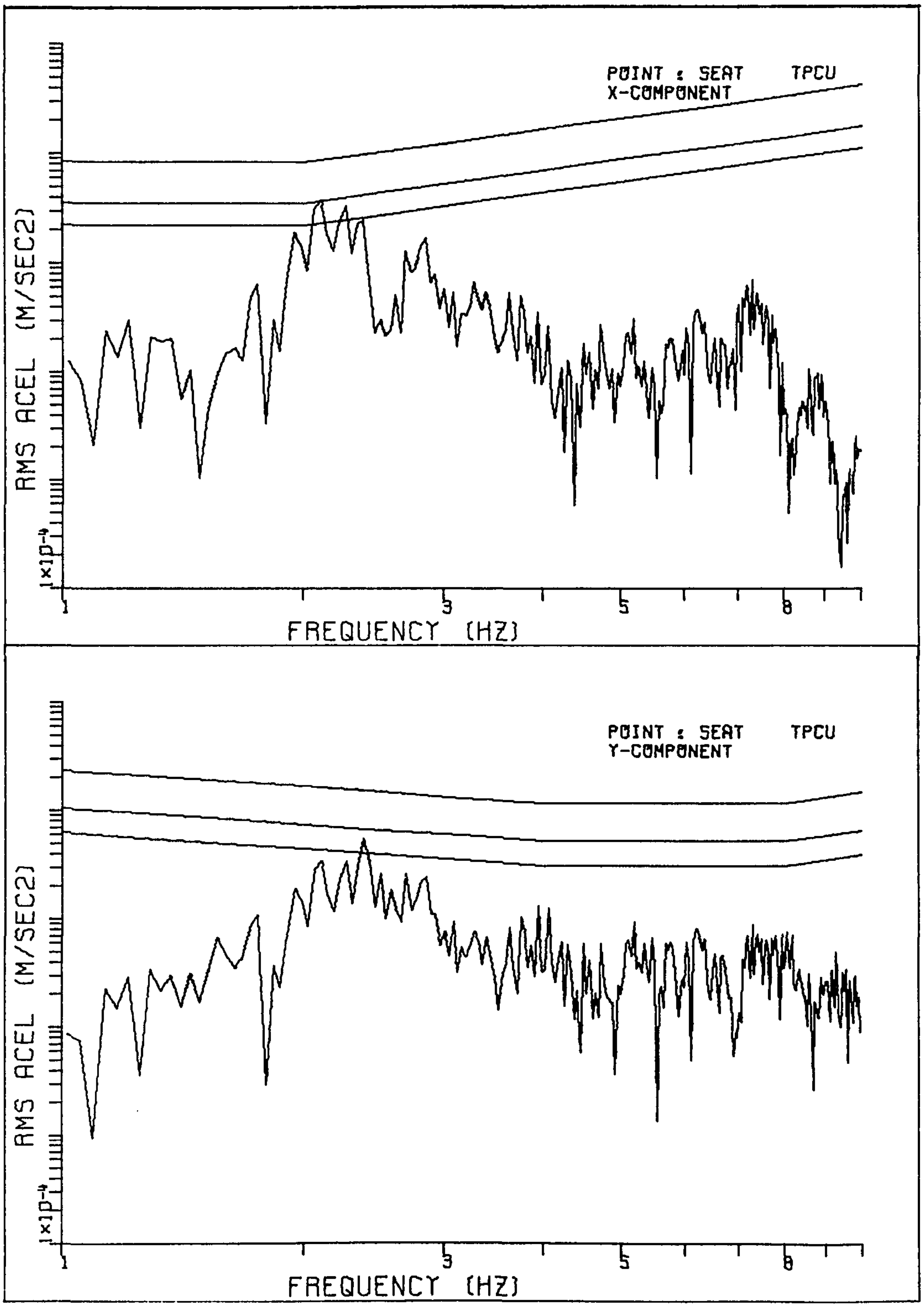


FIGURE 48. RMS acceleration responses of the unsuspended forward cab tractor-plow model

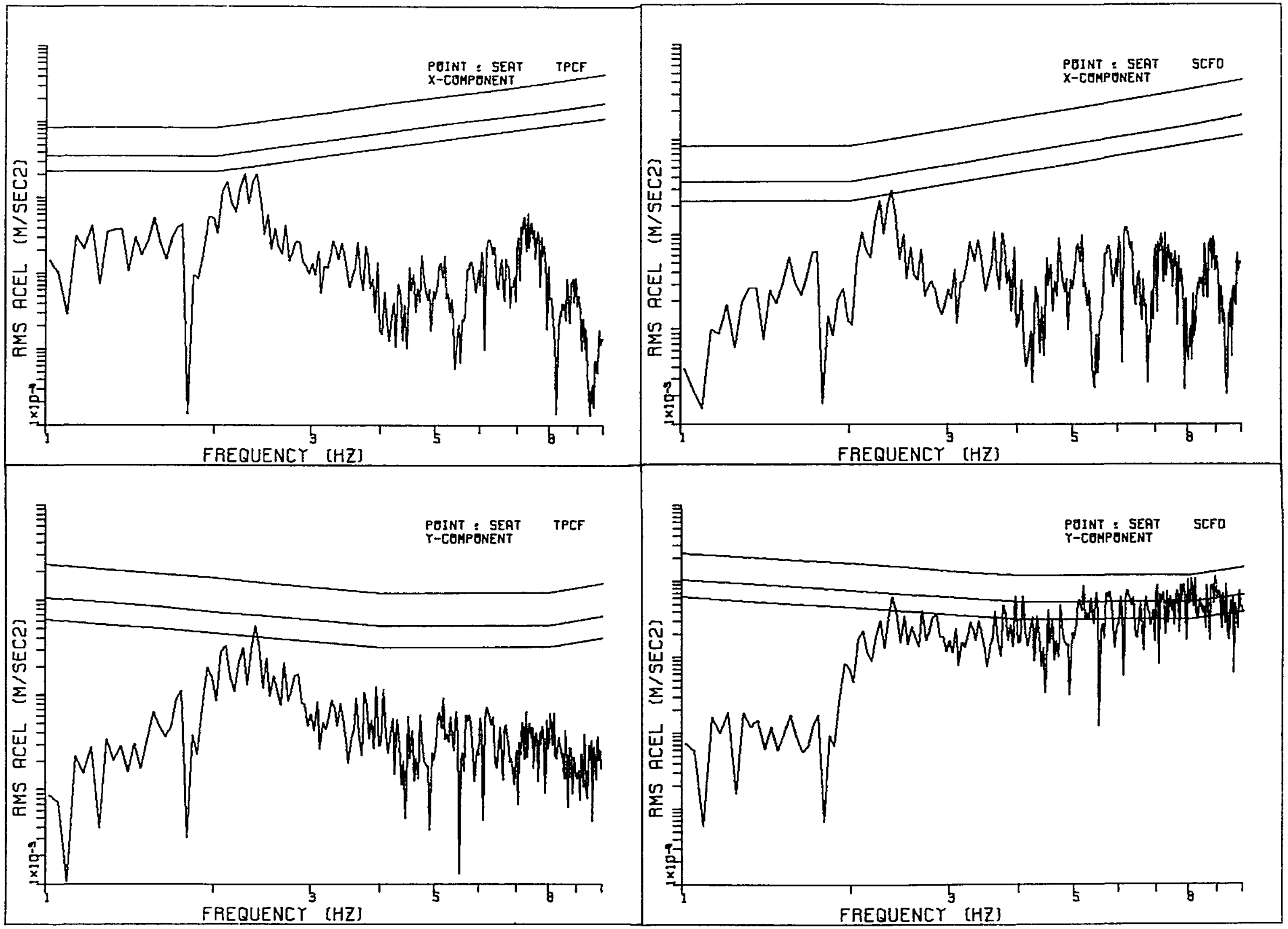


FIGURE 49. RMS acceleration responses of the suspended front axle conventional tractor-plow models

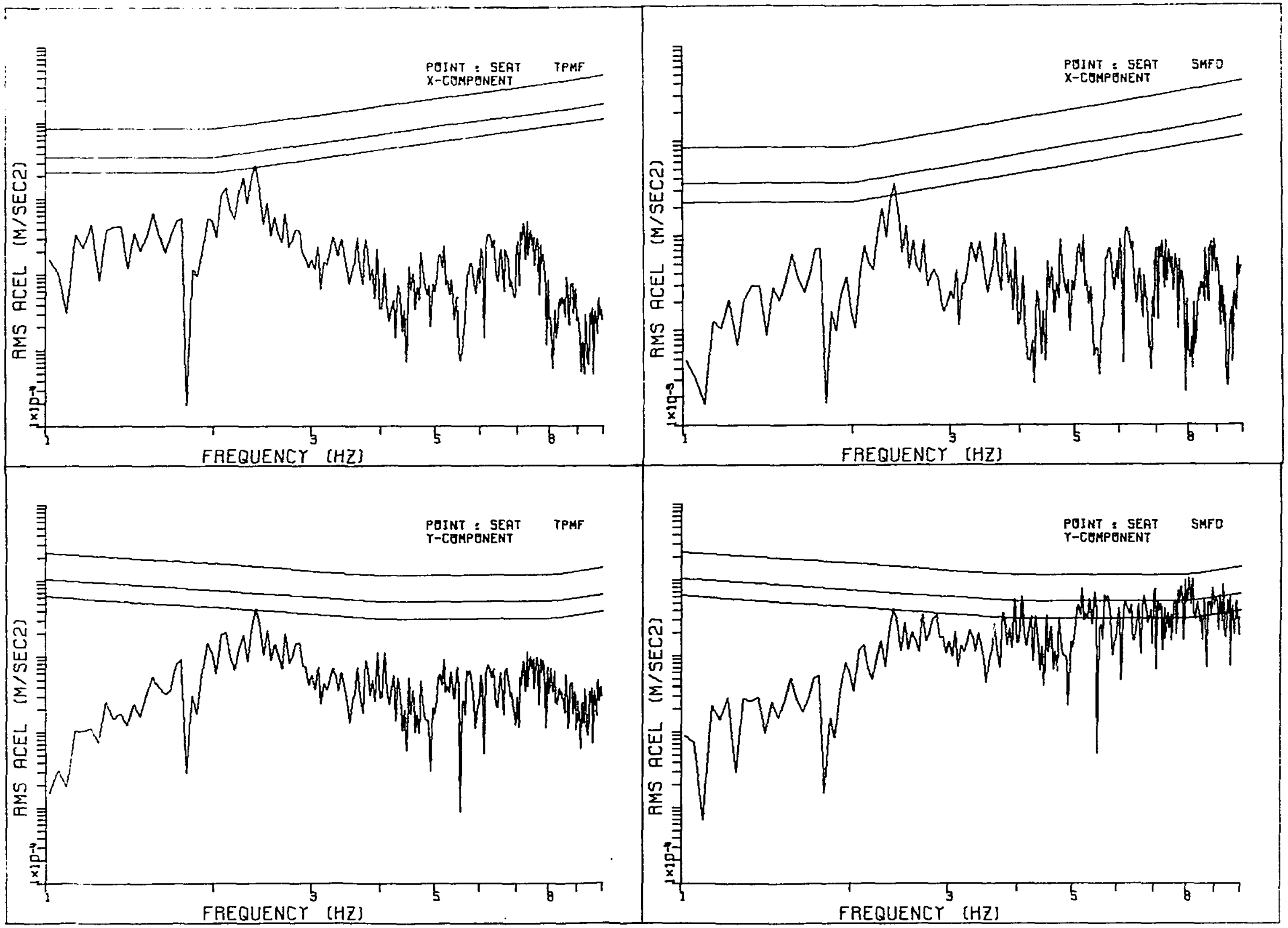


FIGURE 50. RMS acceleration responses of the suspended front axle midchassis cab tractor-plow models

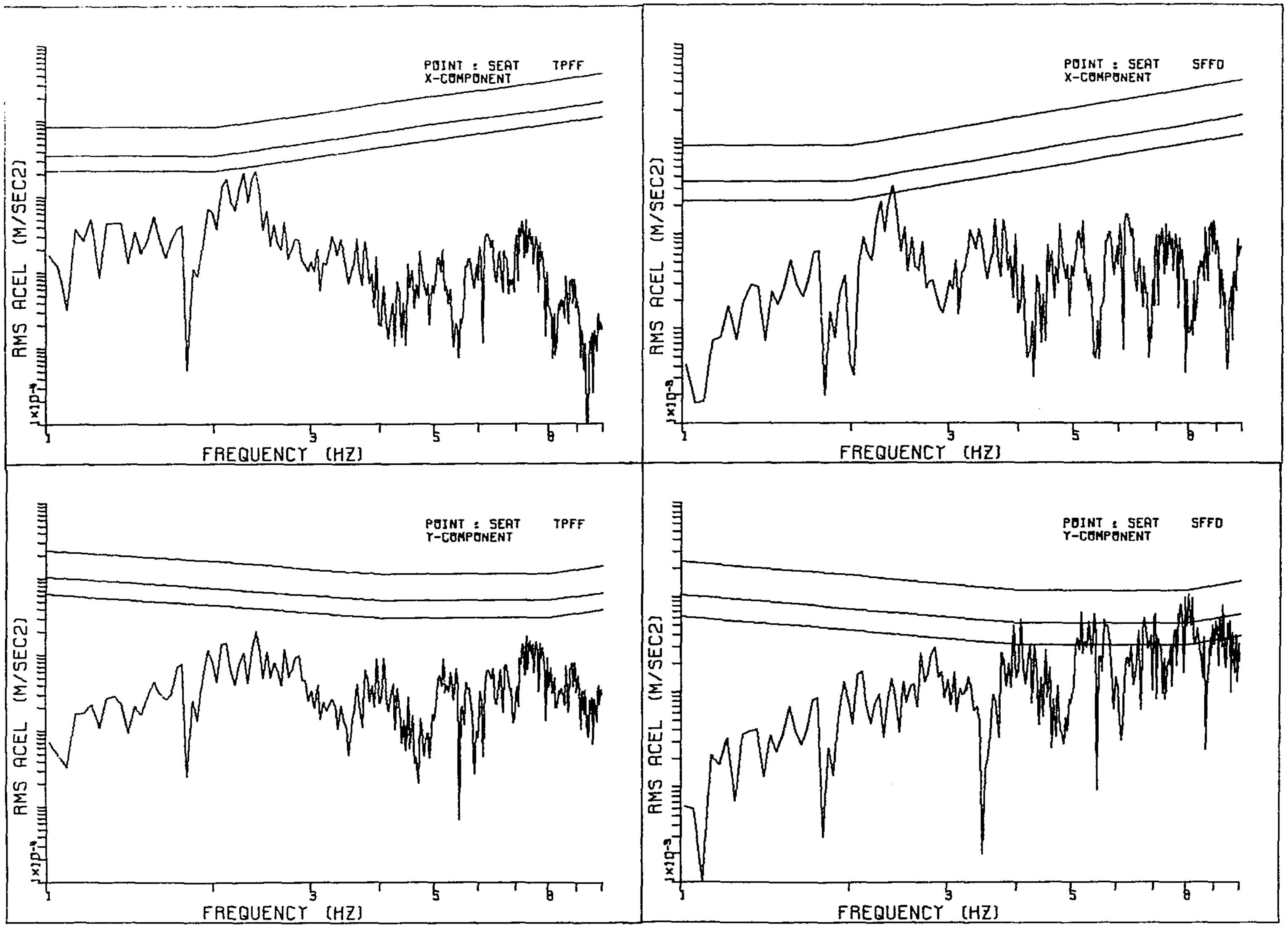


FIGURE 51. RMS acceleration responses of the suspended front axle forward cab tractor-plow models

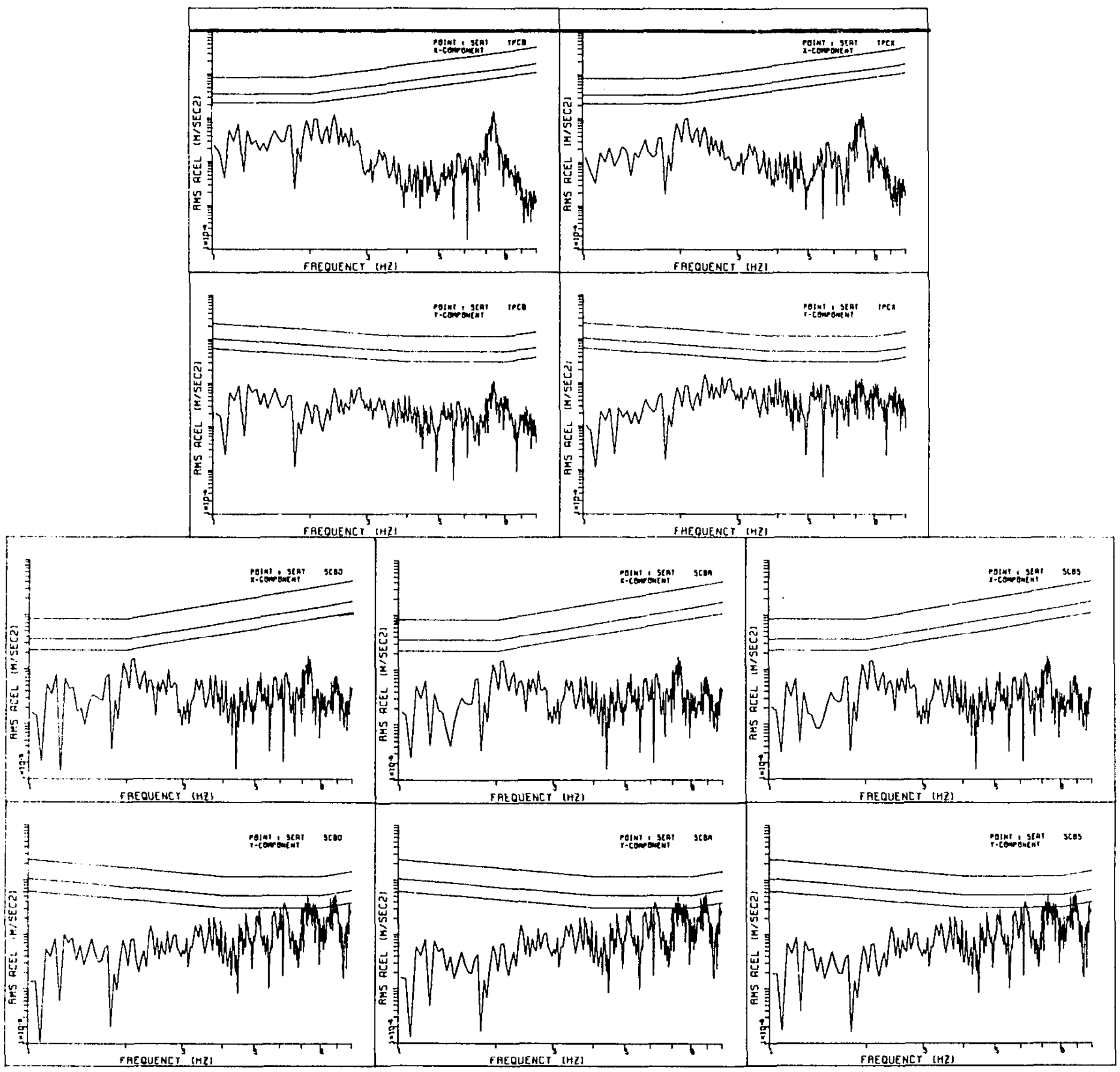


FIGURE 52. RMS acceleration responses of the fully suspended conventional tractor-plow models

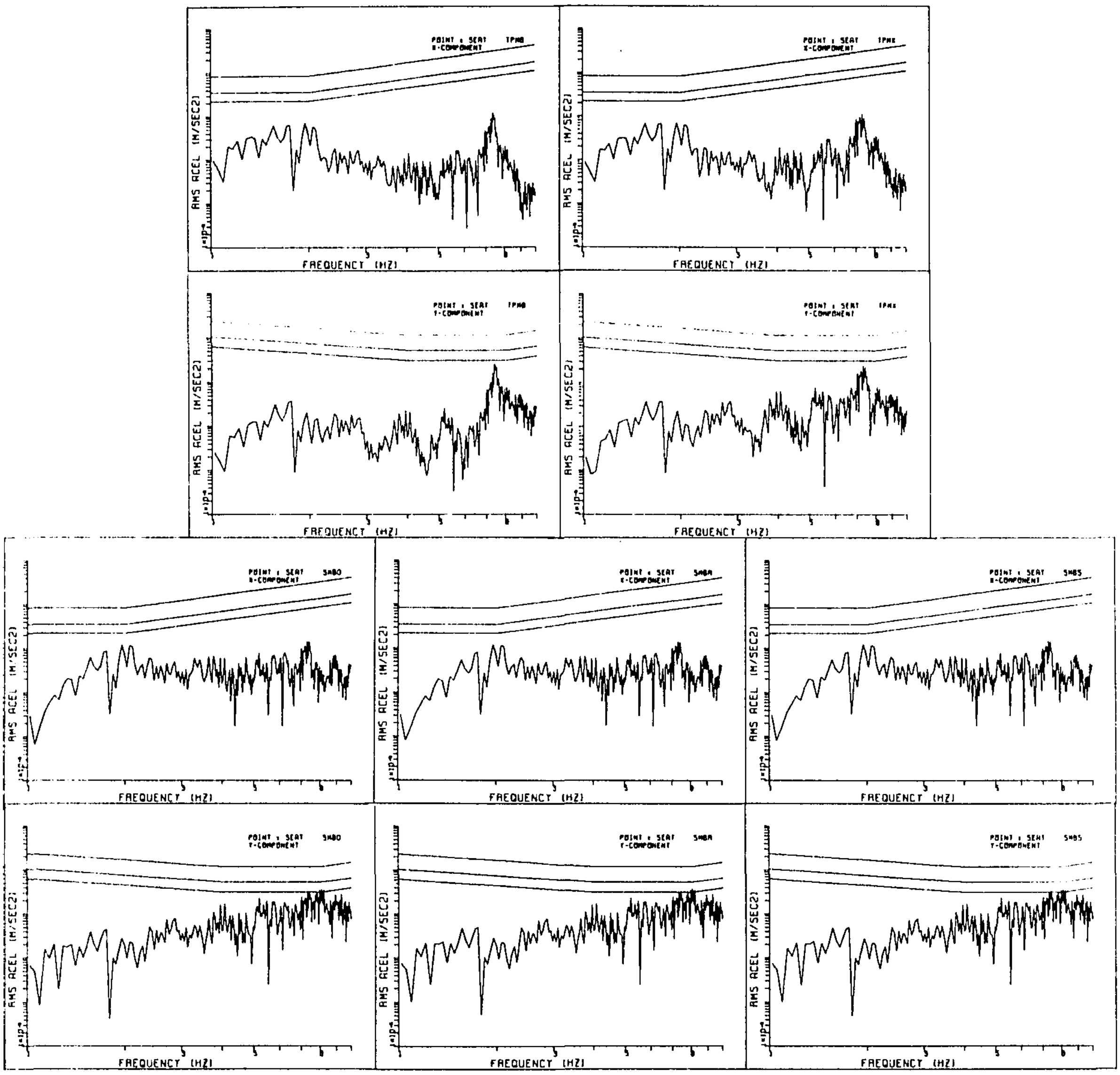


FIGURE 53. RMS acceleration responses of the fully suspended midchassis cab tractor-plow models

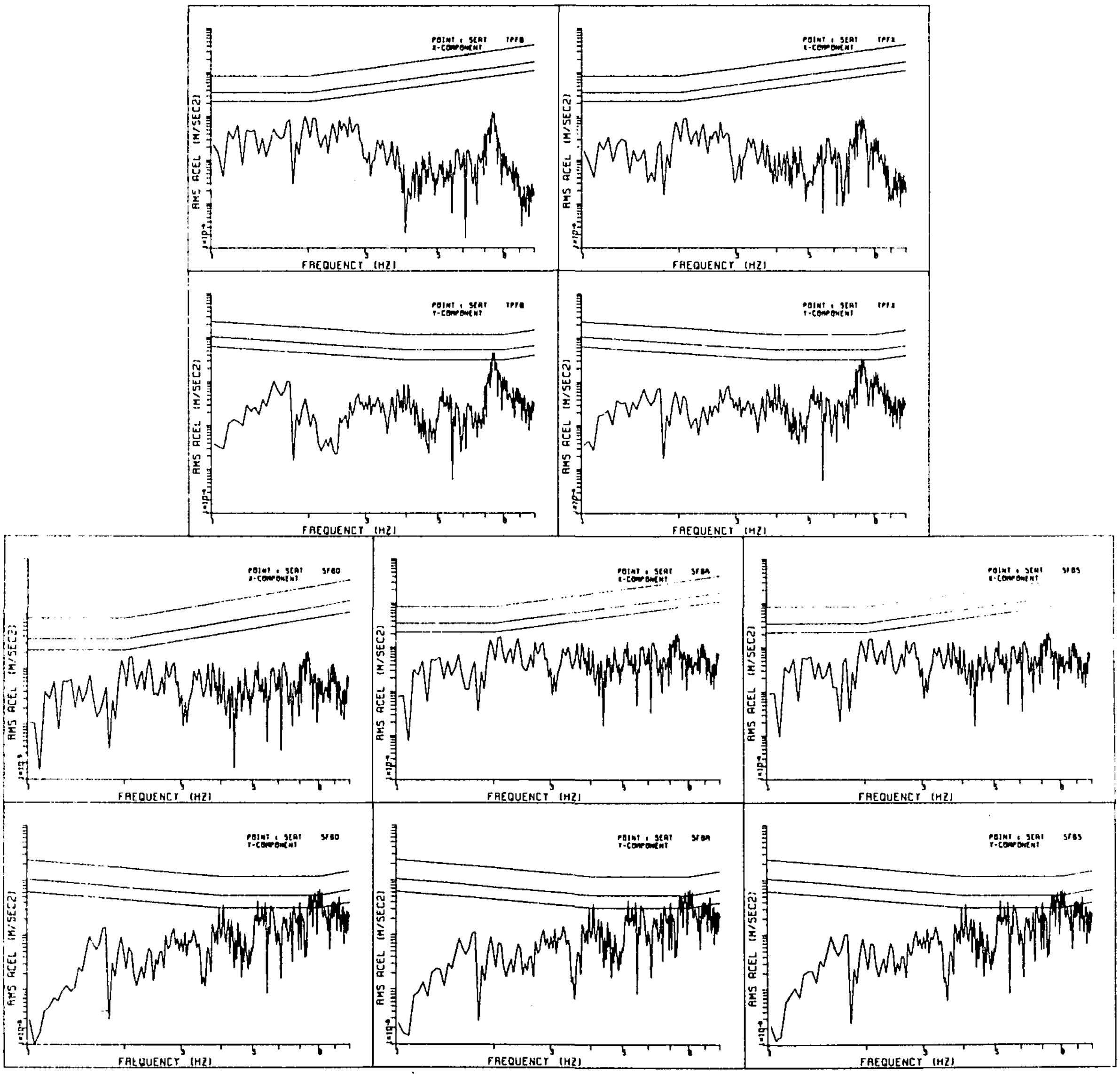


FIGURE 54. RMS acceleration responses of the fully suspended forward cab tractor-plow models

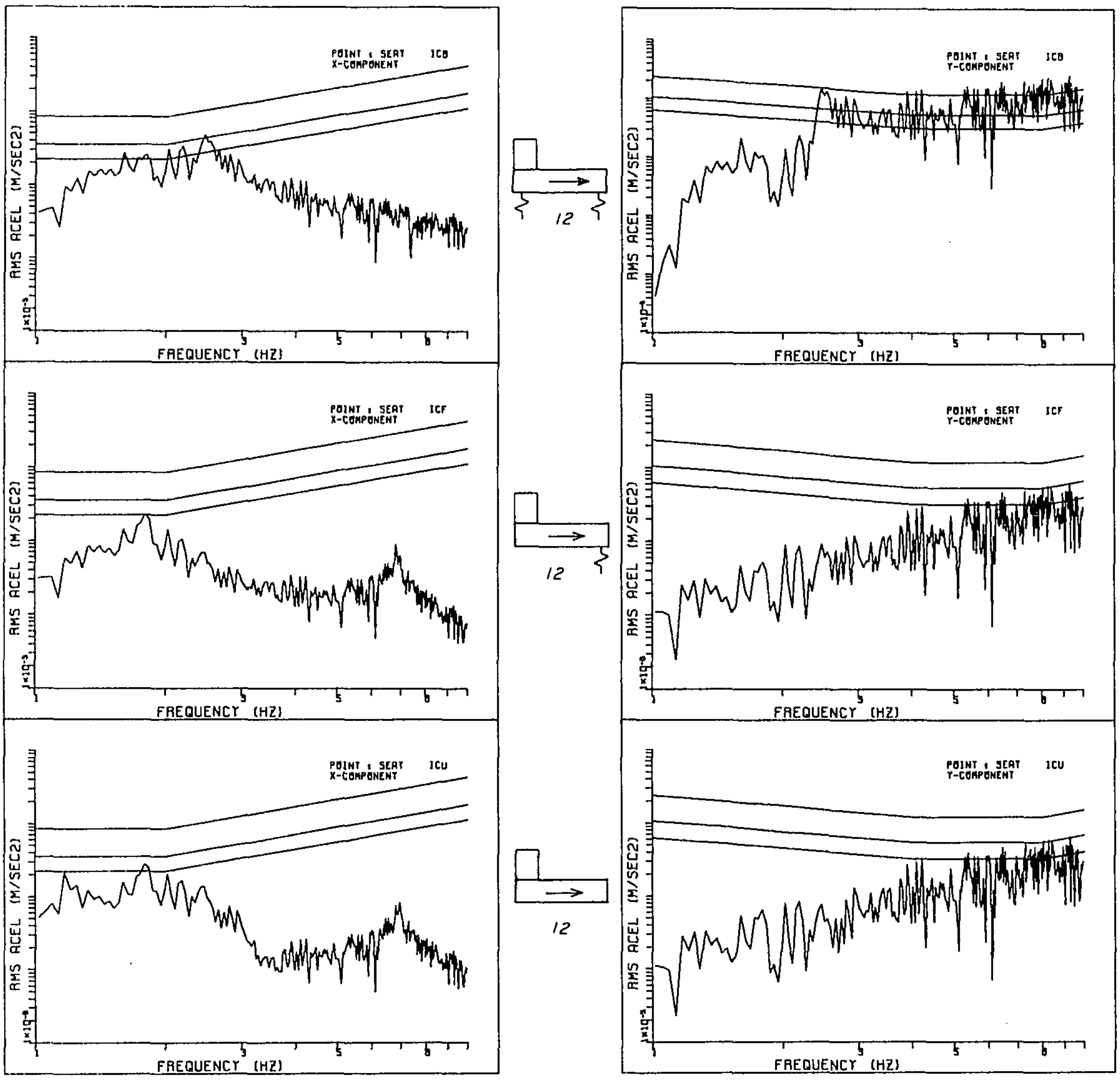


FIGURE 55. RMS acceleration responses of the conventional tractor-trailer (single-axle) models (12.5 km/h)

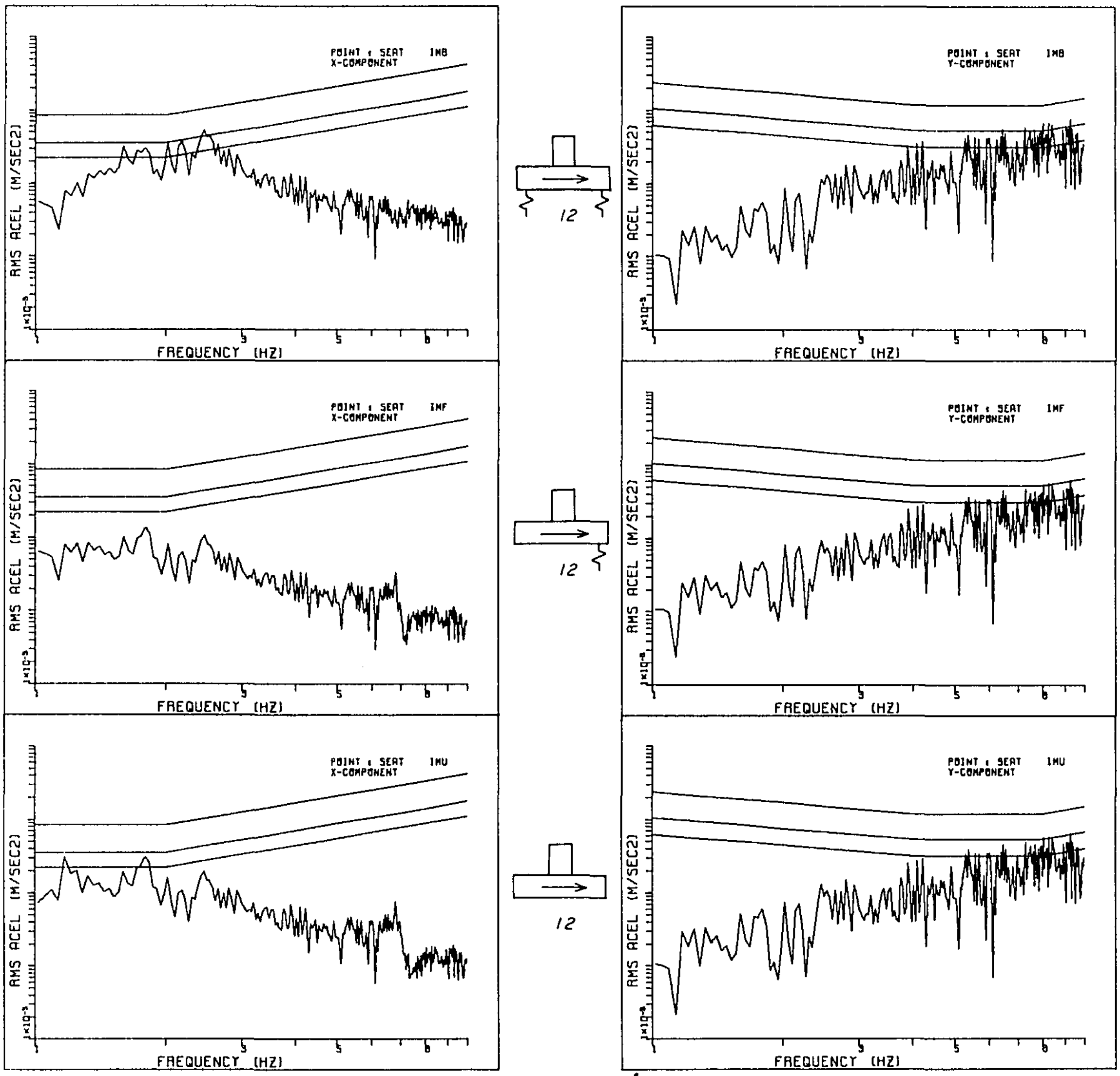


FIGURE 56. RMS acceleration responses of the midchassis cab tractor-trailer (single-axle) models (12.5 km/h)

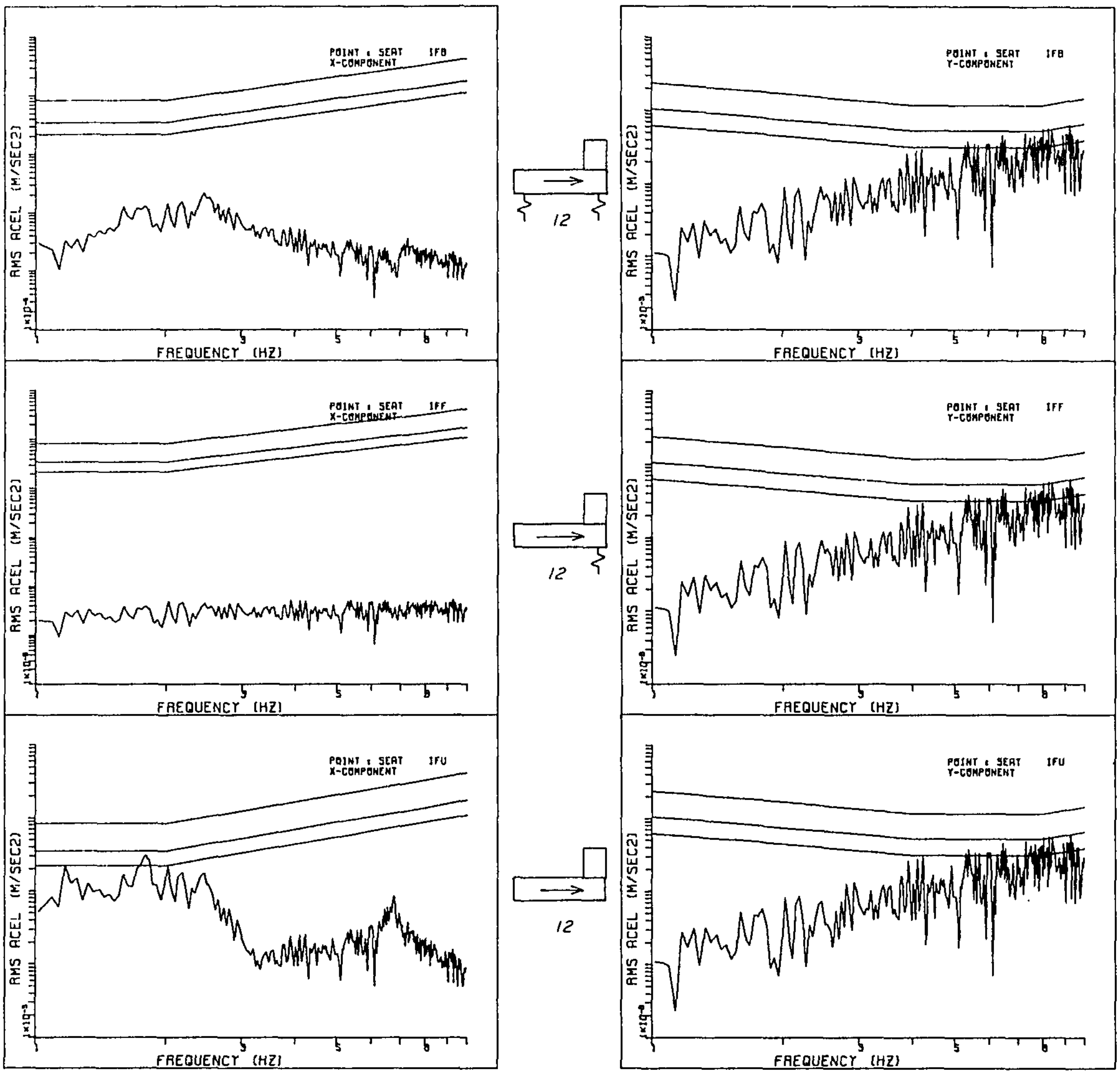


FIGURE 57. RMS acceleration responses of the forward cab tractor-trailer (single-axle) models (12.5 km/h)

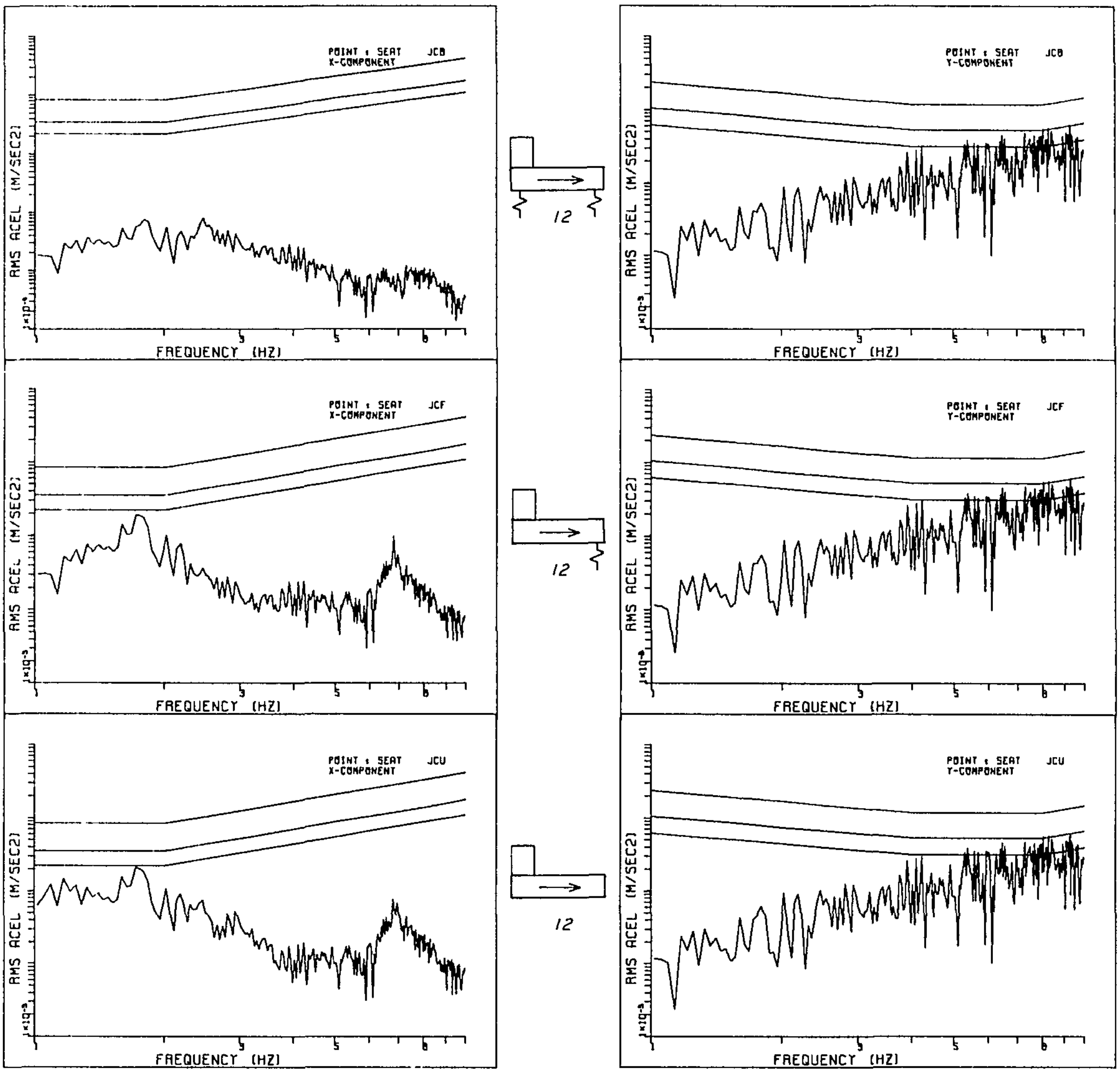


FIGURE 58. RMS acceleration responses of the conventional tractor-trailer (tandem-axle) models (12.5 km/h)

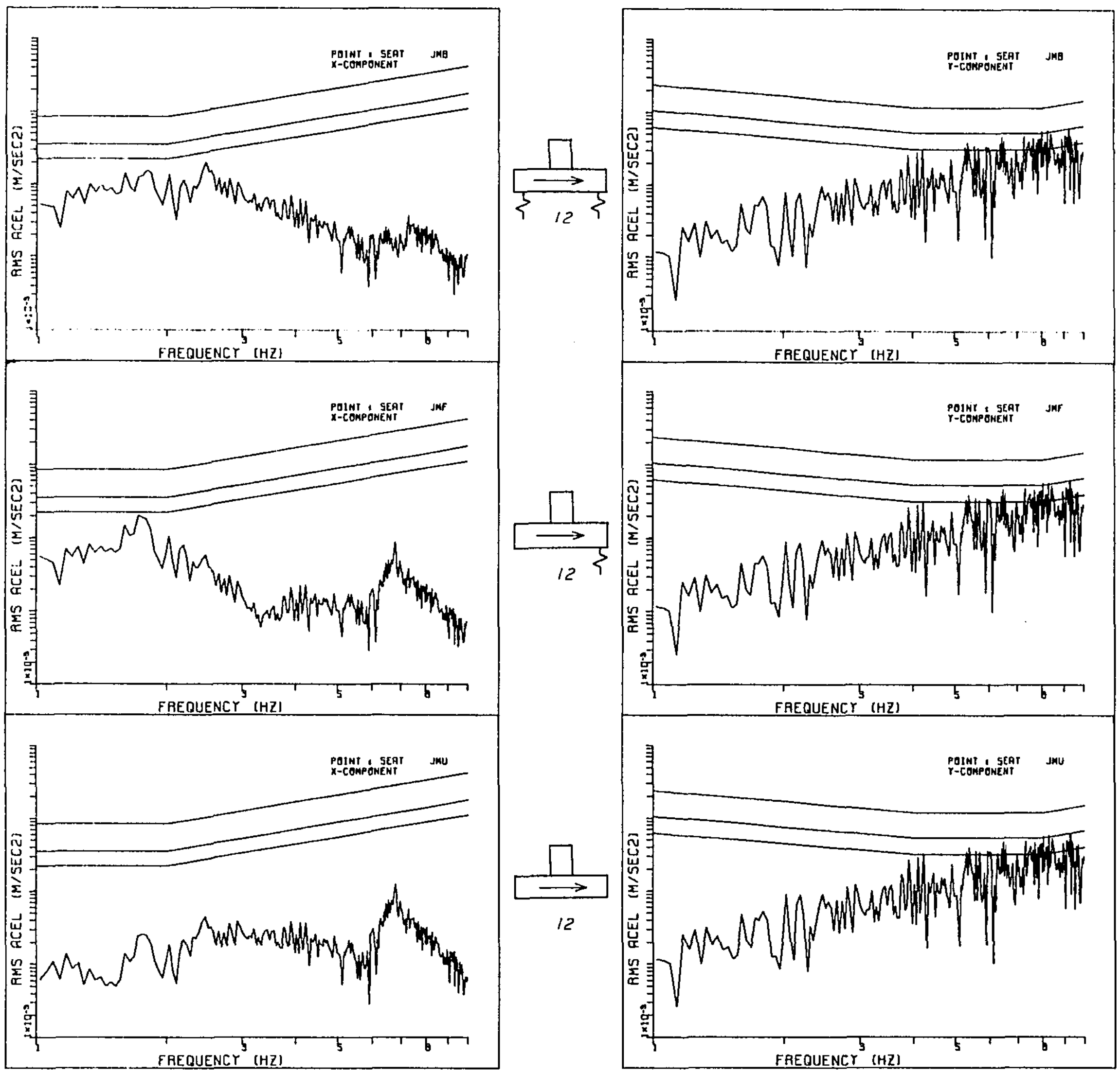


FIGURE 59. RMS acceleration responses of the midchassis cab tractor-trailer (tandem-axle) models (12.5 km/h)

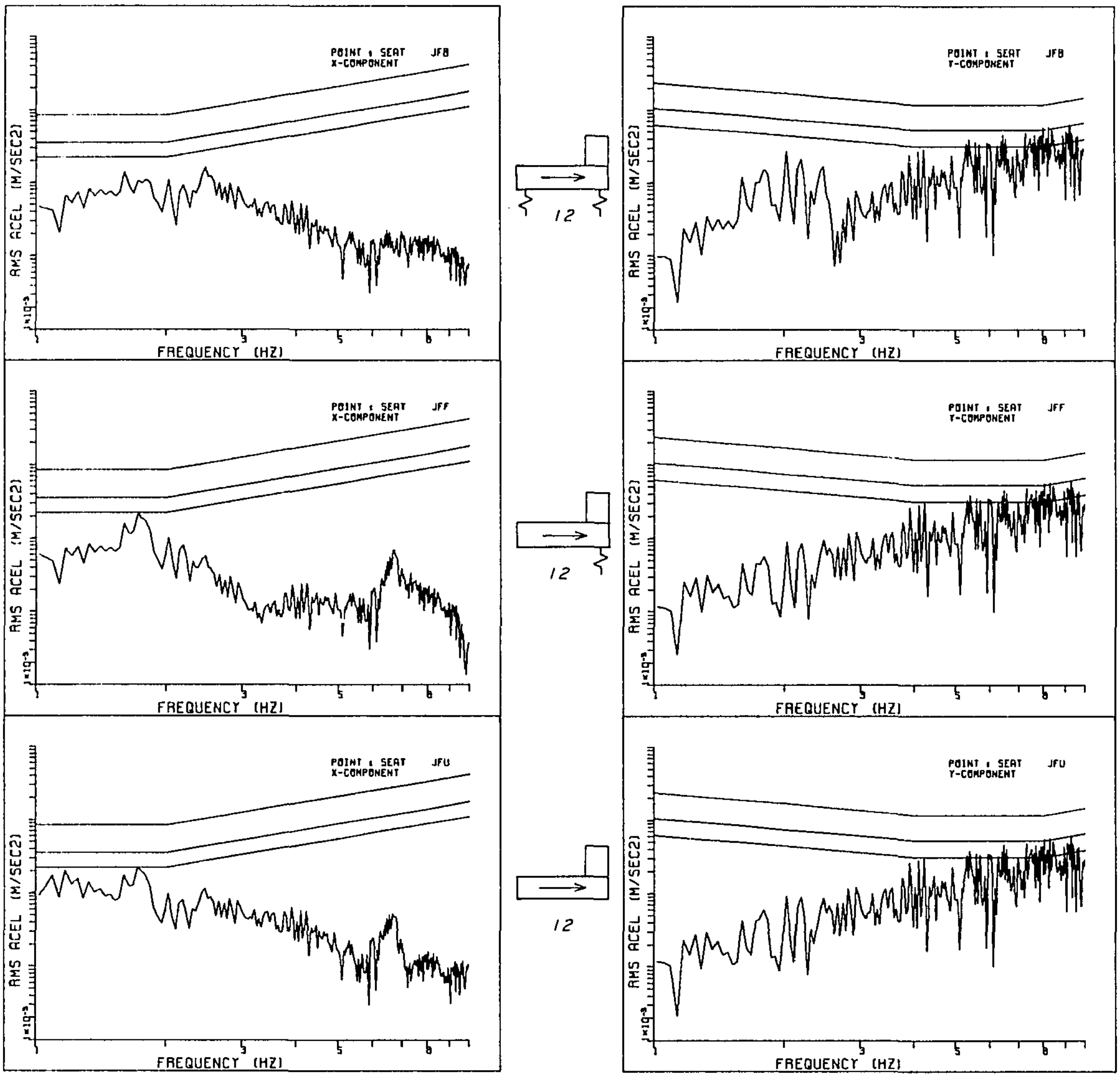


FIGURE 60. RMS acceleration responses of the forward cab tractor-trailer (tandem-axle) models (12.5 km/h)

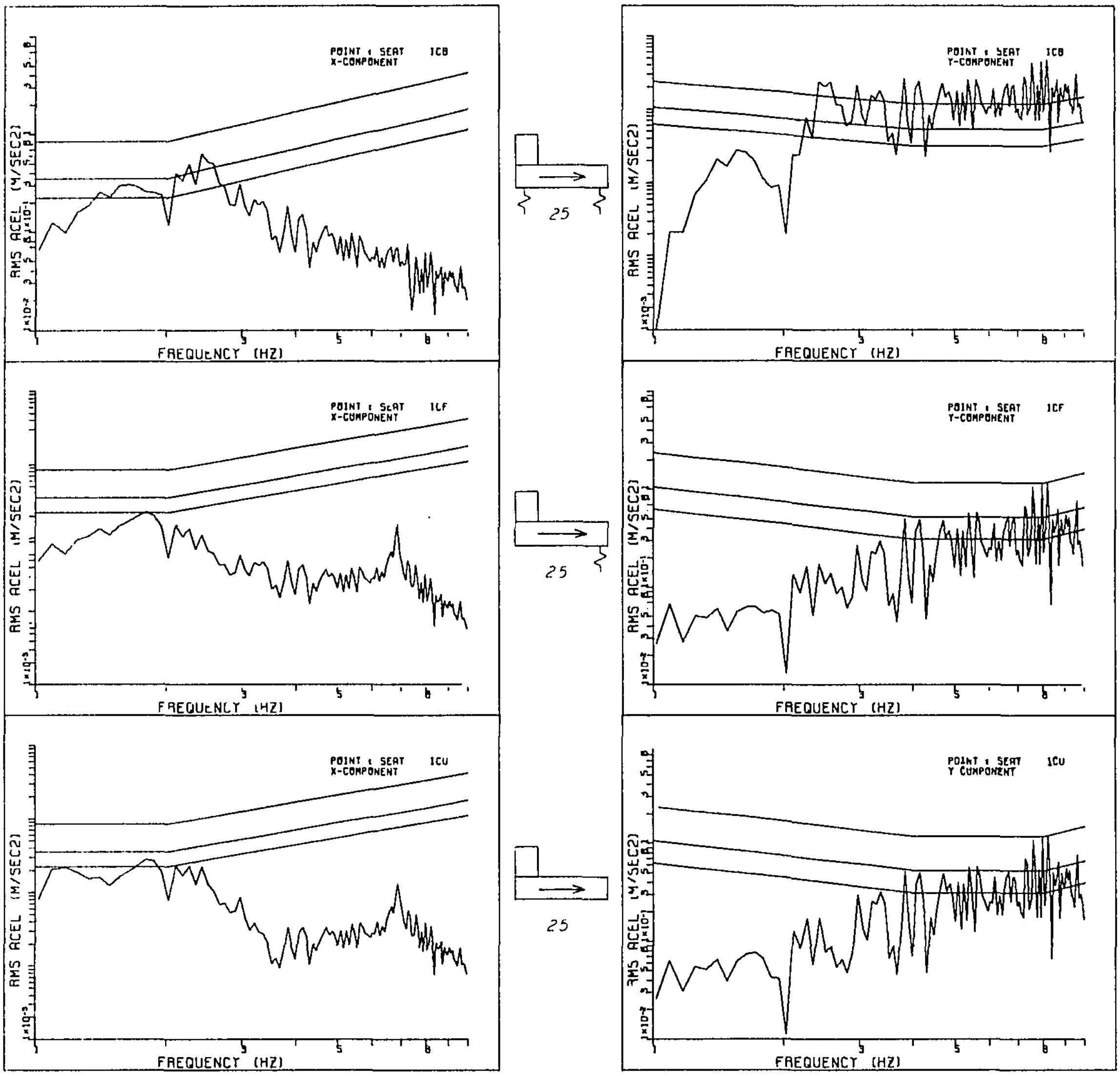


FIGURE 61. RMS acceleration responses of the conventional tractor-trailer (single-axle) models (25.0 km/h)

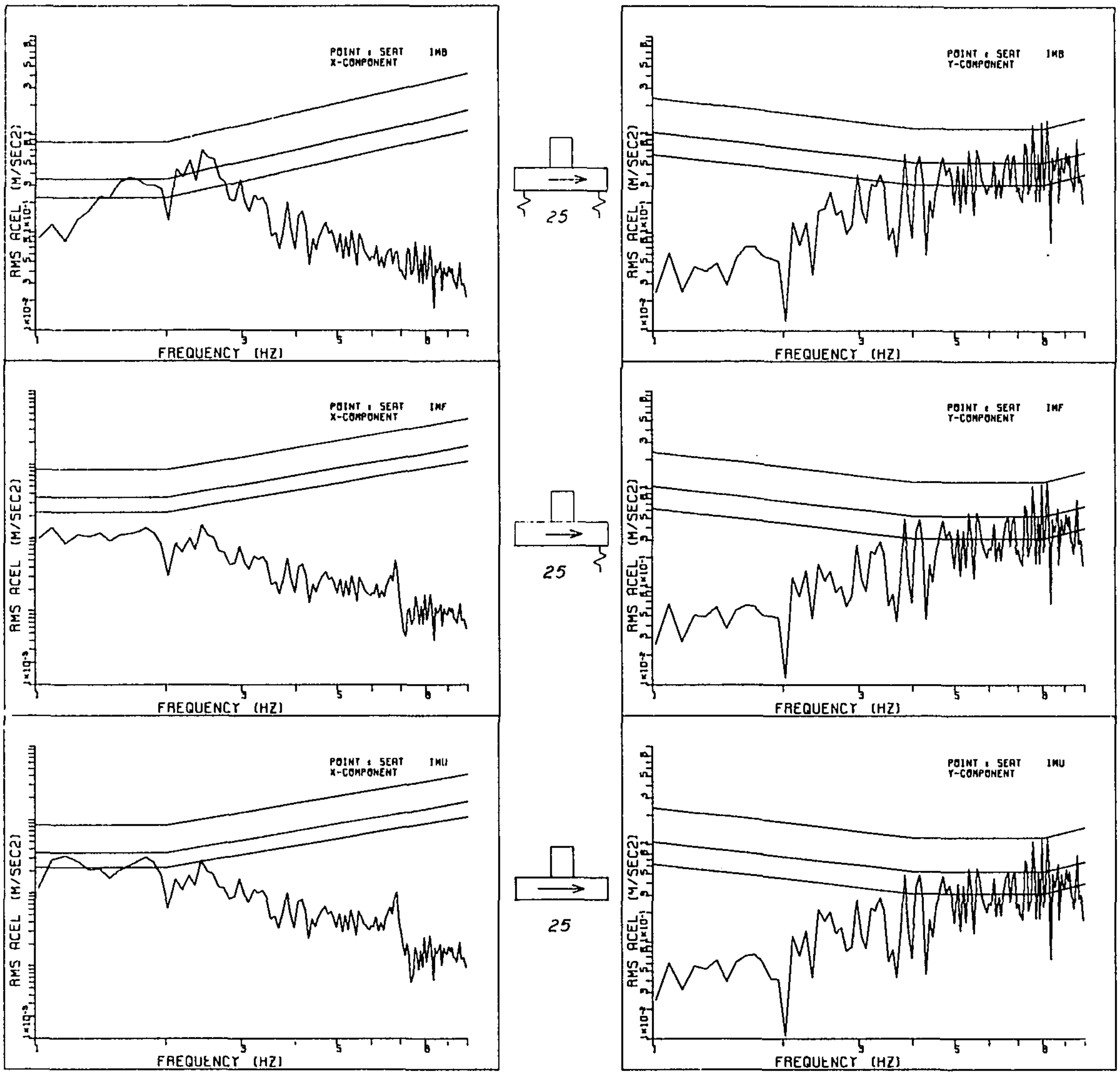


FIGURE 62. RMS acceleration responses of the midchassis cab tractor-trailer (single-axle) models (25.0 km/h)

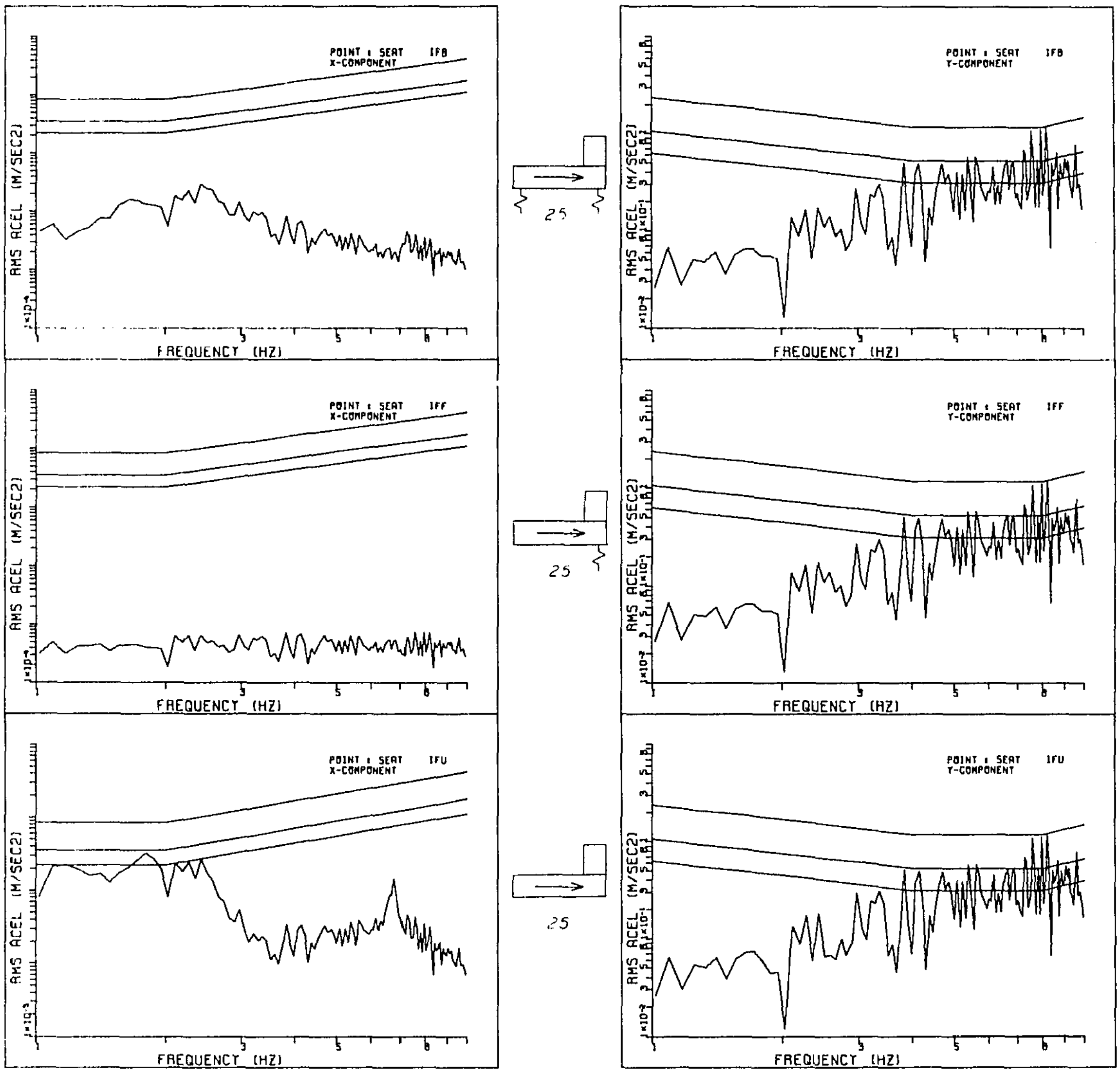


FIGURE 63. RMS acceleration responses of the forward cab tractor-trailer (single-axle) models (25.0 km/h)

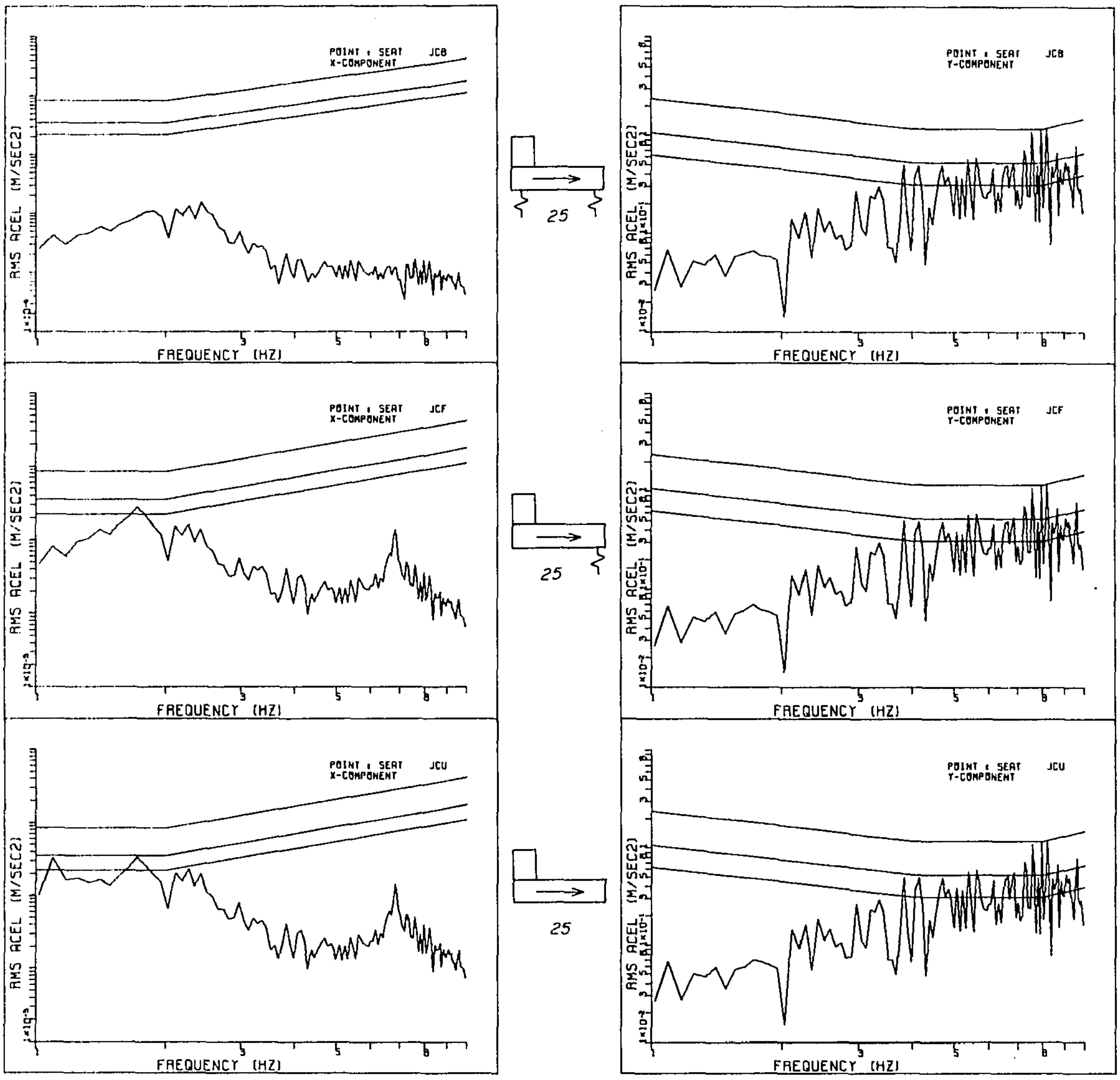


FIGURE 64. RMS acceleration responses of the conventional tractor-trailer (tandem-axle) models (25.0 km/h)

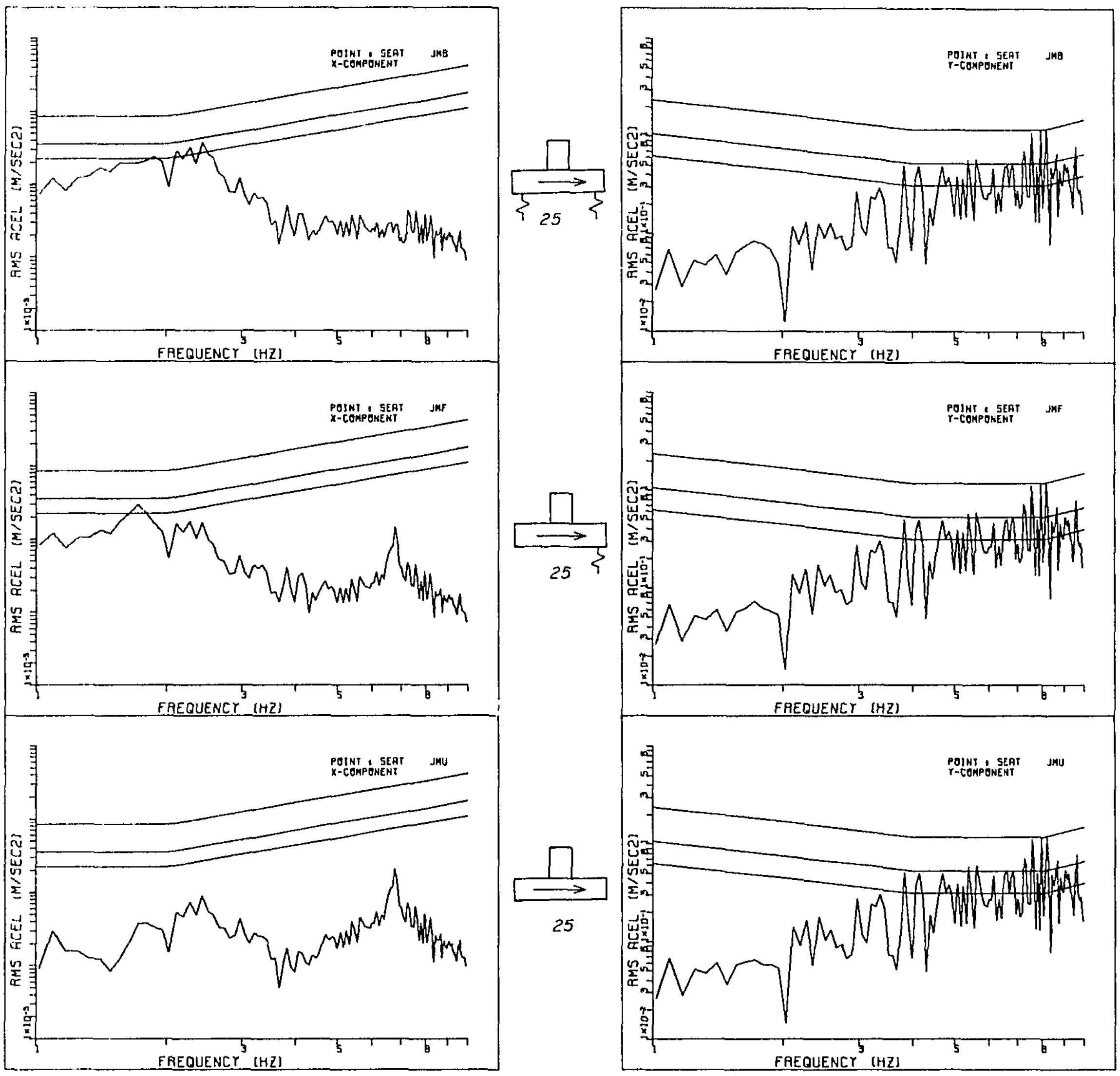


FIGURE 65. RMS acceleration responses of the midchassis cab tractor-trailer (tandem-axle) models (25.0 km/h)

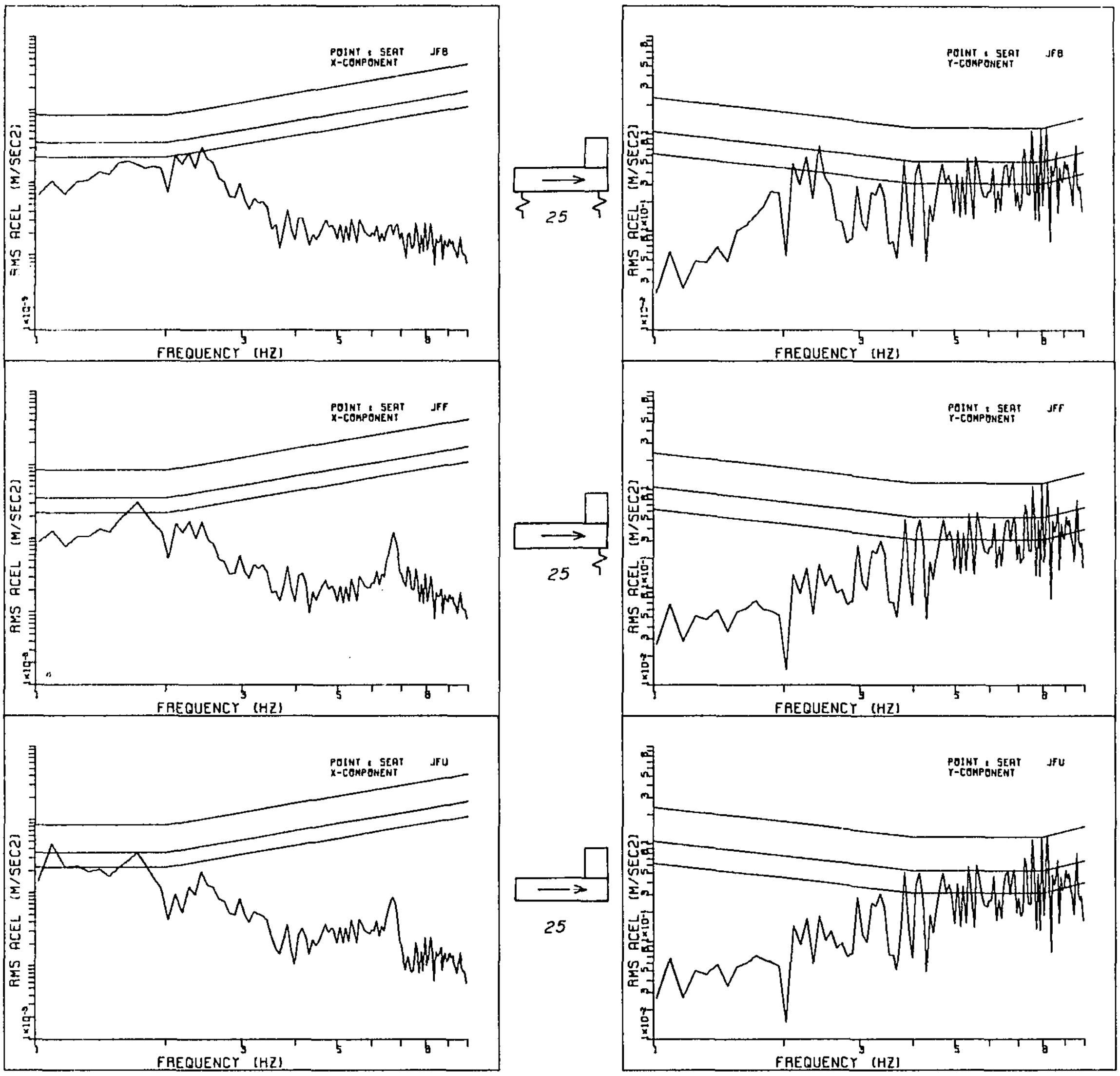


FIGURE 66. RMS acceleration responses of the forward cab tractor-trailer (tandem-axle) models (25.0 km/h)

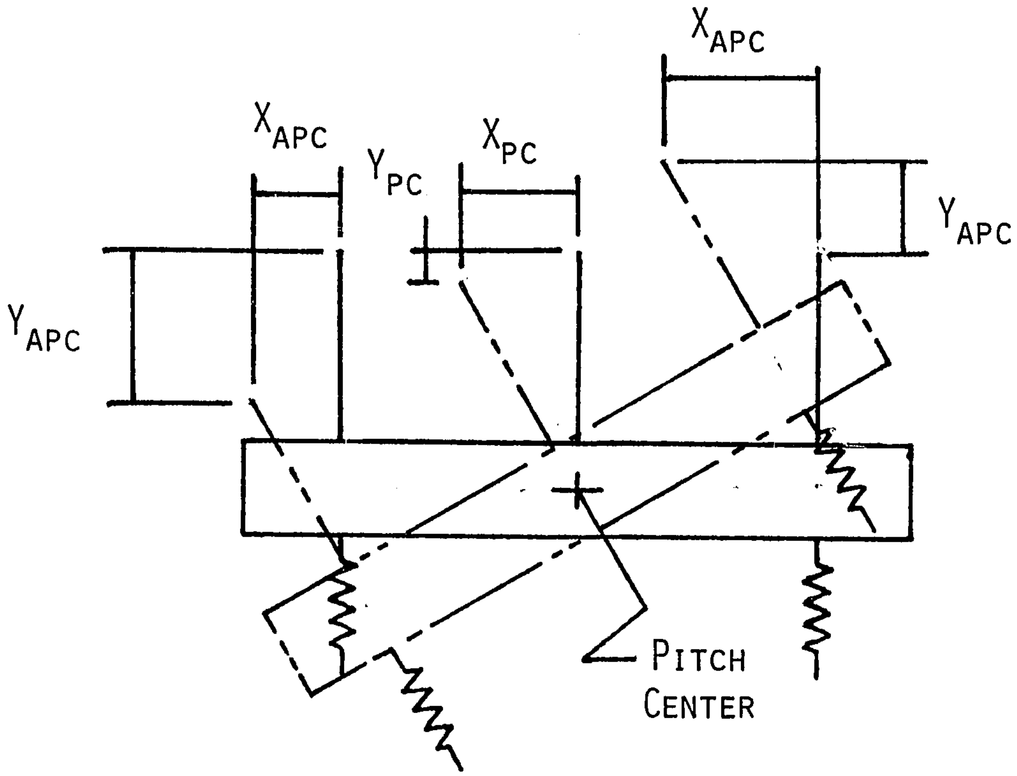


FIGURE 67. Influence of the tractor pitch center on ride comfort

APPENDIX: IMP ANALYSIS OF THE UNSUSPENDED CONVENTIONAL TRACTOR-PLOW MODEL

```
**********************************************************************
*                               IMP-75                               *
*                  THE INTEGRATED MECHANISMS PROGRAM                  *
**********************************************************************

REMARK/  CONVENTIONAL TRACTOR - PLOW MODEL

SYSTEM=  CONV. TRACTOR - PLOW MODEL

GROUND=TRACK

REMARK/  JOHN DEERE 4020 AGRICULTURE ROW-CROP WHEEL TRACTOR

REMARK/  TRACTOR CHASIS

REVLUT(CHASIS,RWHEL)=RAXLE

    DATA/LINK(CHAS,RAXL)=0,33.454,0/$
                         0,33.454,1/$
                         1,33.454,0

    DATA/LINK(RWHE,RAXL)=0,33.454,0/$
                         0,33.454,1/$
                         1,33.454,0

REVLUT(CHAS,FWHEL)=FAXLE

    DATA/LINK(CHAS,FAXLE)=100.25,18.825,0.0/$
                          100.25,18.825,1.0/$
                          101.25,18.825,0.0

    DATA/LINK(FWHE,FAXLE)=100.25,18.825,0.0/$
                          100.25,18.825,1.0/$
                          101.25,18.825,0.0

REMARK/  TRACTOR CHASIS CENTER OF GRAVITY

POINT(CHASIS)=CGCH

    DATA/POINT(CGCH,RAXL)=38.28,-0.258,0

REMARK/  TRACTOR CHASSIS SEAT BASE

POINT(CHASIS)=BZPT

    DATA/POINT(BZPT,RAXL)=6.2,12.7,0.0

REMARK/  TRACTOR CHASIS WEIGHT & INERTIAL PROPERTIES

    DATA/WEIGHT(CHAS,RAXL)=7368,38.28,-0.258,0

    DATA/INERTIA(CHAS,RAXL)=4109183.5,24602734.5,23971369,-69804,0,0

REMARK/  REAR TRACTOR TIRES

REMARK/  V-VERTICAL(BOUNCE)     R-ROLL

PRISM(RWHEL,RTIRE)=VRTR
```

```
    DATA/LINK(RWHE,VRTR)=0,3.938,0/$
                         0,4.938,0/$
                         0,3.938,1

    DATA/LINK(RTIR,VRTR)=0,3.938,0/$
                         0,4.938,0/$
                         0,3.938,1

PRISM(RTIRE,RTLK)=RJNT

    DATA/LINK(RTIR,RJNT)=0,3.938,0/$
                         1,3.938,0/$
                         0,4.938,0

    DATA/LINK(RTLK,RJNT)=0,3.938,0/$
                         1,3.938,0/$
                         0,4.938,0

PRISM(RTLK,TRACK)=RSMP

    DATA/LINK(RTLK,RSMP)=0,3.938,0/$
                         0,4.938,0/$
                         0,3.938,1

    DATA/LINK(TRAC,RSMP)=0,3.938,0/$
                         0,4.938,0/$
                         0,3.938,1

REMARK/  TIRE SPRING & DAMPING RATES

    DATA/SPRING(VRTR)=3958.0,-2.76

    DATA/DAMPER(VRTR)=46.85

    DATA/DAMPER(RJNT)=46.85

    DATA/SPRING(RJNT)=5915.0,0.192

REMARK/  WHEEL WEIGHT & INERTIAL PROPERTIES

    DATA/WEIGHT(RWHE,RAXL)=1332,0,0,0

    DATA/INERTIA(RWHEL,RAXL)=340530,340530,681060,0,0,0

REMARK/  FRONT TRACTOR TIRES

PRISM(FWHEL,FTIRE)=VFTR

    DATA/LINK(FWHE,VFTR)=100.25,3.938,0/$
                         100.25,4.938,0/$
                         100.25,3.938,1

    DATA/LINK(FTIR,VFTR)=100.25,3.938,0/$
                         100.25,4.938,0/$
                         100.25,3.938,1

PRISM(FTIRE,DTLK)=DJNT

    DATA/LINK(FTIR,DJNT)=100.25,3.938,0/$
                         101.25,3.938,0/$
```

```
                          100.25,4.938,0

    DATA/LINK(DTLK,DJNT)=100.25,3.938,0/$
                          101.25,3.938,0/$
                          100.25,4.938,0

PRISM(DTLK,TRACK)=FSMP

    DATA/LINK(DTLK,FSMP)=100.25,3.938,0/$
                          100.25,4.938,0/$
                          100.25,3.938,1

    DATA/LINK(TRAC,FSMP)=100.25,3.938,0/$
                          100.25,4.938,0/$
                          100.25,3.938,1

REMARK/  TIRE SPRING & DAMPING RATES

    DATA/SPRING(VFTR)=2372.0,-1.09

    DATA/DAMPER(VFTR)=17.64

REMARK/  WHEEL WEIGHT & INERTIAL PROPERTIES

    DATA/WEIGHT(FWHE,FAXL)=150,0,0,0

    DATA/INERTIA(FWHEL,FAXLE)=5280,5280,10560,0,0,0

REMARK/ TRACTOR OPERATOR ENCLOSURE CAB

REMARK/  I-VERTICAL(BOUNCE)    R-PITCH

REMARK/  REAR ISOMOUNT PAD

REVLUT(CAB ,RCLK)=RRCB

    DATA/LINK(CAB ,RRCB)=0,46.132,0/$
                          0,46.132,1/$
                          1,46.132,0

    DATA/LINK(RCLK,RRCB)=0,46.132,0/$
                          0,46.132,1/$
                          1,46.132,0

PRISM(RCLK,LCLK)=RIPD

    DATA/LINK(RCLK,RIPD)=0,46.132,0/$
                          0,47.132,0/$
                          1,46.132,0

    DATA/LINK(LCLK,RIPD)=0,46.132,0/$
                          0,47.132,0/$
                          1,46.132,0

PRISM(LCLK,CHASIS)=LJNT

    DATA/LINK(LCLK,LJNT)=0,46.132,0/$
                          1,46.132,0/$
                          0,47.132,0
```

```
    DATA/LINK(CHAS,LJNT)=0,46.132,0/$
                         1,46.132,0/$
                         0,47.132,0

REMARK/  CAB ISOMOUNT PAD SPRING & DAMPING RATE

    DATA/SPRING(RIPD)=15050,0.05

    DATA/DAMPER(RIPD)=1.7

    DATA/SPRING(LJNT)=15050,0

    DATA/DAMPER(LJNT)=1.7

REMARK/  FRONT ISOMOUNT PAD

REVLUT(CAB ,FCLK)=RFCB

    DATA/LINK(CAB ,RFCB)=36,46.52,0.0/$
                         36,46.52,1.0/$
                         37,46.52,0.0

    DATA/LINK(FCLK,RFCB)=36,46.52,0.0/$
                         36,46.52,1.0/$
                         37,46.52,0.0

PRISM(FCLK,DCLK)=FIPD

    DATA/LINK(FCLK,FIPD)=36,46.52,0.0/$
                         36,47.52,0.0/$
                         36,46.52,1.0

    DATA/LINK(DCLK,FIPD)=36,46.52,0.0/$
                         36,47.52,0.0/$
                         36,46.52,1.0

PRISM(DCLK,CHASIS)=CJNT

    DATA/LINK(DCLK,CJNT)=36,46.52,0.0/$
                         37,46.52,0.0/$
                         36,47.52,0.

    DATA/LINK(CHAS,CJNT)=36,46.52,0.0/$
                         37,46.52,0.0/$
                         36,47.52,0.

REMARK/  CAB CENTER OF GRAVITY & SEAT POINT

POINT(CAB)=CGCB,SEAT

    DATA/POINT(CGCB,RRCB)=18,30,0

    DATA/POINT(SEAT,RRCB)=6.75,20.3,0

REMARK/  CAB ISOMOUNT PAD SPRING & DAMPING RATE

    DATA/SPRING(FIPD)=15050,0.05

    DATA/DAMPER(FIPD)=1.7
```

```
    DATA/SPRING(CJNT)=15050,0
    DATA/DAMPER(CJNT)=1.7
REMARK/  CAB WEIGHT & INERTIAL PROPERTIES
    DATA/WEIGHT(CAB ,RRCB)=1505,18,30,0
    DATA/INERTIA(CAB ,RRCB)=2403130,1562285,2800750,812700,0,0
REMARK/  TRACTOR 3-POINT IMPLEMENT HITCH
REVLUT(CHAS,LARM)=APIN
    DATA/LINK(CHAS,APIN)=2.78,44.378,0.0/$
                         2.78,44.378,1.0/$
                         -8.188,47.804,0.0
    DATA/LINK(LARM,APIN)=2.78,44.378,0.0/$
                         2.78,44.378,1.0/$
                         -8.188,47.804,0.0
REVLUT(LARM,LLNK)=LPIN
    DATA/LINK(LARM,LPIN)=-8.188,47.804,0.0/$
                         -8.188,47.804,1.0/$
                         -7.188,47.804,0.0
    DATA/LINK(LLNK,LPIN)=-8.188,47.804,0.0/$
                         -8.188,47.804,1.0/$
                         -7.188,47.804,0.0
REVLUT(LLNK,DRFT)=PPIN
    DATA/LINK(LLNK,PPIN)=-26.802,21.734,0.0/$
                         -26.802,21.734,1.0/$
                         -25.802,21.734,1.0
    DATA/LINK(DRFT,PPIN)=-26.802,21.734,0.0/$
                         -26.802,21.734,1.0/$
                         -25.802,21.734,1.0
REVLUT(CHAS,DRFT)=DPIN
    DATA/LINK(CHAS,DPIN)=-4.5,23.448,0.0/$
                         -4.5,23.448,1.0/$
                         -3.5,23.448,0.0
    DATA/LINK(DRFT,DPIN)=-4.5,23.448,0.0/$
                         -4.5,23.448,1.0/$
                         -3.5,23.448,0.0
REMARK/  IMPLEMENT POSITION CONTROL
    DATA/POSITN(APIN)=0.0
REMARK/  HITCH WEIGHT AND INERTIAL PROPERTIES.
    DATA/WEIGHT(LARM,APIN)=29.6,5.75,0.0,0.0
```

```
    DATA/INERTIA(LARM,APIN)=34.4,1339.2,1335.0,0.0,0.0,0.0

    DATA/WEIGHT(LLNK,PPIN)=44.0,16.14,0.0,0.0

    DATA/INERTIA(LLNK,PPIN)=0.0,15283.0,15283.0,0.0,0.0,0.0

    DATA/WEIGHT(DRFT,DPIN)=109.0,18.33,0.0,0.0

    DATA/INERTIA(DRFT,DPIN)=131.8,48948.8,48858.0,0.0,0.0,0.0

REMARK/  ALLIS-CHALMERS 3-BOTTOM MOLDBOARD SEMI-MOUNTED PLOW

REVLUT(DRFT,PLOW)=HPIN

    DATA/LINK(DRFT,HPIN)=-40.956,20.381,0/$
                          -40.956,20.381,1/$
                          -4.397,23.219,0

    DATA/LINK(PLOW,HPIN)=-40.956,20.381,0/$
                          -40.956,20.381,1/$
                          -39.956,20.381,0

REVLUT(PLOW,PWHEL)=PAXLE

    DATA/LINK(PLOW,PAXL)=-205.956,-4.062,0/$
                          -205.956,-4.062,1/$
                          -204.956,-4.062,0

    DATA/LINK(PWHE,PAXL)=-205.956,-4.062,0/$
                          -205.956,-4.062,1/$
                          -204.956,-4.062,0

PRISM(PWHEL,PTIRE)=VPWHEL

    DATA/LINK(PWHE,VPWH)=-205.956,-4.062,0/$
                          -205.956,-3.062,0/$
                          -205.956,-4.062,1

    DATA/LINK(PTIR,VPWH)=-205.956,-4.062,0/$
                          -205.956,-3.062,0/$
                          -205.956,-4.062,1

PRISM(PTIRE,TRACK)=HPWHEL

    DATA/LINK(PTIR,HPWH)=-205.956,-4.062,0/$
                          -204.956,-4.062,0/$
                          -205.956,-3.062,0

    DATA/LINK(TRAC,HPWH)=-205.956,-4.062,0/$
                          -204.956,-4.062,0/$
                          -205.956,-3.062,0

REMARK/  PLOW WEIGHT & INERTIAL PROPERTIES

    DATA/WEIGHT(PLOW,HPIN)=1725,-50,10,0

    DATA/INERTIA(PLOW,HPIN)=441989,3325483,6525089,-862500,0,0

REMARK/  PLOW TAIL WHEEL
```

```
    DATA/SPRING(VPWHEL)=1125.0,-0.5
    DATA/DAMPER(VPWHEL)=8.25
REMARK/  PLOW CENTER OF GRAVITY AND MOLDBOARD POINTS
POINT(PLOW)=CGPLOW,BTM1,BTM2,BTM3,HPB1,HPB2,HPB3,VPB1,VPB2,VPB3
    DATA/POINT(CGPL,HPIN)=-50,10,0
    DATA/POINT(BTM1,HPIN)=-34.75,-20.0,0.0
    DATA/POINT(BTM2,HPIN)=-62.75,-20.0,0.0
    DATA/POINT(BTM3,HPIN)=-90.75,-20.0,0.0
    DATA/POINT(HPB1,HPIN)=-33.75,-20.0,0.0
    DATA/POINT(HPB2,HPIN)=-61.75,-20.0,0.0
    DATA/POINT(HPB3,HPIN)=-89.75,-20.0,0.0
    DATA/POINT(VPB1,HPIN)=-34.75,-15.5,0.0
    DATA/POINT(VPB2,HPIN)=-62.75,-15.5,0.0
    DATA/POINT(VPB3,HPIN)=-90.75,-15.5,0.0
REMARK/  DRAFT FORCES ON THE MOLDBOARDS
FORCE(BTM1,HPB1,BTM1)=DFT1
    DATA/FORCE(DFT1)=1440
FORCE(BTM2,HPB2,BTM2)=DFT2
    DATA/FORCE(DFT2)=1440
FORCE(BTM3,HPB3,BTM3)=DFT3
    DATA/FORCE(DFT3)=1440
REMARK/  SUCTION FORCES ON THE MOLDBOARDS
FORCE(BTM1,VPB1,BTM1)=SKF1
    DATA/FORCE(SKF1)=655
FORCE(BTM2,VPB2,BTM2)=SKF2
    DATA/FORCE(SKF2)=655
FORCE(BTM3,VPB3,BTM3)=SKF3
    DATA/FORCE(SKF3)=655
REMARK/  TRACTIVE ROLLING RESISTANCE
    DATA/FORCE(RJNT)=-874.0  $$  REAR TRACTOR AXLE
```

```
    DATA/FORCE(DJNT)=-205.6  $$  FRONT TRACTOR AXLE
    DATA/FORCE(HPWHEL)=-58.0   $$ PLOW
REMARK/  SIMULATOR PADS - INPUT TRACK PROFILE
    DATA/POSITN(RSMP)=0.0
    DATA/POSITN(FSMP)=0.0
REMARK/  GRAVITATIONAL CONSTANTS
    DATA/GRAV=0,-386.088,0
REMARK/  IMP TOLERANCES
ZERO(POSITION)=0.001
REMARK/  REQUEST STATEMENTS
FIND/EQULIB
PRINT/FREQ
PRINT/POSITN(ALL)
PRINT/FORCE(ALL)
PRINT/DYNAM/MOTION/ACCEL/SEAT,CGCH/
EXECUTE/HOLD
```

```
**********************************************************************
*                           ANALYSIS OF                              *
*                     CONV. TRACTOR - PLOW MODEL                     *
*                           STATIC MODE                              *
*              ON          i             AT    -                     *
**********************************************************************
```

POSITION 1

THE GENERALIZED COORDINATES ARE

JOINT		SET BY	POSITION DEG, IN	VELOCITY RAD, IN/SEC	ACCELERATION RAD, IN/SEC2
APIN		USER	0.0	0.0	0.0
RSMP		USER	0.0	0.0	0.0
FSMP		USER	0.0	0.0	0.0
CJNT	1	IMP	-0.000	0.0	0.0
VPWH	1	IMP	0.171	0.0	0.0
VFTR	1	IMP	-0.007	0.0	0.0
VRTR	1	IMP	-0.004	0.0	0.0
FIPD	1	IMP	0.100	0.0	0.0
RJNT	1	IMP	0.730	0.0	0.0
RIPD	1	IMP	0.100	0.0	0.0

DEGREE OF FREEDOM = 10. QUALITY INDEX = 0.327E 09

DYNAMIC CHARACTERISTICS

FGC JOINT ---	NATURAL FREQUENCY RAD/SEC	DAMPING RATIO ---	DAMPED FREQUENCY RAD/SEC	DECAY RATE 1/SEC
RIPD	148.334	0.009	148.329	-1.282
RJNT	95.064	0.008	95.061	-0.806
FIPD	45.414	0.134	45.004	-6.086
VRTR	43.290	0.041	43.254	-1.764
VFTR	17.037	0.062	17.004	-1.061
VPWH	13.015	0.070	12.983	-0.917
CJNT	13.598	0.057	13.576	-0.778

POSITION RESULTS (DEG, IN)		X	Y	Z
JNT. RAXL	-0.002			
JNT. FAXL	-0.002			
JNT. VRTR	-0.004			
JNT. RJNT	0.730			
JNT. RSMP	0.0			
JNT. VFTR	-0.007			
JNT. DJNT	0.729			
JNT. FSMP	0.0			
JNT. RRCB	0.000			
JNT. RIPD	0.100			

JNT. LJNT	0.000			
JNT. RFCB	0.000			
JNT. FIPD	0.100			
JNT. CJNT	-0.000			
JNT. APIN	0.0			
JNT. LPIN	0.001			
JNT. PPIN	-0.000			
JNT. DPIN	0.000			
JNT. HPIN	-4.381			
JNT. PAXL	-0.060			
JNT. VPWH	0.171			
JNT. HPWH	0.703			
PNT. CGCH	50.123	37.550	33.201	0.0
PNT. BZPT	46.481	5.470	46.158	0.0
PNT. CGCB	77.973	17.269	76.036	0.0
PNT. SEAT	66.609	6.019	66.336	0.0
PNT. CGPL	96.582	-91.696	30.331	0.0
PNT. BTM1	76.415	-76.414	0.347	0.0
PNT. BTM2	104.414	-104.414	0.317	0.0
PNT. BTM3	132.414	-132.414	0.288	0.0
PNT. HPB1	75.415	-75.414	0.348	0.0
PNT. HPB2	103.414	-103.414	0.318	0.0
PNT. HPB3	131.414	-131.414	0.289	0.0
PNT. VPB1	76.572	-76.419	4.847	0.0
PNT. VPB2	104.530	-104.419	4.817	0.0
PNT. VPB3	132.505	-132.419	4.788	0.0
FORCE RESULTS (IN-LB,LB)		X	Y	Z
JNT. RAXL FROM CHAS ONTO RWHE				
FORC	10399.641	-4054.258	-9576.824	----
TORQ	0.079	----	----	0.079
JNT. FAXL FROM CHAS ONTO FWHE				
FORC	2426.608	-205.655	-2417.878	----

TORQ	3.463	----	----	3.463
JNT. VRTR FROM RWHE ONTO RTIR				
FORC	11637.848	----	-4054.261	-10908.824
TORQ	119680.875	119680.875	----	----
JNT. RJNT FROM RTIR ONTO RTLK				
FORC	11637.848	-10908.824	----	-4054.261
TORQ	127640.813	----	127640.813	----
JNT. RSMP FROM RTLK ONTO TRAC				
FORC	11637.848	----	-4054.261	-10908.824
TORQ	127640.625	127640.625	----	----
JNT. VFTR FROM FWHE ONTO FTIR				
FORC	2576.099	----	-205.652	-2567.878
TORQ	3063.333	3063.333	----	----
JNT. DJNT FROM FTIR ONTO DTLK				
FORC	2576.099	-2567.878	----	-205.652
TORQ	4935.414	----	4935.414	----
JNT. FSMP FROM DTLK ONTO TRAC				
FORC	2576.099	----	-205.652	-2567.878
TORQ	4934.902	4934.902	----	----
JNT. RRCB FROM CAB ONTO RCLK				
FORC	752.547	-0.381	-752.547	----
TORQ	0.004	----	----	-0.004
JNT. RIPD FROM RCLK ONTO LCLK				
FORC	752.547	-0.381	----	-752.547
TORQ	0.042	----	0.042	----
JNT. LJNT FROM LCLK ONTO CHAS				
FORC	752.547	-752.547	----	-0.381
TORQ	0.023	----	-0.023	----
JNT. RFCB FROM CAB ONTO FCLK				
FORC	752.451	0.329	-752.451	----
TORQ	0.023	----	----	0.023
JNT. FIPD FROM FCLK ONTO DCLK				
FORC	752.451	----	0.329	-752.451
TORQ	0.053	0.053	----	----
JNT. CJNT FROM DCLK ONTO CHAS				
FORC	752.451	-752.451	----	0.329
TORQ	0.053	----	0.053	----
JNT. APIN FROM CHAS ONTO LARM				
FORC	5430.820	-1656.893	-5171.898	----
TORQ	59267.527	----	----	-59267.527
JNT. LPIN FROM LARM ONTO LLNK				
FORC	5406.629	3123.622	4413.008	----
TORQ	1.415	----	----	-1.415
JNT. PPIN FROM LLNK ONTO DRFT				
FORC	5370.770	3123.605	4369.012	----

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      TORQ          0.732          ----           ----          -0.732

 JNT. DPIN FROM CHAS ONTO DRFT
      FORC       1742.558      1136.439      -1320.990           ----
      TORQ          0.073          ----           ----          -0.073

 JNT. HPIN FROM DRFT ONTO PLOW
      FORC       5175.484      4263.000       2934.702           ----
      TORQ          3.271          ----           ----          -3.271

 JNT. PAXL FROM PLOW ONTO PWHE
      FORC        757.561       -57.993       -755.338           ----
      TORQ         15.424          ----           ----         -15.424

 JNT. VPWH FROM PWHE ONTO PTIR
      FORC        757.565          ----        -57.993        -755.342
      TORQ         25.363       -25.363           ----           ----

 JNT. HPWH FROM PTIR ONTO TRAC
      FORC        757.565      -755.342           ----         -57.993
      TORQ        504.618          ----        504.618           ----

TRANSFER FUNCTION ANALYSIS : MOTI INPUT(S)

  EIGENVECTORS(2*NDOF(FORCE) OR 2*NFGC(MOTION) :

  COLUMN :  1
  PSI( 1)=( -0.331111E 02,  0.119069E 01)
  PSI( 2)=( -0.488582E 01,  0.970444E-01)
  PSI( 3)=( -0.344245E 02,  0.111323E 01)
  PSI( 4)=( -0.960285E 00, -0.227522E-01)
  PSI( 5)=(  0.189517E 01, -0.379584E-02)
  PSI( 6)=( -0.482915E 01, -0.455387E 00)
  PSI( 7)=( -0.952151E 02,  0.309739E 01)
  PSI( 8)=(  0.101153E-01,  0.223874E 00)
  PSI( 9)=(  0.941665E-03,  0.329611E-01)
  PSI(10)=(  0.949306E-02,  0.231906E 00)
  PSI(11)=( -0.100210E-03,  0.642272E-02)
  PSI(12)=( -0.139813E-03, -0.128115E-01)
  PSI(13)=( -0.279538E-02,  0.325057E-01)
  PSI(14)=(  0.265181E-01,  0.642255E 00)

  COLUMN :  2
  PSI( 1)=( -0.331111E 02, -0.119069E 01)
  PSI( 2)=( -0.488582E 01, -0.970444E-01)
  PSI( 3)=( -0.344245E 02, -0.111323E 01)
  PSI( 4)=( -0.960285E 00,  0.227522E-01)
  PSI( 5)=(  0.189517E 01,  0.379584E-02)
  PSI( 6)=( -0.482915E 01,  0.455387E 00)
  PSI( 7)=( -0.952151E 02, -0.309739E 01)
  PSI( 8)=(  0.101153E-01, -0.223874E 00)
  PSI( 9)=(  0.941665E-03, -0.329611E-01)
  PSI(10)=(  0.949306E-02, -0.231906E 00)
  PSI(11)=( -0.100210E-03, -0.642272E-02)
  PSI(12)=( -0.139813E-03,  0.128115E-01)
  PSI(13)=( -0.279538E-02, -0.325057E-01)
  PSI(14)=(  0.265181E-01, -0.642255E 00)

  COLUMN :  3
  PSI( 1)=(  0.193333E 01,  0.663114E 01)
```

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PSI( 2)=( -0.538030E-02, -0.428024E-01)
PSI( 3)=( -0.186121E 01, -0.628893E 01)
PSI( 4)=( -0.262233E 00, -0.104768E 01)
PSI( 5)=( -0.162463E 00, -0.538703E 00)
PSI( 6)=( -0.125635E 00, -0.186475E 01)
PSI( 7)=(  0.167782E-01,  0.341550E-01)
PSI( 8)=(  0.693903E-01, -0.208467E-01)
PSI( 9)=( -0.461072E-03,  0.632703E-04)
PSI(10)=( -0.660036E-01,  0.201537E-01)
PSI(11)=( -0.109861E-01,  0.284828E-02)
PSI(12)=( -0.564352E-02,  0.175347E-02)
PSI(13)=( -0.195922E-01,  0.148273E-02)
PSI(14)=(  0.116348E-03, -0.100732E-03)

COLUMN :  4
PSI( 1)=(  0.193333E 01, -0.663114E 01)
PSI( 2)=( -0.538030E-02,  0.428024E-01)
PSI( 3)=( -0.186121E 01,  0.628893E 01)
PSI( 4)=( -0.262233E 00,  0.104768E 01)
PSI( 5)=( -0.162463E 00,  0.538703E 00)
PSI( 6)=( -0.125635E 00,  0.186475E 01)
PSI( 7)=(  0.167782E-01, -0.341550E-01)
PSI( 8)=(  0.693903E-01,  0.208467E-01)
PSI( 9)=( -0.461072E-03, -0.632703E-04)
PSI(10)=( -0.660036E-01, -0.201537E-01)
PSI(11)=( -0.109861E-01, -0.284828E-02)
PSI(12)=( -0.564352E-02, -0.175347E-02)
PSI(13)=( -0.195922E-01, -0.148273E-02)
PSI(14)=(  0.116348E-03,  0.100732E-03)

COLUMN :  5
PSI( 1)=( -0.484523E 00,  0.611743E 00)
PSI( 2)=(  0.197460E 00, -0.117747E 00)
PSI( 3)=( -0.654426E 00,  0.503375E 00)
PSI( 4)=(  0.453496E 00,  0.286645E-01)
PSI( 5)=( -0.607765E 00,  0.245322E 00)
PSI( 6)=(  0.780362E 01,  0.296754E 01)
PSI( 7)=(  0.170042E 00, -0.220978E 00)
PSI( 8)=(  0.148448E-01,  0.881660E-02)
PSI( 9)=( -0.313829E-02, -0.399021E-02)
PSI(10)=(  0.128717E-01,  0.126826E-01)
PSI(11)=( -0.713944E-03, -0.997958E-02)
PSI(12)=(  0.712878E-02,  0.125603E-01)
PSI(13)=(  0.418136E-01, -0.178997E 00)
PSI(14)=( -0.518316E-02, -0.347453E-02)

COLUMN :  6
PSI( 1)=( -0.484523E 00, -0.611743E 00)
PSI( 2)=(  0.197460E 00,  0.117747E 00)
PSI( 3)=( -0.654426E 00, -0.503375E 00)
PSI( 4)=(  0.453496E 00, -0.286645E-01)
PSI( 5)=( -0.607765E 00, -0.245322E 00)
PSI( 6)=(  0.780362E 01, -0.296754E 01)
PSI( 7)=(  0.170042E 00,  0.220978E 00)
PSI( 8)=(  0.148448E-01, -0.881660E-02)
PSI( 9)=( -0.313829E-02,  0.399021E-02)
PSI(10)=(  0.128717E-01, -0.126826E-01)
PSI(11)=( -0.713944E-03,  0.997958E-02)
PSI(12)=(  0.712878E-02, -0.125603E-01)
PSI(13)=(  0.418136E-01,  0.178997E 00)
```

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PSI(14)=( -0.518316E-02,  0.347453E-02)

COLUMN :  7
PSI( 1)=(  0.512085E 01, -0.851607E 01)
PSI( 2)=( -0.153388E 01,  0.192279E 01)
PSI( 3)=(  0.564854E 01, -0.827183E 01)
PSI( 4)=( -0.188384E 01,  0.575735E 00)
PSI( 5)=(  0.420633E 01, -0.444226E 01)
PSI( 6)=( -0.185304E 02, -0.220933E 02)
PSI( 7)=( -0.185503E 01,  0.316042E 01)
PSI( 8)=( -0.202013E 00, -0.110492E 00)
PSI( 9)=(  0.457364E-01,  0.336329E-01)
PSI(10)=( -0.196510E 00, -0.122026E 00)
PSI(11)=(  0.151255E-01,  0.429586E-01)
PSI(12)=( -0.106471E 00, -0.928902E-01)
PSI(13)=( -0.492256E 00,  0.448859E 00)
PSI(14)=(  0.730150E-01,  0.407247E-01)

COLUMN :  8
PSI( 1)=(  0.512085E 01,  0.851607E 01)
PSI( 2)=( -0.153388E 01, -0.192279E 01)
PSI( 3)=(  0.564854E 01,  0.827183E 01)
PSI( 4)=( -0.188384E 01, -0.575735E 00)
PSI( 5)=(  0.420633E 01,  0.444226E 01)
PSI( 6)=( -0.185304E 02,  0.220933E 02)
PSI( 7)=( -0.185503E 01, -0.316042E 01)
PSI( 8)=( -0.202013E 00,  0.110492E 00)
PSI( 9)=(  0.457364E-01, -0.336329E-01)
PSI(10)=( -0.196510E 00,  0.122026E 00)
PSI(11)=(  0.151255E-01, -0.429586E-01)
PSI(12)=( -0.106471E 00,  0.928902E-01)
PSI(13)=( -0.492256E 00, -0.448859E 00)
PSI(14)=(  0.730150E-01, -0.407247E-01)

COLUMN :  9
PSI( 1)=( -0.200598E 00, -0.173034E 00)
PSI( 2)=(  0.669812E 00,  0.514193E 00)
PSI( 3)=( -0.323246E 00, -0.290753E 00)
PSI( 4)=(  0.575987E 00,  0.535259E 00)
PSI( 5)=(  0.721892E 01,  0.503329E 01)
PSI( 6)=(  0.360140E 00,  0.207069E 00)
PSI( 7)=(  0.731716E-01,  0.772919E-01)
PSI( 8)=( -0.915337E-02,  0.118687E-01)
PSI( 9)=(  0.276834E-01, -0.411809E-01)
PSI(10)=( -0.158014E-01,  0.195661E-01)
PSI(11)=(  0.292112E-01, -0.356638E-01)
PSI(12)=(  0.268454E 00, -0.441272E 00)
PSI(13)=(  0.104399E-01, -0.214598E-01)
PSI(14)=(  0.465107E-02, -0.600440E-02)

COLUMN : 10
PSI( 1)=( -0.200598E 00,  0.173034E 00)
PSI( 2)=(  0.669812E 00, -0.514193E 00)
PSI( 3)=( -0.323246E 00,  0.290753E 00)
PSI( 4)=(  0.575987E 00, -0.535259E 00)
PSI( 5)=(  0.721892E 01, -0.503329E 01)
PSI( 6)=(  0.360140E 00, -0.207069E 00)
PSI( 7)=(  0.731716E-01, -0.772919E-01)
PSI( 8)=( -0.915337E-02, -0.118687E-01)
PSI( 9)=(  0.276834E-01,  0.411809E-01)
```

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PSI(10)=( -0.158014E-01, -0.195661E-01)
PSI(11)=(  0.292112E-01,  0.356638E-01)
PSI(12)=(  0.268454E 00,  0.441272E 00)
PSI(13)=(  0.104399E-01,  0.214598E-01)
PSI(14)=(  0.465107E-02,  0.600440E-02)

COLUMN : 11
PSI( 1)=(  0.354973E 00,  0.218050E 00)
PSI( 2)=(  0.144396E 01,  0.162515E 01)
PSI( 3)=(  0.110656E 00,  0.119865E 00)
PSI( 4)=(  0.631190E 01,  0.170633E 01)
PSI( 5)=( -0.100299E 01, -0.698289E 00)
PSI( 6)=( -0.113229E 01, -0.444719E 00)
PSI( 7)=( -0.118575E 00, -0.911123E-01)
PSI( 8)=(  0.146822E-01, -0.274882E-01)
PSI( 9)=(  0.116758E 00, -0.119541E 00)
PSI(10)=(  0.905418E-02, -0.105688E-01)
PSI(11)=(  0.966475E-01, -0.493214E 00)
PSI(12)=( -0.480462E-01,  0.806345E-01)
PSI(13)=( -0.278044E-01,  0.885478E-01)
PSI(14)=( -0.586110E-02,  0.863844E-02)

COLUMN : 12
PSI( 1)=(  0.354973E 00, -0.218050E 00)
PSI( 2)=(  0.144396E 01, -0.162515E 01)
PSI( 3)=(  0.110656E 00, -0.119865E 00)
PSI( 4)=(  0.631190E 01, -0.170633E 01)
PSI( 5)=( -0.100299E 01,  0.698289E 00)
PSI( 6)=( -0.113229E 01,  0.444719E 00)
PSI( 7)=( -0.118575E 00,  0.911123E-01)
PSI( 8)=(  0.146822E-01,  0.274882E-01)
PSI( 9)=(  0.116758E 00,  0.119541E 00)
PSI(10)=(  0.905418E-02,  0.105688E-01)
PSI(11)=(  0.966475E-01,  0.493214E 00)
PSI(12)=( -0.480462E-01, -0.806345E-01)
PSI(13)=( -0.278044E-01, -0.885478E-01)
PSI(14)=( -0.586110E-02, -0.863844E-02)

COLUMN : 13
PSI( 1)=(  0.396114E-01,  0.127848E 00)
PSI( 2)=( -0.272187E 00,  0.443397E 01)
PSI( 3)=( -0.263231E-01,  0.207957E 00)
PSI( 4)=(  0.126730E 01, -0.191346E 01)
PSI( 5)=( -0.110894E 00, -0.725329E 00)
PSI( 6)=( -0.224771E 00,  0.890015E-01)
PSI( 7)=(  0.161589E-02, -0.106766E 00)
PSI( 8)=(  0.951822E-02, -0.322938E-02)
PSI( 9)=(  0.326670E 00,  0.132179E-02)
PSI(10)=(  0.154362E-01,  0.808261E-03)
PSI(11)=( -0.145839E 00, -0.850341E-01)
PSI(12)=( -0.526950E-01,  0.111730E-01)
PSI(13)=(  0.742096E-02,  0.160202E-01)
PSI(14)=( -0.770327E-02,  0.208396E-03)

COLUMN : 14
PSI( 1)=(  0.396114E-01, -0.127848E 00)
PSI( 2)=( -0.272187E 00, -0.443397E 01)
PSI( 3)=( -0.263231E-01, -0.207957E 00)
PSI( 4)=(  0.126730E 01,  0.191346E 01)
PSI( 5)=( -0.110894E 00,  0.725329E 00)
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PSI( 6)=( -0.224771E 00, -0.890015E-01)
PSI( 7)=(  0.161589E-02,  0.106766E 00)
PSI( 8)=(  0.951822E-02,  0.322938E-02)
PSI( 9)=(  0.326670E 00, -0.132179E-02)
PSI(10)=(  0.154362E-01, -0.808261E-03)
PSI(11)=( -0.145839E 00,  0.850341E-01)
PSI(12)=( -0.526950E-01, -0.111730E-01)
PSI(13)=(  0.742096E-02, -0.160202E-01)
PSI(14)=( -0.770327E-02, -0.208396E-03)

EIGENVALUES - COMMON DENOMINATORS
   D( 1) = S-( -0.128259E 01,  0.148350E 03)
   D( 2) = S-( -0.128259E 01, -0.148350E 03)
   D( 3) = S-( -0.806641E 00,  0.950309E 02)
   D( 4) = S-( -0.806641E 00, -0.950309E 02)
   D( 5) = S-( -0.606255E 01,  0.450166E 02)
   D( 6) = S-( -0.606255E 01, -0.450166E 02)
   D( 7) = S-( -0.178746E 01,  0.432520E 02)
   D( 8) = S-( -0.178746E 01, -0.432520E 02)
   D( 9) = S-( -0.106128E 01,  0.170045E 02)
   D(10) = S-( -0.106128E 01, -0.170045E 02)
   D(11) = S-( -0.917436E 00,  0.129831E 02)
   D(12) = S-( -0.917436E 00, -0.129831E 02)
   D(13) = S-( -0.777634E 00,  0.135759E 02)
   D(14) = S-( -0.777634E 00, -0.135759E 02)

ACCELERATION TRANSFER FUNCTION : POIN - SEAT
   MULTIPLY BY OMEGA**2 FOR FREQUENCY RESPONSE

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 1) : RIPD
WMAT(11) =   0.128958E 01  WMAT(21) =  -0.979718E 00  WMAT(31) =   0.0
WMAT(12) =   0.0           WMAT(22) =   0.277778E-01  WMAT(32) =   0.0
WMAT(13) =  -0.277778E-01  WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0           WMAT(24) =   0.0           WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 2) : RJNT
WMAT(11) =  -0.999997E 00  WMAT(21) =   0.193796E-05  WMAT(31) =   0.0
WMAT(12) =   0.0           WMAT(22) =   0.112872E-06  WMAT(32) =   0.0
WMAT(13) =  -0.112872E-06  WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0           WMAT(24) =   0.0           WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 3) : FIPD
WMAT(11) =   0.128955E 01  WMAT(21) =   0.202819E-01  WMAT(31) =   0.0
WMAT(12) =   0.0           WMAT(22) =   0.277778E-01  WMAT(32) =   0.0
WMAT(13) =  -0.277778E-01  WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0           WMAT(24) =   0.0           WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 4) : VRTR
WMAT(11) =   0.333742E 00  WMAT(21) =  -0.992718E 00  WMAT(31) =   0.0
WMAT(12) =   0.0           WMAT(22) =   0.997503E-02  WMAT(32) =   0.0
WMAT(13) =  -0.997503E-02  WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0           WMAT(24) =   0.0           WMAT(34) =   0.0
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FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 5) : VFTR
WMAT(11) =  -0.333743E 00  WMAT(21) =  -0.727934E-02  WMAT(31) =   0.0
WMAT(12) =   0.0           WMAT(22) =  -0.997502E-02  WMAT(32) =   0.0
WMAT(13) =   0.997502E-02  WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0           WMAT(24) =   0.0           WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 6) : VPWH
WMAT(11) =   0.0           WMAT(21) =   0.190735E-05  WMAT(31) =   0.0
WMAT(12) =   0.0           WMAT(22) =   0.0           WMAT(32) =   0.0
WMAT(13) =   0.0           WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0           WMAT(24) =   0.0           WMAT(34) =   0.0

FIRST PARTIAL DERIVATIVES FOR POINT : SEAT WRT -

FGC( 7) : CJNT
WMAT(11) =   0.100000E 01  WMAT(21) =   0.345244E-04  WMAT(31) =   0.0
WMAT(12) =   0.0           WMAT(22) =  -0.232831E-09  WMAT(32) =   0.0
WMAT(13) =   0.232831E-09  WMAT(23) =   0.0           WMAT(33) =   0.0
WMAT(14) =   0.0           WMAT(24) =   0.0           WMAT(34) =   0.0

X-COMPONENT NUMERATORS(2*NFGC) :

BASE INPUT EXCITATION( 1) : APIN
NC( 1) = ( -0.125090E-11, -0.347726E-08)
NK( 1) = (  0.668006E-02, -0.880724E-01)
NM( 1) = ( -0.652364E-04,  0.347367E-03)
NC( 2) = ( -0.125090E-11,  0.347726E-08)
NK( 2) = (  0.668006E-02,  0.880724E-01)
NM( 2) = ( -0.652364E-04, -0.347367E-03)
NC( 3) = ( -0.262530E-13, -0.775173E-13)
NK( 3) = ( -0.499387E-03,  0.376920E-02)
NM( 3) = (  0.536830E-05,  0.646682E-04)
NC( 4) = ( -0.262530E-13,  0.775173E-13)
NK( 4) = ( -0.499387E-03, -0.376920E-02)
NM( 4) = (  0.536830E-05, -0.646682E-04)
NC( 5) = ( -0.107199E-09,  0.158620E-09)
NK( 5) = ( -0.158129E 01, -0.173173E 00)
NM( 5) = (  0.212045E-01,  0.100864E-01)
NC( 6) = ( -0.107199E-09, -0.158620E-09)
NK( 6) = ( -0.158129E 01,  0.173173E 00)
NM( 6) = (  0.212045E-01, -0.100864E-01)
NC( 7) = (  0.813801E-10, -0.732571E-09)
NK( 7) = (  0.154294E 01, -0.205341E 00)
NM( 7) = ( -0.204267E-01, -0.311904E-01)
NC( 8) = (  0.813801E-10,  0.732571E-09)
NK( 8) = (  0.154294E 01,  0.205341E 00)
NM( 8) = ( -0.204267E-01,  0.311904E-01)
NC( 9) = (  0.339887E-11, -0.353030E-10)
NK( 9) = (  0.334841E-01,  0.960698E 00)
NM( 9) = (  0.116681E-02,  0.182367E-01)
NC(10) = (  0.339887E-11,  0.353030E-10)
NK(10) = (  0.334841E-01, -0.960698E 00)
NM(10) = (  0.116681E-02, -0.182367E-01)
NC(11) = (  0.329863E-10, -0.540459E-10)
NK(11) = ( -0.388595E 00,  0.216033E 01)
NM(11) = ( -0.122258E-01,  0.906264E-01)
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NC(12) = (  0.329863E-10,  0.540459E-10)
NK(12) = ( -0.388595E 00, -0.216033E 01)
NM(12) = ( -0.122258E-01, -0.906264E-01)
NC(13) = ( -0.175928E-10, -0.642861E-10)
NK(13) = (  0.343059E 00, -0.725555E 00)
NM(13) = (  0.106799E-01, -0.512479E-01)
NC(14) = ( -0.175928E-10,  0.642861E-10)
NK(14) = (  0.343059E 00,  0.725555E 00)
NM(14) = (  0.106799E-01,  0.512479E-01)

BASE INPUT EXCITATION( 2) : RSMP
NC( 1) = (  0.400560E-13,  0.111330E-09)
NK( 1) = (  0.353649E-03, -0.110138E-01)
NM( 1) = (  0.311288E-05, -0.355323E-05)
NC( 2) = (  0.400560E-13, -0.111330E-09)
NK( 2) = (  0.353649E-03,  0.110138E-01)
NM( 2) = (  0.311288E-05,  0.355323E-05)
NC( 3) = (  0.840570E-15,  0.248185E-14)
NK( 3) = ( -0.275713E-04,  0.195009E-03)
NM( 3) = ( -0.842955E-06,  0.748846E-06)
NC( 4) = (  0.840570E-15, -0.248185E-14)
NK( 4) = ( -0.275713E-04, -0.195009E-03)
NM( 4) = ( -0.842955E-06, -0.748846E-06)
NC( 5) = (  0.343215E-11, -0.507846E-11)
NK( 5) = ( -0.873059E-01, -0.101600E-01)
NM( 5) = ( -0.222131E-03, -0.364943E-04)
NC( 6) = (  0.343215E-11,  0.507846E-11)
NK( 6) = ( -0.873059E-01,  0.101600E-01)
NM( 6) = ( -0.222131E-03,  0.364943E-04)
NC( 7) = ( -0.260552E-11,  0.234544E-10)
NK( 7) = (  0.842914E-01, -0.824047E-02)
NM( 7) = (  0.318705E-03, -0.138682E-03)
NC( 8) = ( -0.260552E-11, -0.234544E-10)
NK( 8) = (  0.842914E-01,  0.824047E-02)
NM( 8) = (  0.318705E-03,  0.138682E-03)
NC( 9) = ( -0.108820E-12,  0.113028E-11)
NK( 9) = (  0.971129E-03,  0.174705E-01)
NM( 9) = (  0.227374E-03, -0.987712E-03)
NC(10) = ( -0.108820E-12, -0.113028E-11)
NK(10) = (  0.971129E-03, -0.174705E-01)
NM(10) = (  0.227374E-03,  0.987712E-03)
NC(11) = ( -0.105611E-11,  0.173036E-11)
NK(11) = ( -0.102610E-01,  0.668975E-01)
NM(11) = ( -0.402130E-02,  0.258621E-01)
NC(12) = ( -0.105611E-11, -0.173036E-11)
NK(12) = ( -0.102610E-01, -0.668975E-01)
NM(12) = ( -0.402130E-02, -0.258621E-01)
NC(13) = (  0.563260E-12,  0.205822E-11)
NK(13) = (  0.956043E-02, -0.279835E-01)
NM(13) = (  0.368551E-02, -0.107407E-01)
NC(14) = (  0.563260E-12, -0.205822E-11)
NK(14) = (  0.956043E-02,  0.279835E-01)
NM(14) = (  0.368551E-02,  0.107407E-01)

BASE INPUT EXCITATION( 3) : FSMP
NC( 1) = (  0.121201E-12,  0.336892E-09)
NK( 1) = ( -0.136692E-03,  0.890617E-02)
NM( 1) = ( -0.263814E-05,  0.362205E-05)
NC( 2) = (  0.121201E-12, -0.336892E-09)
NK( 2) = ( -0.136692E-03, -0.890617E-02)
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NM( 2) = ( -0.263814E-05, -0.362205E-05)
NC( 3) = (  0.254354E-14,  0.750995E-14)
NK( 3) = (  0.998465E-05, -0.742424E-04)
NM( 3) = ( -0.167299E-06,  0.223626E-06)
NC( 4) = (  0.254354E-14, -0.750995E-14)
NK( 4) = (  0.998465E-05,  0.742424E-04)
NM( 4) = ( -0.167299E-06, -0.223626E-06)
NC( 5) = (  0.103859E-10, -0.153678E-10)
NK( 5) = (  0.328044E-01, -0.727214E-03)
NM( 5) = (  0.170755E-03, -0.855277E-04)
NC( 6) = (  0.103859E-10,  0.153678E-10)
NK( 6) = (  0.328044E-01,  0.727214E-03)
NM( 6) = (  0.170755E-03,  0.855277E-04)
NC( 7) = ( -0.788446E-11,  0.709746E-10)
NK( 7) = ( -0.313137E-01,  0.226829E-01)
NM( 7) = ( -0.313973E-03,  0.529003E-03)
NC( 8) = ( -0.788446E-11, -0.709746E-10)
NK( 8) = ( -0.313137E-01, -0.226829E-01)
NM( 8) = ( -0.313973E-03, -0.529003E-03)
NC( 9) = ( -0.329296E-12,  0.342031E-11)
NK( 9) = ( -0.605552E-03, -0.123397E-01)
NM( 9) = ( -0.106361E-03, -0.723726E-02)
NC(10) = ( -0.329296E-12, -0.342031E-11)
NK(10) = ( -0.605552E-03,  0.123397E-01)
NM(10) = ( -0.106361E-03,  0.723726E-02)
NC(11) = ( -0.319585E-11,  0.523619E-11)
NK(11) = (  0.511990E-02, -0.323143E-01)
NM(11) = (  0.123528E-02, -0.280147E-02)
NC(12) = ( -0.319585E-11, -0.523619E-11)
NK(12) = (  0.511990E-02,  0.323143E-01)
NM(12) = (  0.123528E-02,  0.280147E-02)
NC(13) = (  0.170446E-11,  0.622830E-11)
NK(13) = ( -0.481660E-02,  0.130670E-01)
NM(13) = ( -0.974407E-03, -0.182737E-02)
NC(14) = (  0.170446E-11, -0.622830E-11)
NK(14) = ( -0.481660E-02, -0.130670E-01)
NM(14) = ( -0.974407E-03,  0.182737E-02)
BASE( 1) - INPUT DISPL WVECT( 1) :      -0.000
BASE( 2) - INPUT DISPL WVECT( 4) :      -0.328
BASE( 3) - INPUT DISPL WVECT( 7) :       0.328

Y-COMPONENT NUMERATORS(2*NFGC) :

BASE INPUT EXCITATION( 1) : APIN
NC( 1) = (  0.362882E-11,  0.142247E-08)
NK( 1) = ( -0.265371E-02,  0.360344E-01)
NM( 1) = (  0.263753E-04, -0.142158E-03)
NC( 2) = (  0.362882E-11, -0.142247E-08)
NK( 2) = ( -0.265371E-02, -0.360344E-01)
NM( 2) = (  0.263753E-04,  0.142158E-03)
NC( 3) = (  0.335018E-11,  0.982135E-11)
NK( 3) = (  0.622751E-01, -0.478048E 00)
NM( 3) = ( -0.698528E-03, -0.819786E-02)
NC( 4) = (  0.335018E-11, -0.982135E-11)
NK( 4) = (  0.622751E-01,  0.478048E 00)
NM( 4) = ( -0.698528E-03,  0.819786E-02)
NC( 5) = (  0.495168E-11,  0.126684E-09)
NK( 5) = ( -0.911929E 00,  0.527358E 00)
NM( 5) = (  0.152718E-01, -0.292735E-02)
NC( 6) = (  0.495168E-11, -0.126684E-09)
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NK( 6) = ( -0.911929E 00, -0.527358E 00)
NM( 6) = (  0.152718E-01,  0.292735E-02)
NC( 7) = ( -0.139383E-10, -0.405196E-09)
NK( 7) = (  0.823475E 00, -0.234413E 00)
NM( 7) = ( -0.135972E-01, -0.153527E-01)
NC( 8) = ( -0.139383E-10,  0.405196E-09)
NK( 8) = (  0.823475E 00,  0.234413E 00)
NM( 8) = ( -0.135972E-01,  0.153527E-01)
NC( 9) = ( -0.325272E-11,  0.182103E-10)
NK( 9) = (  0.230233E-01, -0.500857E 00)
NM( 9) = (  0.160891E-03, -0.953000E-02)
NC(10) = ( -0.325272E-11, -0.182103E-10)
NK(10) = (  0.230233E-01,  0.500857E 00)
NM(10) = (  0.160891E-03,  0.953000E-02)
NC(11) = (  0.199585E-10, -0.774101E-10)
NK(11) = (  0.325373E 00,  0.275215E 01)
NM(11) = (  0.185723E-01,  0.113954E 00)
NC(12) = (  0.199585E-10,  0.774101E-10)
NK(12) = (  0.325373E 00, -0.275215E 01)
NM(12) = (  0.185723E-01, -0.113954E 00)
NC(13) = ( -0.120570E-10,  0.305669E-10)
NK(13) = ( -0.349804E 00,  0.184919E 00)
NM(13) = ( -0.193602E-01,  0.170663E-01)
NC(14) = ( -0.120570E-10, -0.305669E-10)
NK(14) = ( -0.349804E 00, -0.184919E 00)
NM(14) = ( -0.193602E-01, -0.170663E-01)

BASE INPUT EXCITATION( 2) : RSMP
NC( 1) = ( -0.116183E-12, -0.455425E-10)
NK( 1) = ( -0.134797E-03,  0.450581E-02)
NM( 1) = ( -0.127022E-05,  0.145633E-05)
NC( 2) = ( -0.116183E-12,  0.455425E-10)
NK( 2) = ( -0.134797E-03, -0.450581E-02)
NM( 2) = ( -0.127022E-05, -0.145633E-05)
NC( 3) = ( -0.107261E-12, -0.314446E-12)
NK( 3) = (  0.344186E-02, -0.247337E-01)
NM( 3) = (  0.106673E-03, -0.951798E-04)
NC( 4) = ( -0.107261E-12,  0.314446E-12)
NK( 4) = (  0.344186E-02,  0.247337E-01)
NM( 4) = (  0.106673E-03,  0.951798E-04)
NC( 5) = ( -0.158533E-12, -0.405599E-11)
NK( 5) = ( -0.505838E-01,  0.287968E-01)
NM( 5) = ( -0.132872E-03,  0.675855E-04)
NC( 6) = ( -0.158533E-12,  0.405599E-11)
NK( 6) = ( -0.505838E-01, -0.287968E-01)
NM( 6) = ( -0.132872E-03, -0.675855E-04)
NC( 7) = (  0.446253E-12,  0.129730E-10)
NK( 7) = (  0.452234E-01, -0.111855E-01)
NM( 7) = (  0.162442E-03, -0.100816E-03)
NC( 8) = (  0.446253E-12, -0.129730E-10)
NK( 8) = (  0.452234E-01,  0.111855E-01)
NM( 8) = (  0.162442E-03,  0.100816E-03)
NC( 9) = (  0.104141E-12, -0.583031E-12)
NK( 9) = (  0.230375E-03, -0.912341E-02)
NM( 9) = ( -0.159775E-03,  0.503921E-03)
NC(10) = (  0.104141E-12,  0.583031E-12)
NK(10) = (  0.230375E-03,  0.912341E-02)
NM(10) = ( -0.159775E-03, -0.503921E-03)
NC(11) = ( -0.639005E-12,  0.247840E-11)
NK(11) = (  0.122163E-01,  0.845720E-01)
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NM(11) = (  0.465692E-02,  0.327150E-01)
NC(12) = ( -0.639005E-12, -0.247840E-11)
NK(12) = (  0.122163E-01, -0.845720E-01)
NM(12) = (  0.465692E-02, -0.327150E-01)
NC(13) = (  0.386024E-12, -0.978646E-12)
NK(13) = ( -0.120428E-01,  0.821689E-02)
NM(13) = ( -0.462865E-02,  0.314912E-02)
NC(14) = (  0.386024E-12,  0.978646E-12)
NK(14) = ( -0.120428E-01, -0.821689E-02)
NM(14) = ( -0.462865E-02, -0.314912E-02)

BASE INPUT EXCITATION( 3) : FSMP
NC( 1) = ( -0.351573E-12, -0.137815E-09)
NK( 1) = (  0.479337E-04, -0.364343E-02)
NM( 1) = (  0.107595E-05, -0.148406E-05)
NC( 2) = ( -0.351573E-12,  0.137815E-09)
NK( 2) = (  0.479337E-04,  0.364343E-02)
NM( 2) = (  0.107595E-05,  0.148406E-05)
NC( 3) = ( -0.324580E-12, -0.951531E-12)
NK( 3) = ( -0.124531E-02,  0.941588E-02)
NM( 3) = (  0.211511E-04, -0.284005E-04)
NC( 4) = ( -0.324580E-12,  0.951531E-12)
NK( 4) = ( -0.124531E-02, -0.941588E-02)
NM( 4) = (  0.211511E-04,  0.284005E-04)
NC( 5) = ( -0.479732E-12, -0.122737E-10)
NK( 5) = (  0.172251E-01, -0.132460E-01)
NM( 5) = (  0.576216E-04, -0.112580E-03)
NC( 6) = ( -0.479732E-12,  0.122737E-10)
NK( 6) = (  0.172251E-01,  0.132460E-01)
NM( 6) = (  0.576216E-04,  0.112580E-03)
NC( 7) = (  0.135040E-11,  0.392570E-10)
NK( 7) = ( -0.152405E-01,  0.148351E-01)
NM( 7) = ( -0.128840E-03,  0.312885E-03)
NC( 8) = (  0.135040E-11, -0.392570E-10)
NK( 8) = ( -0.152405E-01, -0.148351E-01)
NM( 8) = ( -0.128840E-03, -0.312885E-03)
NC( 9) = (  0.315138E-12, -0.176429E-11)
NK( 9) = ( -0.204500E-03,  0.644062E-02)
NM( 9) = ( -0.249285E-03,  0.376698E-02)
NC(10) = (  0.315138E-12,  0.176429E-11)
NK(10) = ( -0.204500E-03, -0.644062E-02)
NM(10) = ( -0.249285E-03, -0.376698E-02)
NC(11) = ( -0.193367E-11,  0.749981E-11)
NK(11) = ( -0.570358E-02, -0.409119E-01)
NM(11) = (  0.461375E-03, -0.383797E-02)
NC(12) = ( -0.193367E-11, -0.749981E-11)
NK(12) = ( -0.570358E-02,  0.409119E-01)
NM(12) = (  0.461375E-03,  0.383797E-02)
NC(13) = (  0.116813E-11, -0.296145E-11)
NK(13) = (  0.576249E-02, -0.373280E-02)
NM(13) = ( -0.155558E-03,  0.100906E-02)
NC(14) = (  0.116813E-11,  0.296145E-11)
NK(14) = (  0.576249E-02,  0.373280E-02)
NM(14) = ( -0.155558E-03, -0.100906E-02)
BASE( 1) - INPUT DISPL WVECT( 2) :       0.000
BASE( 2) - INPUT DISPL WVECT( 5) :      -0.933
BASE( 3) - INPUT DISPL WVECT( 8) :      -0.067

Z-COMPONENT NUMERATORS(2*NFGC) :
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BASE INPUT EXCITATION( 1) : APIN
NC( 1) = (  0.0          ,  0.0          )
NK( 1) = (  0.0          ,  0.0          )
NM( 1) = (  0.0          ,  0.0          )
NC( 2) = (  0.0          ,  0.0          )
NK( 2) = (  0.0          ,  0.0          )
NM( 2) = (  0.0          ,  0.0          )
NC( 3) = (  0.0          ,  0.0          )
NK( 3) = (  0.0          ,  0.0          )
NM( 3) = (  0.0          ,  0.0          )
NC( 4) = (  0.0          ,  0.0          )
NK( 4) = (  0.0          ,  0.0          )
NM( 4) = (  0.0          ,  0.0          )
NC( 5) = (  0.0          ,  0.0          )
NK( 5) = (  0.0          ,  0.0          )
NM( 5) = (  0.0          ,  0.0          )
NC( 6) = (  0.0          ,  0.0          )
NK( 6) = (  0.0          ,  0.0          )
NM( 6) = (  0.0          ,  0.0          )
NC( 7) = (  0.0          ,  0.0          )
NK( 7) = (  0.0          ,  0.0          )
NM( 7) = (  0.0          ,  0.0          )
NC( 8) = (  0.0          ,  0.0          )
NK( 8) = (  0.0          ,  0.0          )
NM( 8) = (  0.0          ,  0.0          )
NC( 9) = (  0.0          ,  0.0          )
NK( 9) = (  0.0          ,  0.0          )
NM( 9) = (  0.0          ,  0.0          )
NC(10) = (  0.0          ,  0.0          )
NK(10) = (  0.0          ,  0.0          )
NM(10) = (  0.0          ,  0.0          )
NC(11) = (  0.0          ,  0.0          )
NK(11) = (  0.0          ,  0.0          )
NM(11) = (  0.0          ,  0.0          )
NC(12) = (  0.0          ,  0.0          )
NK(12) = (  0.0          ,  0.0          )
NM(12) = (  0.0          ,  0.0          )
NC(13) = (  0.0          ,  0.0          )
NK(13) = (  0.0          ,  0.0          )
NM(13) = (  0.0          ,  0.0          )
NC(14) = (  0.0          ,  0.0          )
NK(14) = (  0.0          ,  0.0          )
NM(14) = (  0.0          ,  0.0          )

BASE INPUT EXCITATION( 2) : RSMP
NC( 1) = (  0.0          ,  0.0          )
NK( 1) = (  0.0          ,  0.0          )
NM( 1) = (  0.0          ,  0.0          )
NC( 2) = (  0.0          ,  0.0          )
NK( 2) = (  0.0          ,  0.0          )
NM( 2) = (  0.0          ,  0.0          )
NC( 3) = (  0.0          ,  0.0          )
NK( 3) = (  0.0          ,  0.0          )
NM( 3) = (  0.0          ,  0.0          )
NC( 4) = (  0.0          ,  0.0          )
NK( 4) = (  0.0          ,  0.0          )
NM( 4) = (  0.0          ,  0.0          )
NC( 5) = (  0.0          ,  0.0          )
NK( 5) = (  0.0          ,  0.0          )
NM( 5) = (  0.0          ,  0.0          )
```

```
NC( 6) = (  0.0          ,  0.0          )
NK( 6) = (  0.0          ,  0.0          )
NM( 6) = (  0.0          ,  0.0          )
NC( 7) = (  0.0          ,  0.0          )
NK( 7) = (  0.0          ,  0.0          )
NM( 7) = (  0.0          ,  0.0          )
NC( 8) = (  0.0          ,  0.0          )
NK( 8) = (  0.0          ,  0.0          )
NM( 8) = (  0.0          ,  0.0          )
NC( 9) = (  0.0          ,  0.0          )
NK( 9) = (  0.0          ,  0.0          )
NM( 9) = (  0.0          ,  0.0          )
NC(10) = (  0.0          ,  0.0          )
NK(10) = (  0.0          ,  0.0          )
NM(10) = (  0.0          ,  0.0          )
NC(11) = (  0.0          ,  0.0          )
NK(11) = (  0.0          ,  0.0          )
NM(11) = (  0.0          ,  0.0          )
NC(12) = (  0.0          ,  0.0          )
NK(12) = (  0.0          ,  0.0          )
NM(12) = (  0.0          ,  0.0          )
NC(13) = (  0.0          ,  0.0          )
NK(13) = (  0.0          ,  0.0          )
NM(13) = (  0.0          ,  0.0          )
NC(14) = (  0.0          ,  0.0          )
NK(14) = (  0.0          ,  0.0          )
NM(14) = (  0.0          ,  0.0          )

BASE INPUT EXCITATION( 3) : FSMP
NC( 1) = (  0.0          ,  0.0          )
NK( 1) = (  0.0          ,  0.0          )
NM( 1) = (  0.0          ,  0.0          )
NC( 2) = (  0.0          ,  0.0          )
NK( 2) = (  0.0          ,  0.0          )
NM( 2) = (  0.0          ,  0.0          )
NC( 3) = (  0.0          ,  0.0          )
NK( 3) = (  0.0          ,  0.0          )
NM( 3) = (  0.0          ,  0.0          )
NC( 4) = (  0.0          ,  0.0          )
NK( 4) = (  0.0          ,  0.0          )
NM( 4) = (  0.0          ,  0.0          )
NC( 5) = (  0.0          ,  0.0          )
NK( 5) = (  0.0          ,  0.0          )
NM( 5) = (  0.0          ,  0.0          )
NC( 6) = (  0.0          ,  0.0          )
NK( 6) = (  0.0          ,  0.0          )
NM( 6) = (  0.0          ,  0.0          )
NC( 7) = (  0.0          ,  0.0          )
NK( 7) = (  0.0          ,  0.0          )
NM( 7) = (  0.0          ,  0.0          )
NC( 8) = (  0.0          ,  0.0          )
NK( 8) = (  0.0          ,  0.0          )
NM( 8) = (  0.0          ,  0.0          )
NC( 9) = (  0.0          ,  0.0          )
NK( 9) = (  0.0          ,  0.0          )
NM( 9) = (  0.0          ,  0.0          )
NC(10) = (  0.0          ,  0.0          )
NK(10) = (  0.0          ,  0.0          )
NM(10) = (  0.0          ,  0.0          )
NC(11) = (  0.0          ,  0.0          )
```

```
NK(11) = (  0.0         ,  0.0          )
NM(11) = (  0.0         ,  0.0          )
NC(12) = (  0.0         ,  0.0          )
NK(12) = (  0.0         ,  0.0          )
NM(12) = (  0.0         ,  0.0          )
NC(13) = (  0.0         ,  0.0          )
NK(13) = (  0.0         ,  0.0          )
NM(13) = (  0.0         ,  0.0          )
NC(14) = (  0.0         ,  0.0          )
NK(14) = (  0.0         ,  0.0          )
NM(14) = (  0.0         ,  0.0          )
BASE( 1) - INPUT DISPL WVECT( 3) :       0.0
BASE( 2) - INPUT DISPL WVECT( 6) :       0.0
BASE( 3) - INPUT DISPL WVECT( 9) :       0.0
```

GENERAL SUMMARY

1. Analytical computer simulation techniques have been proven to be valuable design analysis tools for modelling the dynamic behavior of off-road vehicles.
2. Sixty-six computer simulation models were developed for studying the dynamic behavior of unsuspended and suspended agricultural tractors with different cab locations, and with and without attached implements.
3. The transfer function analysis procedure which was incorporated into the generalized articulated mechanical systems simulation program IMP was proven to be valuable for evaluating the dynamic behavior of agricultural tractor-implement systems.
4. The development of a chassis suspension system for an agricultural tractor was found to be effective for improving operator ride comfort from terrain-induced vibrations.
5. The operator cab position on the tractor was found to affect the operator ride comfort.
6. The tractor-trailer systems were found to have the highest levels of operator ride vibration in comparison to the tractor alone and the tractor-plow systems.

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